PARAMETRIC ADAPTATION OF ASSOCIATIVE CONTROL

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ABSTRACT

Adaptation methods for solving the control problem of the complex object are considered by identification of object's parameters. The model is based on associative approach of using precedents in a fixed protocol. Algorithms of associative control and acceptance of associative decisions under the protocol of precedents are described and examples of its applications for solving the control problem of complex dynamic objects are resulted. To perform the control efficiently, it is necessary to correct the model's parameters constantly, that is, to have the associative control with parametric adaptation.

INTRODUCTION

A complicated controllable object differs from a simple one primarily by the lack of its formal description, that is, by the absence of its mathematical model. Therefore it is usual (see, for example Moore (1979)), for intensive efficient control of such object, first to identify it, i.e. to synthesize its mathematical model, which is a simplified image of a complex object. However, this conventional approach requires a preliminary definition of the object's model structure, which for complex objects is realized in the expert way, i.e. the model depends substantially on the individual preferences of an expert. The proposed associative approach eliminates, or, in any case, restricts the expert's role in the processes of behaviour modelling and control of complex objects.

Few authors have studied the associative algorithm (Rastrigin and Vulf, 1997; Grabis, 1998). They have discussed some issues related to associative algorithm models. The most complete description of the algorithm itself is by Rastrigin and Vulfs (1997).

Identification of the dependencies of unknown parameters obtained by observation is a longstanding problem (Moore (1979)). From among the basic approaches to its solution it is possible to distinguish a parametric and a non-parametric approach. The parametric approach was considered by Rastrigin and Vulf, (1997). The parametric approach supposes the choice of a class of models based on previous information about the object. The nonparametric approach accepts regression as a model and the probability characteristics are replaced by their nuclear estimations. This circumstance allows solving problems under conditions of scant previous information, which is rather attractive when practical problems are dealt with.

The basic problem of nonparametric algorithms is the underlying idea of a local approximation of the function's value of function at some point. The required value is searched as the weighted average of the nearest elements of the training sample. The nearest elements are the ones the values of whose arguments are in some neighbourhood of a considered point. As a consequence, the algorithm is not sensitive to a possible significant dispersal of the measured values of the function among the sample units that have appeared in a neighbourhood of the given point.

ASSOCIATIVE PROCESSING OF INFORMATION

The processing of information implies, in general case, its transformation from one form to another. We consider a case when initial data X convert to a result Y by means of operator F :

$$Y = F(X).$$

The challenge is to achieve that the result Y be a solution of the stated problem with initial data X. Obviously, in this case F is the solution algorithm for problem P.

Consider the problem of F synthesis. Traditionally, algorithm F is a product of the activities of an expert

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- specialist in solving problems of the class P. Formalization of this procedure is, as of yet, in embryo, and is now a prerogative of the artificial intellect. However, this problem can be solved formally if we possess precedents of its solution.

Let the i-th precedent be described by doublet

$$< X(i), Y(i) > ,$$

which fixes result of solving problem P with initial data $X(i)$. N such pairs are accumulated in the precedents protocol $\mathbf{Pr}$:

$$\Pr = < X(i), Y(i);\ i = 1, \dots, N > \qquad (1)$$

containing the information on algorithm F by means of which the protocol was obtained.

Now, it is natural to state the problem of reconstructing the algorithm F by the precedents protocol available (where this algorithm was used). Assume $\hat{F}$ to be an estimate of algorithm F according to protocol $\mathbf{Pr}$. It is obvious that this estimate is bound to possess the property of restoring the protocol by the initial data:

$$Y(i) = \hat{F}(X(i)) \quad (i = 1, \dots, N). \qquad (2)$$

However, the structure of estimate $\hat{F}$ is, naturally, considerably different from that of algorithm F. The true is that operator $\hat{F}$ (in the absence of information about F) should have an "imitative" (regarding the precedents) character, that is, it should simulate the behavior F reflected in protocol $\mathbf{Pr}$ (1). To do so, this operator should own, apart from property (2), a possibility to extrapolate protocol $\mathbf{Pr}$, that is, to extend the latter to some intermediate data (not fixed in the protocol). Evidently, such an extrapolation, in view of smallness of the protocol information, should possess a local rather than a global character (Rastrigin and Vulf, 1997). This means that operator F may be written as

$$F(X) = \alpha(X \mid \Pr(X)),$$

where α is a sufficiently universal algorithm (not connected directly with the problem P that has led to creation of protocol); $\mathbf{Pr}(X)$ is the part of protocol $\mathbf{Pr}$ that influences the solution of problem P with initial data X. Just here the idea of association arises, that is, the idea of closeness in a certain (depending on the problem) sense. This means that subprotocol $\mathbf{Pr}(X)$ is formed by the data from protocol $\mathbf{Pr}$ that are associated with X, i.e. are "close" to X.

Therefore algorithm ϕ of associative information processing, i.e. of solving a problem with initial data X by protocol $\mathbf{Pr}$, is defined by algorithm φ of determining subprotocol $\mathbf{Pr}(X) = \varphi(\mathbf{Pr})$ and by algorithm α of processing the data from this subprotocol:

$$\phi = < \varphi, \alpha > .$$

Solution of the problem (represented by protocol $\mathbf{Pr}$ and having initial data X) by the algorithm of the associative information processing takes the form:

$$Y = \phi\ (X, \Pr) .$$

To describe algorithms ϕ and α, it is convenient to resort to the terminology of the decision making procedure: we represent the initial data X as situation S, and solution Y as the made decision R.

PARAMETRIC ADAPTATION OF THE ASSOCIATIVE CONTROL

The process of control over a complex object is hampered by the fact that such an object is usually evolving in some uncertain way; these are its parameters that change randomly. To solve this task, it is common to involve the adaptation methods for identification of the object's changing parameters. This type of control we call the control with an adaptable model.

Consider the associative control of an object, which parameters are changing in an uncertain way. To perform the control efficiently, it is necessary to correct the model's parameters constantly, that is, to have the associative control with the parametric adaptation.

Let an object have a known operator F with unknown parameters A:

$$Y(i) = F(X(i), Y(i-j), U(i), A), \qquad (3)$$

where Y(i) is the state of the object at the i-th time moment (n x 1), X(i) is the environment state, or the external perturbation, U(i) is the control (n x 1), and A is vector of the changing parameters, $A = (a_1, \dots, a_q)$.

The information on the object's state Y(i) is fixed by a linear observer

$$Z(i) = C\,Y(i), \qquad (4)$$

where $\dim Z = l \le n$; C is the given matrix (l x n) of the observer. The control is formed by algorithm α in the limits of the given resource R:

$$U(i+1) = \alpha(Z(i), Y^*(i+1), A, R), \qquad (5)$$

where $Y^*(i+1)$ is the predetermined target state of the object, while the adaptability of the algorithm manifests itself as its control dependence on the unknown parameters A of the object.

We decompose the control algorithm α to the algorithm α' of estimation of parameters A and algorithm α'' of control with known (estimated) parameters A:

$$A^{(i)} = \alpha'(A^{(i-1)}, Z(i)) \tag{6}$$

$$U(i+1) = \alpha''(Z(i), Y^*(i+1), A^{(i)}, R). \tag{7}$$

Therefore, the system of adaptive control is described by the following expressions:

$$\begin{aligned} &Y(i) = F(X(i), Y(i-1), U(i), A), \\ &Z(i) = C\,Y(i), \\ &A^{(i)} = \alpha'(A^{(i-1)}, Z(i)), \\ &U(i+1) = \alpha''(Z(i), Y^*(i+1), A^{(i)}, R). \end{aligned} \tag{8}$$

Thus, the task of synthesizing this type of system gets reduced to the process of defining the algorithms α' and α''. We seek them in the class of associative algorithms.

We start with the algorithm α' of an adaptive estimation of unknown parameters A. The imitation model appears here in the following way :

$$\begin{aligned} &A^{(i)} = Nor(A^{(i-1)}, DA) \\ &Y(i) = F(X(i), Y(i-1), U(i), A(i)) \\ &Z(i) = C\,Y(i), \end{aligned} \tag{9}$$

where $A^{(i)}$ is a random normally-distributed value with an average $A^{(i-1)}$ and a diagonal disperse matrix DA (predetermined by a priori reasoning). In the model (9) there are supposed to be known: X(i), U(i), and $Y(i-1)$ - the preceding estimate of the object's state.

As a result of the imitation we get a precedents protocol of the form:

$$\Pr PA =< A_j(i), Z_j(i);\ j = 1, \dots, N >, \tag{10}$$

where j is the number of the model's run. This gives an opportunity to estimate parameters on the i-th step of control of associative algorithm φ :

$$A^{(i)} = \varphi\ (Z'(i), \Pr PA). \tag{11}$$

The estimate will be efficient, that is, precise, at $N \to \infty$ only if $l \ge q$. When $l < q$, the estimate will be approximate, but on average better, than $A^{(i-1)}$.

To evaluate the A efficiently, in the latter case $l < q$ we should resort to a deeper prehistory in (9):

$$A^{(i)} = Nor(A^{(i-1)}, DA)$$

$$\begin{aligned} &Z(v) = C\,F(X(v), Y(v-1), U(v), A(i)), \\ &v = i, i-1, \dots, i-k), \end{aligned} \tag{12}$$

where $k = \frac{q}{l}$ if $\frac{q}{l}$ is integer, and $k =]\frac{q}{l}[+1$ if $\frac{q}{l}$ is fractional (where $]w[$ is the integer part of w.)

Introduce the vector

$$\overline{Z}(i) =< Z(i), Z(i-1), \dots, Z(i-k) >.$$

Using (12) we obtain the protocol:

$$\Pr P\overline{A} =< A_j(i), \overline{Z}_j(i);\ j = 1, \dots, N >$$

and an efficient associative estimate of $A^{(i)}$:

$$A^{(i)} = \varphi\ (\overline{Z}'(i), \Pr P\overline{A}), \tag{13}$$

where the meanings of $\overline{Z}$ are the results of preceding real observations:

$$\overline{Z}'(i) =< Z'(i), Z'(i-1), \dots, Z'(i-k) >.$$

Example 1.

Assume that a linear object has $q = 2,\ l = n = 1$:

$$\begin{aligned} &y(i) = a_1 y(i-1) + a_2\, x(i) + a_3 u(i) \\ &z(i) = c\, y(i). \end{aligned}$$

We obtain $k = \frac{q}{l} = 2$ and the imitation model (12) in the following form:

$$\begin{aligned} &A^{(i)} = Nor(A^{(i-1)}, DA), \\ &z(i) = c\,(a_1 y(i-1) + a_2\, x(i) + a_3 u(i)), \\ &z(i-1) = c\,(a_1 y(i-2) + a_2\, x(i-1) + a_3 u(i-1)), \\ &z(i-2) = c\,(a_1 y(i-3) + a_2\, x(i-2) + a_3 u(i-2)). \end{aligned}$$

It is easy to notice that in this (linear) case it suffices to have on1y three precedents (there is a zero precedent) in the protocol in order to precisely define the parameters by associative algorithm (13) according to the measured values z'(i), z'(i-1), and z'(i-2) (it is assumed, naturally, that these parameters did not change in the interval from i-2 to i).

Now we will consider the control algorithm α'' (7), which transfers the object into the target state Y*(i+1). We should form an imitation model as

$$\begin{aligned} &U(i+1) = rnd(R) \\ &Y(i+1) = F(X(i+1), Y(i), U(i+1), A(i)), \end{aligned} \tag{14}$$

where *rnd*(R) is the generator of random controls limited by resource R,

$$Y(i) = F(X(i), Y(i-1), U(i), A^{(i)}).$$

As a result of simulation we get the protocol of precedents:

$$\Pr PU =< U_j(i+1), Y_j(i+1);\ j = 1, \dots, N >, \tag{15}$$

which gives an opportunity to apply associative algorithm φ to synthesis of control:

$$U(i+1) = \varphi\ (Y^*(i+1)\ /\ \Pr PU).$$

This estimate will be efficient (that is, precise) at $N \to \infty$ only if $n \geq m$. To achieve the goal at $n < m$ there will be required $k =]\frac{m}{n}[$ control steps, which corresponds to the conventional control schedule (when control dimensions are less than those of the object's state). That is why we determine the control of k steps in advance, by extrapolating the environment state, i.e., if we possess the estimates $X(i+2), \dots, X(i+k)$. In this case (n < m) we obtain an imitation model of the form:

$$\begin{aligned}
&U(i+1) = rnd(R)\\
&\dots\dots\dots\dots\dots\dots\dots\dots\\
&U(i+k) = rnd(R) \qquad (16)\\
&Y(i+1) = F(X(i+1), Y(i), U(i+1), A^{(i)}),\\
&Y(i+2) = F(X(i+2), Y(i+1), U(i+2), A^{(i)}),\\
&\dots\dots\dots\dots\dots\dots\dots\dots\\
&Y(i+k) = F(X(i+k), Y(i+k-1), U(i+k), A^{(i)}),
\end{aligned}$$

which gives a precedents protocol of the following form:

$$\Pr P\overline{U} = < \overline{U}_j(i+1), \overline{Y}_j(i+1);\ j = 1, \dots, N >,$$

where

$$\begin{aligned}
\overline{U} &= < U(i+1), \dots, U(i+k) >,\\
\overline{Y} &= < Y(i+1), \dots, Y(i+k) >. \qquad (17)
\end{aligned}$$

This makes it possible to use an associative algorithm α'' for synthesizing a future control:

$$\overline{U}(i+1) = \alpha''(Y^*(i+k)\ /\ \Pr P\overline{U}),$$

where Y*(i+k) is the end goal.

We exemplify the said as follows.

Example 2.

Assume a linear object to have n=1, m=2:

$$y(i+1) = a\,y(i) + b_1 u_1(i+1) + b_2\,u_2(i+1).$$

We obtain k=m/n=2 and imitation model (16) of the form:

$$\begin{aligned}
&u_1(i+1) = rnd(R),\\
&u_1(i+2) = rnd(R),\\
&u_2(i+1) = rnd(R), \qquad (18)\\
&u_2(i+2) = rnd(R),\\
&y(i+1) = a\,y(i) + b_1 u_1(i+1) + b_2\,u_2(i+1),\\
&y(i+2) = a\,y(i+1) + b_1 u_1(i+2) + b_2\,u_2(i+2).
\end{aligned}$$

In our case the vector of control is

$$U(i+1) = \alpha''(Z(i), Y^*(i+1), A^{(i)}, R),$$
$$\dim U = m.$$

Our target state is $Y^*(i+2)$ and from (18) we have protocol

$$\Pr PU = < U_j(i+1), Y_j(i+1);\ j = 1,2 >$$

i.e.

$$\begin{aligned}
\overline{U} &= < u(i+1),\ u(i+2) >,\\
\overline{Y} &= < y(i+1),\ y(i+2) >. \qquad (19)
\end{aligned}$$

Therefore the algorithm α'' can be rewritten in the form

$$\overline{U}(i+1) = \alpha''\ (Y^*(i+2)\ \Big|\ \Pr P\overline{U}).$$

As is seen, in this (linear) case it suffices to have only three precedents (zero precedent is absent here) in order to determine, with the help of associative algorithm (19), the control that transfers the object exactly to a state preset by target

$$Y^*(i+2) = < y^*(i+1), y^*(i+2) >.$$

Therefore, the adaptive associative control is reduced, on every control step, to a consequent application of the algorithm associative estimation of object's changing parameters as well as of the algorithm for associative synthesis of control. In the linear case both the tasks are solved precisely.

CONCLUSIONS

As we see, the algorithm of associative modelling consists in consecutive application of algorithms α' and α''. The algorithm α' allows to estimate the unknown parameters of considered object F while the algorithm and α'' solves a problem of recognition of object with already estimated parameters, according to algorithm α'. We see, that using protocols of precedents, this problem yields satisfactory results under the associative control of observable object. The received results are new in the field of associative control and continue results of Matvejevs An., Vulfs G. 2003.

In view of the said above it is worth touching the traditional procedure of decision making. Such a procedure requires *a priory* information on the object under decision and the goals being pursued. In the simplest case the information on the object should be represented by its model f, while the goals - as a criterion of efficiency Q of the decision R being made. This efficiency depends on the object's model f and decision R, i.e. $Q = Q(f, R)$. Such being a case, decision R is determined by maximization of criterion Q:

$$Q(f,R) \to \max_{R} \Rightarrow R^*,$$

where R is the solution optimal in criterion Q for the object with model f. (Possible deviations from this traditional pattern do not exclude the necessity to have model f of the object as well as criterion Q of decision R).

The major difficulty that should be overcome in the theory and practice of decision making is that of objectivity model f and criterion Q that are concealed behind the realities of a situation and preferences of the decision making person.

So the difficulties of realizing such a traditional procedure are proportional to the complexity of the object whose model f must be available.

However there is a wide class of objects for which such models do not exist at all (to this class belong majority of socially-economical, psychological, political, and the like objects). Still, humans are capable of making decisions for objects of the kind.

Another difficulty we face when realizing the traditional pattern of decision making is necessity of objectivity the goals in the form of criterion. In complicated situations when the expert's intuition is the determining factor, she/he is not capable of defining accurately (that is, formally) the efficiency criterion Q. In many cases the expert would not be apt to doing so (even if he could). For example, in case there are non-prestigious (egoistic, hedonistic, etc.) goals.

The said requires, in complicated cases of the decision making process, first, to abandon the decision optimality (in favor of rationality, acceptability, rightness, and so on), that is, the explicit formulation of criterion Q, and, second, to give up the necessity to have model f of the decision R object.

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