

# APPLICATIONS OF STOCHASTIC OPTIMISATION IN NON-LINEAR AND DISCONTINUOUS DESIGN SPACES

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## ABSTRACT

The application of a stochastic optimiser to two problems in engineering design is presented. The benefits of using such an optimiser in conjunction with a calculus based method are discussed, and its ability to succeed in non-linear and discontinuous design spaces is shown in light of two aerospace design optimisation problems: the design of quiet and efficient propellers and the design of a manoeuvre controller for a satellite structure.

## INTRODUCTION

Successful engineering design processes of complex systems require that numerous design variables and constraints be taken into account across multiple disciplines. The systematic modification of design parameters relying on judgement in a manual design process is often ineffective, and the benefits of computer-aided design optimisation can reduce design time, improve design through improved methodology, solve complex interactions and ultimately reduce the cost of design.

Multidisciplinary design problems require an optimiser capable of efficiently handling local minima in non-linear and discontinuous design spaces of high dimensionality. Traditionally optimisers rely on a good starting point to obtain a solution or even to converge, thus an additional requirement is that an acceptable solution be found without a good initial starting point to the optimisation. Furthermore, the optimiser must be robust, as the computational expense of objective function calculation makes convergence on non-optimal solutions unacceptably.

The application of a two-stage optimisation process that meets these requirements is discussed here. The first stage uses the unconstrained stochastic optimisation method Simulated Annealing (SA) (Ingber, 1989) to obtain a good solution. Once the region of an acceptable minimum has been found, a constrained non-linear programming method is used to converge on the final solution. Two very different aerospace design problems to which this optimiser was successfully applied are discussed, these being the design of high performance propellers subject to noise constraints, and the design of a manoeuvre controller for a satellite.

## OPTIMISATION METHODOLOGY

SA amounts to a stochastic search over a cost landscape, being directed by noise that is gradually reduced during convergence. The origins of SA lie in the statistical mechanics of condensed matter physics. A cooling liquid will solidify into an optimal crystalline structure when the lowest atomic energy

state is attained. The optimal state is achieved through specific cooling schedules - the progress of temperatures known as annealing. The reader is referred to Ingber (Ingber, 1989) and Drack, Zadeh et al (Drack, Zadeh, Wharington, Herszberg and Wood, 1999) for details of the implementation used in the applications discussed here.

There are several reasons for not adopting a more traditional, calculus-based approach, to the design optimisation problem. Firstly, they rely on gradient information that is supplied either analytically or calculated numerically. For engineering applications, analytical gradients are rarely available, thus numerical methods must be employed introducing the possibility of numerical error. Furthermore, the cost function must be continuous to the first or second derivative, depending on the method used. These optimisers are not suitable for design variables of integer value, as gradients become infinite. SA does not calculate or estimate gradients, thus it is free from these restrictions.

Another problem that afflicts calculus-based methods is the need for a good initial guess to the solution (Arora, 1989), in which convergence is often not possible or reliable if the starting point is poor. This becomes a serious issue in design spaces of higher dimensionality, where an unsuitable initial guess made by the designer may not seem unrealistic, or intuition fails due to the large number of variables. In addition, an unsuitable starting point combined with a design space of high dimensionality compounds the problem by lessening the possibility of convergence, slowing the optimisation process considerably.

The tendency of gradient-based optimisers to become trapped in local minima is well known (Gage, 1994). One of the most attractive features of SA is that it is less susceptible to becoming trapped in local minima, since escape from these minima is still possible at non-zero temperature, thus affording great robustness to the optimiser in finding a good solution. Nevertheless, it must be said that in non-linear programming problems SA does not always result in a global minimum (Van Laarhoven, 1987), something that is offset by the fact that for most practical engineering applications, a global minimum is not required and a near global solution is sufficient.

SA is well suited to problems of high dimensionality with performance improvements over calculus-based methods becoming more pronounced as the dimensionality increases. The stochastic nature of the algorithm ensures it is suitable for discontinuous design spaces. For this reason, it also does not require a good initial guess for successful convergence. A

variant of SA known as Adaptive Simulated Annealing (ASA) has been shown to be very efficient in terms of computational effort (Ingber, 1992), and has the feature of specifying a 'bad data' flag for unrealistic designs. This results in the optimiser excluding these data points from any future searches.

The main disadvantage of SA is its poor convergence in shallow valleys, which may well occur in the vicinity of optima. This property is a consequence of the algorithm's stochastic sampling. In these regions, calculus-based optimisation is superior in converging to the exact value of the optimum. Thus, best performance is achieved by obtaining a good solution to the problem using SA, which is halted when reannealing produces negligible changes in the objective. The solution is then used to seed a calculus-based method to obtain a refined solution quickly.

The second optimisation stage is a constrained non-linear programming method. The Kuhn-Tucker (KT) conditions are solved for using Sequential Quadratic Programming (SQP) (Fletcher, 1987). A constrained optimiser was chosen, as these are considered more efficient when compared to unconstrained problems based on the KT equations. The primary advantage of using Newton methods lies in their super linear convergence, resulting in very rapid convergence in the region of the minimum. Rather than calculating the Hessian directly, a quasi-Newton method known as BFGS was chosen, as it is reliable across a broad range of problems (NEOS, 1998). This optimisation stage comes into play at the very end of the design process, and forms a small part of the total computation time.

The correct combination of objectives and constraints is the key to optimiser convergence; the more effective their application, the greater the speed at which the optimiser converges. This need is amplified by the computational cost of calculation of the objective function, that is, the constraints must be applied in such a way as to minimise calculation time on unrealistic or unsuitable solutions. This process will be described separately for each application discussed below, as it constitutes much of the art in design optimisation and is unique to each design problem.

## **THE DESIGN OF QUIET AND EFFICIENT PROPELLERS**

### *The Propeller Design Problem*

Aircraft noise is of concern to the community, and propellers are the predominant source of general aviation aircraft noise. The development of a quiet and efficient propeller poses several design difficulties. The first is that these requirements are often in conflict, such that, for example, a reduction in noise level is almost always accompanied by a reduction of propeller efficiency, and vice versa. The most effective noise reduction comes from a reduction in the strength of sources near the blade tips, which equates to reducing the blade helical tip Mach number (using RPM or diameter reduction for example), or by shifting the blade loading inboard. All of these methods reduce overall efficiency. In addition, the propeller must perform well over a range of operating conditions, from static operation through to maximum speed flight. There are many design variables to consider, and in addition, the objective function requires a considerable amount

of processing time due to numerous noise, performance and structural calculations that must be evaluated at several design conditions. The propeller design code SPONOP (Drack and Wood, 1999; Drack, 2000) was written to implement the aforementioned optimisation methodology for the purpose of designing propellers subject to noise constraints.

### *Optimisation Variables and Designer Inputs*

The optimisation variables are a combination of geometrical and operational parameters, including parameterised distributions of thickness, twist, sweep and chord, and radius and RPM. The designer specifies the propeller operation; be it fixed pitch or constant speed, engine performance curves, aircraft performance curves and material properties.

### *Design Point Calculations*

At the climb, maximum speed, cruise, static and takeoff design points the propeller RPM (fixed pitch case) or the blade pitch angle (constant speed case) is solved for by calculating propeller performance over several velocities and RPMs (blade pitch angles). Finally, the propeller's structure is tested at each operating point. The noise calculation portion of the design program carries out a fly-over noise certification test as per ICAO Annex 16 (ICAO, 1993). In this test the observer is positioned at a certain distance from the takeoff point, and the noise of the aircraft is calculated for several points as it flies overhead. The noise levels are A-weighted and the maximum level is then compared to the aircraft weight based maximum allowable noise level specified by regulation.

### *Analytical Methods Used*

The propeller's geometry is mapped to a two dimensional plane using an inverse Joukowski transformation for greater accuracy in performance and noise calculations. Aerodynamics are represented by neural networks that were trained to produce lift and drag coefficients on input of section type and angle of attack. The neural networks were trained with aerodynamic data consisting of lift coefficient obtained from the panel method and drag coefficient obtained from the integral displacement, momentum and energy equations for boundary layer thickness (Eppler, 1990). Propeller performance is calculated using blade element/vortex theory, according to Lock's method, with an iteration scheme developed by Larrabee (Larrabee, 1979).

The discrete tone noise of the propeller is calculated by the method of Farassat (Farassat, 1980), using the subsonic formulation of his equations. The noise at the observer due to blade loading and thickness sources is produced in the time domain, and spectral analysis is then carried out on the combined pressure history of the propeller resulting in sound pressure levels. The flyover noise of the propeller is calculated by applying the above theory at many points during a flyover. The sound emitted is corrected for spherical spreading and atmospheric and ground effects according to ANOPP (Zorumski, 1986).

Structural analysis is carried out for centrifugal force stress, structural stress, bending stress due to torque load, bending stress due to thrust load and shear stress.

TABLE I

COMPARISON BETWEEN PREDICTED PERFORMANCE OF SPONOP AND CURRENTLY INSTALLED PROPELLERS

Aircraft	Prop	$ROC_{cl}$ %	$LA_{max}$ $\Delta dB(A)$	$x_{to}$ %	$V_{cl}$ %	$V_{cr}$ %	$V_{max}$ %
GA200	30	+6.3	-5.1	-3.3	+4.5	-2.1	-2.3
GA200C	31	+6.7	-8.3	-5.7	0	+2.2	-1.4
GA8	34	+8.9	-5.3	-5.9	-2.7	-0.4	-1.2
	36	+11.3	-6.2	-9.2	0	+2.5	+1
	16	+23.3	-8.3	-11	0	-13.5	-2.8
	16a	+23.3	-13.9	-11	0	-13.5	-2.8

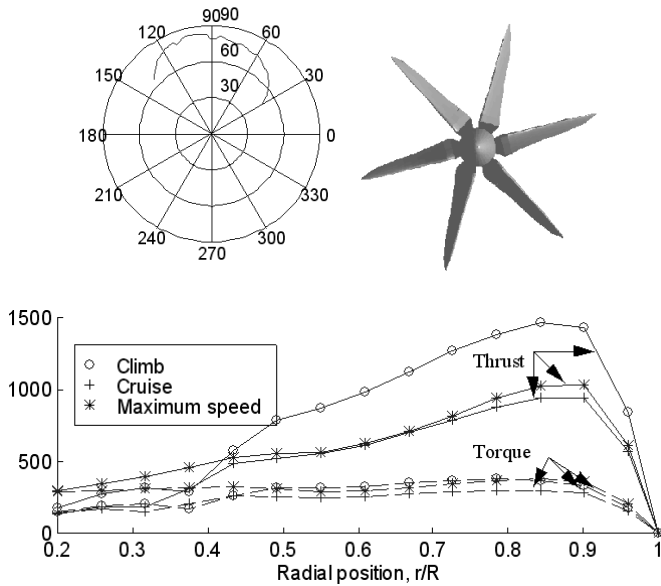


Fig. 1. Flyover Imission (dBA), Thrust (N) and Torque (Nm) for Propeller Configuration 16

### The Cost Function and Constraints

The performance indices for the optimisation process consist of takeoff distance, rate of climb, cruise speed and maximum speed. These indices are ranked by the user and combined into the cost function such that their relative importance affects the change in cost appropriately using a weighted sum approach and the softmax function (Yuille and Geiger, 1995). Being an unconstrained optimisation method, SA requires the constraints to be added to the cost function. Primary constraints are the allowable noise level during a fly-over noise test and structural soundness in all operating conditions.

The maximum allowable noise level is also treated as a constraint. Bound programming (Ignizio, 1976) is used for the noise constraint and both goal and bound programming are available for the performance indices to ensure designs meet minimum performance requirements. A modified SUMT (Fiacco and McCormick, 1968) approach is used to add constraints to the cost function.

Additionally, in ASA certain states can also be labelled as invalid, preventing the optimiser from accepting those generated states, thereby leading the search in another direction. Constraints applied to the problem in this way include incapability of flight in any operating condition due to lack or excess of power, structurally unsound blades and a failed fly-over noise test.

The second stage BFGS optimiser is a constrained optimisation method. Additional constraints are required since the parametric functions used to represent propeller geometry in the first stage are no longer used, with the blade now being discretised, in order to provide the second stage with sufficient freedom to refine the blade. These constraints require the propeller to have a reasonable geometry and are handled implicitly by the optimiser.

### Optimiser Performance and Results

Experiments were performed with SPONOP in order to explore a variety of optimisation and propeller parameter settings on the designs produced, including blade number, section type, propeller type, tip shape, engine type and the ranking of performance indices. The effects of goal and bound programming on performance indices versus the use of a weighted sum of these, as well as the use of weighting factors to favour propellers better suited to climbing and cruising were established. The weighting of the noise constraint was also examined.

The use of the softmax function alone to produce a weighted sum of performance indices was found to result in designs that met the noise constraint but were of relatively low performance. However, the strategy of first implementing bound programming, until desired performance and noise were achieved, followed by the use of the softmax function to further optimise the design, was found to be very successful in achieving improved performance whilst meeting or improving on the noise constraint. A high noise weighting was found to have an adverse dominating effect on the optimisation process, forcing the optimiser to quickly meet the noise constraint at the expense of the correlated performance indicators, which, for example, in the case of fixed pitch propellers, was through the reduction of radius.

Statistical analysis of the many valid designs produced by SPONOP gave insight into its optimisation process. Notably, principal component analysis established that for constant speed propellers, designing blades with the position of maximum chord relatively far inboard with thin sections was the optimiser's primary means of meeting performance and noise requirements. Linear regression identified several known or intuitive relationships between geometry, operation, performance and noise. These include the large influence of helical Mach number on noise, the influence of diameter on thrust, and the effect of increased blade number on reducing noise. These analyses provided confidence in the ability of SPONOP to produce realistic and effective designs.

Several sample propellers are shown in Table I to demonstrate the effectiveness of the designs produced by SPONOP. Changes in rate of climb (ROC), maximum flyover noise level ( $LA_{max}$ ), takeoff distance ( $x_{to}$ ), climb speed ( $V_{cl}$ ), cruise speed ( $V_{cr}$ ) and maximum speed ( $V_{max}$ ) are compared to the reference propellers used on two aircraft produced by Gippsland Aeronautics Ptd Ltd, one an agricultural aircraft (GA200) and another a utility transport aircraft (GA8). All propellers exhibit significant improvements in rate of climb,

takeoff distance and noise. Cruise and maximum speeds are similar to those of the reference propellers, with the exception of the six bladed propeller (16), which has a significantly reduced cruise speed (see Figure 1). Uneven azimuthal blade spacing of the six bladed propeller provides by far the greatest reduction in noise level (16a). The use of ARAD (Eppler, 1990) section types is found to encourage reduced blade thickness, and consequently a significantly reduced thickness noise component.

## MANOEUVRE CONTROL OF A SATELLITE STRUCTURE

### Introduction

Weight is of primary concern in spacecraft design, affecting all performance indicators, with the extensive use of composite materials being testimony to this. Recent satellites use progressively higher transmission power in order to reduce ground segment size and costs. However, increased transmission power also brings about higher power consumption, which in the case of satellites, translates to larger solar panels.

The need for weight-optimised aerospace structures coupled with ever-increasing solar panel sizes results in flexible structures with highly non-linear behaviour. The structures are usually so flexible that they cannot support their own weight at  $1g$  on the surface of the earth. These structures also exhibit very low damping, especially in the absence of aerodynamic damping.

Furthermore, the requirement to minimise the volume occupied by a satellite at launch leads to the use of mechanisms to unfold the vehicle's solar panels and other appendages from their stowed position after launch. The jointed structures required to do this usually exhibit some degree of backlash and nonlinear damping within the joints, which can be significant for a multi-jointed structure. The combination of a very flexible structure, precise pointing demanded by communications and other payloads, and the difficulty of reproducing the space environment to verify system behaviour using conventional test techniques, has made the design of control algorithms and the investigation of system behaviour for attitude control of flexible spacecraft a challenging area.

Although conventional control theory is reasonably well-established and mature, it is mainly limited to rather simple applications. Control of lightweight space structures poses a challenge that limits the usefulness of conventional control techniques. Optimisation of such controllers is a formidable design problem requiring an optimiser capable of efficiently handling local minima and numerous design variables.

### Problem Definition

A structure was chosen to represent a class of aerospace structures that has traditionally been one of the most obscure and difficult problems to analyse and control. The configuration modelled is similar to an Optus Class B satellite. The model has three solar panels on each side of a central hub (see Figure 2).

The development of the equations of motion is based on analytically derived systems, which have been separately validated in the literature by Riseborough (Riseborough, 1993) and Scott (Scott, 1999). Non-linearities such as backlash and hinge stiffness are also considered in the equations of motion. The

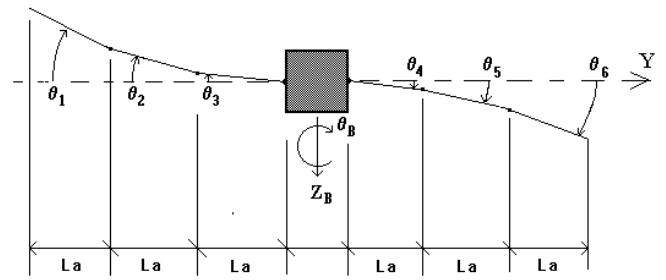


Fig. 2. Schematic of The Model Used For Simulation

derivation is explained in detail by Riseborough (Riseborough, 1993) and Zadeh (Zadeh and Wood, 1999).

The problem is defined as the rotation of the central hub about the axis passing through the hub and perpendicular to the page in Figure 2. The rotation should be carried out in the shortest possible time with minimal vibration induced in the solar panels. If possible, over-shoot should be avoided. The satellite model used by Riseborough (Riseborough, 1993) and Scott (Scott, 1999) can exert maximum torque of  $1 Nm$ , which is the limit that is used in this study as well.

A nominal fuzzy logic controller was designed and implemented for this structure (Zadeh, 2004). The performance of this nominal controller was compared to that of two other controllers, namely  $\mathcal{H}_\infty$  and  $\ell_1$  (Zadeh and Wood, 2000). The current research indicates that when a mathematical model of a system is not known in detail, fuzzy logic controllers can still yield a good design. However, one needs to have physical insight into the system in order to design a reasonable fuzzy logic controller. Furthermore, the nominal fuzzy controller was shown to yield better time domain performance than the  $\mathcal{H}_\infty$  controller (Zadeh, Scott and Wood, 1997).

The nominal controller was used as the basis for designing an optimised controller. The design process and the study of robustness of the fuzzy controller are explained below.

### Optimisation of Fuzzy Logic Controllers (FLC)

Numerical optimisation has been applied to the optimisation of FLCs as early as 1992. PD controllers (Hwang, Tao, Thompson and Paz, 1993), Kalman filters (Ramaswamy, Riese, Edwards and Lee, 1993), Genetic Algorithms (GA) (Krishnakumar, Gonsalves, Satyadas and Zacharias, 1995), reinforcement learning (Kang and Bien, 2000), and Neural Networks (Li, 1994) have been used in the optimisation of FLCs. The use of GAs is still be an active area of research in this field. Surprisingly the use of simulated annealing in FLC optimisation seems to be limited to very few researchers (Akbar and Parra-Loera, 1995).

As discussed earlier, one of the most attractive features of SA is that it is less susceptible to becoming trapped in local minima in the manner of calculus based optimisation methods. Based on this and other favourable characteristics described in (Drack, 2000), SA techniques in general, and ASA in particular, seem to be good candidates for optimisation of FLCs in aerospace domain.

### Application of Cost and Constraints

The Matlab Simulink environment was used to construct a simulation model. Saturation of the inputs and the physical

limitation of maximum available control authority were also implemented.

The satellite model was simulated for 30 seconds. A Runge-Kutta 5<sup>th</sup> order integration with varying step size was used. Minimum and maximum step sizes were set to 0.0001 and 1 respectively. At each interval, rotational position and speed, as well as vibration in each of the panels were calculated.

Excessive movement of the solar panels is known to accelerate fatigue and reduce aiming accuracy of the satellite. Therefore, one of the design goals was to reduce long-term vibration in the panels. In order to achieve this, the amplitude of vibration in the panels were summed in the cost function, and the resulting scalar was multiplied by the simulation time. This reduced the importance of vibration amplitude at the start of the manoeuvre, but heavily penalised long-term panel vibrations.

Following a similar procedure, long-term aiming accuracy was also included in the cost function.

Some of the constraints and goals of the optimisation that were added to the cost function were the available actuator authority, and the desired settling time and overshoot. Bound constraints were implemented for the actuator commands. Because the  $\ell_1$  controller exhibited very good overshoot and settling time characteristics, the performance of the  $\ell_1$  controller was set as the goal for overshoot and settling time in the cost function.

The distribution of the controller's membership functions were optimised using the cost value described above.

### Robustness

Fuel is usually a major part of most satellite payloads. Use of fuel throughout the life of a satellite can have a profound effect on the dynamics of the structure. Therefore, it is desirable that any controller used for this application have satisfactory performance in the presence of plant variations or uncertainties.

In order to study the performance of the optimised fuzzy controller under various plant configurations, a software package for the simulation of articulated rigid body systems, known as Odessa (Wharington, 2003), was used.

Odessa is based on ODE (Open Dynamics Engine), an open-source package developed primarily for game development. The formulation is physics-based, though ODE makes various approximations in order to produce faster simulations with somewhat greater robustness. Some of these approximations, in particular, the use of a 'friction pyramid' instead of a proper friction cone to model static friction, and false restitution in inter-penetration of joints, do not impact on the use of Odessa in the application domain considered here.

The Odessa model was used as basis for deriving a number of other simulation models that have physical properties that deviate from those of the ideal model. These were then used to check the off-design performance of the optimised fuzzy logic controller. These experiments are summarised in Table II, and the findings are discussed below.

### Results

Experiment No. 2, having the same characteristics as the model used for simulation, was chosen as the benchmark.

The behaviour of each Odessa model was observed for 30 seconds following commencement of the manoeuvre. Various parameters of the system were recorded.

TABLE II  
DESCRIPTION OF PLANT VARIATION EXPERIMENTS

Experiment No.	Experiment description
1	Free-play of 11.5 degrees
2	Free-play of 5.7 degrees
3	Free-play of 2.3 degrees
4	Centre panel has 200% inertia (0.0449 kg.m <sup>2</sup> )
5	Centre panel has 50% inertia (0.011225 kg.m <sup>2</sup> )
6	Panel 3 has 133% inertia (0.03 kg.m <sup>2</sup> )
7	Panel 3 has 67% inertia (0.015 kg.m <sup>2</sup> )
8	Panel 1 size is 122% size
9	Panel 1 size is 78% size
10	X velocity of 1 m/s
11	X velocity of 10 m/s
12	Friction coefficient of 0.002 N.m/s
13	Friction coefficient of 0.00005 N.m/s
14	Undergoing $5\pi/8$ radians
15	Undergoing $3\pi/8$ radians
16	Y velocity of 1 m/s
17	Y velocity of 10 m/s
18	Nominal fuzzy logic controller

Seven plots were created for each of the experiments. They were plots of angular position, angular velocity, and angular acceleration of the central hub, actuator torque, and angle of rotation, angular velocity, and angular acceleration of the outer solar panel.

Even though very detailed, this resulted in a large number of plots, making comparison difficult. For the sake of clarity, the following scalar values were calculated for each of the experiments (based on the plots described above).

- integral of angular acceleration of the central hub over 30 seconds
- integral of actuator torque over 30 seconds
- total exerted power (integral of torque times angular speed) over 30 seconds
- integral of position error of the central hub over 30 seconds
- integral of angular acceleration of outer solar panel over 30 seconds

These scalar values were then used to compare performance of the controllers in various experiments with that of the benchmark model.

One of the major drawbacks of some optimal controllers, such as  $\ell_1$ , is that they perform very poorly to input commands other than the design input. Considering that the tested controller was optimised in an environment of no parameter variations, that is, without any explicit regard for robustness, it is remarkable that it was able to satisfactorily perform the requested manoeuvres in all of the experiments. This robustness could be attributed to the underlying fuzzy logic control architecture.

While this work does not provide an analytical proof of stability or robustness, it demonstrates application of the developed optimisation methodology in a real-life example where there exists large degree of plant uncertainties, and

where the dynamics of the system can change under different operating conditions.

## CONCLUSION

From the applications presented we see that the combination of ASA and BFGS results in a very robust optimiser that is capable of finding the optimum in high dimensional design spaces with many minima, even without a good initial guess to the problem. However, the judicious application of constraints to the cost function as well as the development of strategies to effectively combine performance indices is essential in producing a successful optimiser.

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