INVESTIGATION ON SHANNON-KOTELNIK THEOREM IMPACT ON SOMA ALGORITHM PERFORMANCE

Zuzana Oplatková
Ivan Zelinka
Institute of Process Control and Applied Informatics
Faculty of Technology
Tomas Bata University in Zlin
Mostni 5139 Zlin, Czech Republic
E-mail: {oplatkova,zelinka}@ft.utb.cz

KEYWORDS

Self-Organizing Migrating Algorithm (SOMA), sampling, Shannon-Kotelnik theorem

ABSTRACT:

This paper deals with evolutionary algorithm SOMA (SelfOrganizing Migrating Algorithm) and studies the performance impact from Step size point of view based on information theory. For this preliminary study a simple unimodal (with one extreme) function was chosen. The article describes technique of analysis and results are well-arranged shown in graphs and charts.

INTRODUCTION:

Computational performance of algorithms is mostly influenced by sampling. Too dense sampling will task Equipment with a long computational time. On the other hand, thin sampling can disturb the image of original signal. This contribution deals with an analysis of a method of sampling during run of one newer evolutionary algorithm - SelfOrganizing Migrating Algorithm (SOMA) (Zelinka 2000, 2001, 2002, 2004).
The first part describes SOMA itself, then Shannon – Kotelnik sampling theorem and its application on a simple unimodal (with one extreme) will follow. Next section will focus on an analysis of sampling of SOMA on this function. As conclusion a possible improvement of SOMA will be pointed up.

MOTIVATION:

SOMA is an algorithm which uses sampling during its run as is described in next section. Because of its properties we were interested if Step (parameter) of SOMA is set up in suitable way. In other words, if sampling of SOMA comply with known Shannon – Kotelnik theorem (Bracewell R. N., 1999).
Till this time this parameter was set up heuristically on the previous experience with performed simulations (Zelinka 2000, 2001, 2002, 2004).

SOMA

SelfOrganizing Migrating Algorithm (SOMA) is a one newer evolutionary algorithm based on a conception of cooperative – competitive strategy. The construction of new population of individuals is not based on evolution principle (two parents produce offspring) but on the behaviour of social group, e.g. a herd of animals looking for food. During one generation, in the case of SOMA this is called 'Migration loop', only the position of individuals in the search space is changed. From the geometrical point of view the run of SOMA is possible to imagine as movements of individuals on the surface of the cost function.
In every migration loop the best individual is chosen, i.e. individual with the minimum cost value, which is called Leader. An active individual from the population moves in the direction to Leader in the search space. At the end of the movement the position of the individual with minimum cost value is chosen. If the cost value of the new position is better than the cost value of an individual from the old population, the new one appears in new population. Otherwise the old one rests there. The movement is described by Eq. (1).

$$x_{i,j}^{MK+1} = x_{i,j,START}^{MK} + (x_{L,j}^{MK} - x_{i,j,START}^{MK}) * t * PRTVector_j \quad (1)$$

where

$x_{i,j}^{MIL+1}$ - value of i–individual's j–parameter, in step t in next migration loop ML + 1

$x_{i,j,START}^{ML}$ - value of i–individual's j-parameter, Start position in actual migration loop

$x_{L,j}^{ML}$ - value of Leader's j– parameter in migration loop ML

t - step ∈ <0, by Step to, PathLength>

PRTVector - is vector of ones and zeros depended on PRT. If random number from interval <0, 1> is less than PRT, then 1 is saved to PRTVector, otherwise it will be 0.

SOMA works with controlling and stopping parameters which are summarized in Table1.

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Table 1: Parameters of SOMA

Parameter name	**Recommended range**	**Note**
PathLength	<1.1, 3>	Controlling parameter
Step	<.11, PathLength>	Controlling parameter
PRT	<0, 1>	Controlling parameter
Dim	Given by problem	Number of arguments in cost function
PopSize	<10, up to user>	Controlling parameter
Migrations	<10, up to user>	Stopping parameter
MinDiv	<arbitrary negative, up to user >	Stopping parameter

PathLength $\in$ <1.1, 3>. This parameter defines how far an individual stops behind the Leader (PathLength=1: stop at the leader's position, PathLength=2: stop behind the leader's position on the opposite side but at the same distance as the starting point). If it is smaller than 1, then the Leader's position is not overshot, which carries the risk of premature convergence. In that case SOMA may get trapped in a local optimum rather than finding the global optimum. The recommended value is 3.

Step $\in$ <.11, PathLength>. The step size defines the granularity with which the search space is sampled. In case of simple objective functions (convex, one or a few local extremes, etc.), it is possible to use a large Step size in order to speed up the search process. If prior information about the objective function is not known, then the recommended value should be used. For greater diversity of the population, it is better if the distance between the start position of an individual and the Leader is not a multiple of the Step parameter. That means that a Step size of 0.11 is better than a Step size of 0.1, because the active individual will not reach exactly the position of the Leader. The recommended value set up to 0.11 was taken from (Zelinka 2000, 2001, 2002, 2004). But this article will show that the recommended value does not have to be always 0.11. The aim of this article is to show that according to Shannon – Kotelnik theorem the value can changed adaptively during the run.

PRT $\in$ <0, 1>. PRT stands for perturbation. This parameter determines whether an individual will travel directly towards the Leader, or not. It is one of the most sensitive control parameters. The optimal value is near 0.1. When the value for PRT is increased, the convergence speed of SOMA increases as well. In the case of low dimensional functions and a great number of individuals, it is possible to set PRT to 0.7-1.0. If PRT Equals 1 then the stochastic component of SOMA disappears and it performs only deterministic behaviour suitable for local search.

Dim - the dimensionality (number of optimized arguments of cost function) is given by the optimization problem. Its exact value is determined by the cost function and usually cannot be changed unless the user can reformulate the optimization problem.

PopSize $\in$ <10, up to the user>. This is the number of individuals in the population. It may be chosen to be 0.2 to 0.5 times of the dimensionality (Dim) of the given problem. For example, if the optimization function has 100 arguments, then the population should contain approximately 30-50 individuals. In the case of simple functions, a small number of individuals may be sufficient; otherwise larger values for PopSize should be chosen. It is recommended to use at least 10 individuals (two are minimum), because if the population size is smaller than that, SOMA will strongly degrade its performance to the level of simple and classical optimization methods.

Migrations $\in$ <10, up to user>. This parameter represents the maximum number of iterations. It is basically the same as generations for GA or DE. Here, it is called Migrations to refer to the nature of SOMA - individual creatures move over the landscape and search for an optimum solution. 'Migrations' is a stopping criterion, i.e. it tells the optimizing process when to stop.

MinDiv $\in$ <arbitrary negative, up to the user >. The MinDiv defines the largest allowed difference between the best and the worst individual from the actual population. If the difference is too small, then the optimizing process will stop. It is recommended to use a small value. It is safe to use small values for the MinDiv, e.g. MinDiv = 1. In the worst case, the search will stop when the maximum number of migrations is reached. Negative values are also possible for the MinDiv. In this case, the stop condition for MinDiv will not be satisfied and thus SOMA will pass through all migrations.

SHANNON – KOTELNIK THEOREM

SOMA has as a sampling parameter Step. To carry out an analysis, how Step influences the run of SOMA, was necessary to make a study of a cost function according to sampling Shannon - Kotelnik theorem (Abramson N. 1963, Proaksis J. G., 1989, Hamming R.W., 1980).

Shannon – Kotelnik theorem says: If in the process of sampling the information must not lose, a frequency of sampling ω_s and maximal frequency ω_m included in the signal spectrum have to comply with a condition in the Eq. (2).

$$\omega_s \geq 2\,\omega_m \qquad (2)$$

It results from the definition that the aim is to find a maximal frequency in the Fourier analysis of the cost function.

EXPERIMENTS

Fourier transform

As a cost function it was chosen a simple unimodal function (1st DeJong) for 2 arguments Eq. (3) for this purpose.

$$\sum_{i=1}^{Dim} x_i^2 \qquad (3)$$

where
i – i-argument of the function
Dim – dimension (number of optimized arguments of the cost function. In this case Dim = 2.

Decomposition of a function on separate signals solves Fourier transform. It works with periodical signals but some interval of a function can be imagined as one period. Therefore it can be used Fourier transform (Bracewell R. N., 1999) also for such case. Basic Fourier transform is given by Eq. (4) (Bracewell R. N., 1999, Farlow 1993).

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{j\,\omega\, t}\, dt \qquad (4)$$

It was used to make a Fourier transform of the chosen function. The quality of solution of Fourier transform depended on number of expansion members. Graphs with the origin and created function by means of Fourier transform for number of expansion members from 1 till 20 were made. Then it was chosen heuristically which one could correspond with acceptable error.
In the Fig. 1 the blue space shows the difference between original function in Eq. (3) and the reconstructed function based on a few terms of the Fourier transform - for number of expansion members which was equal 1.

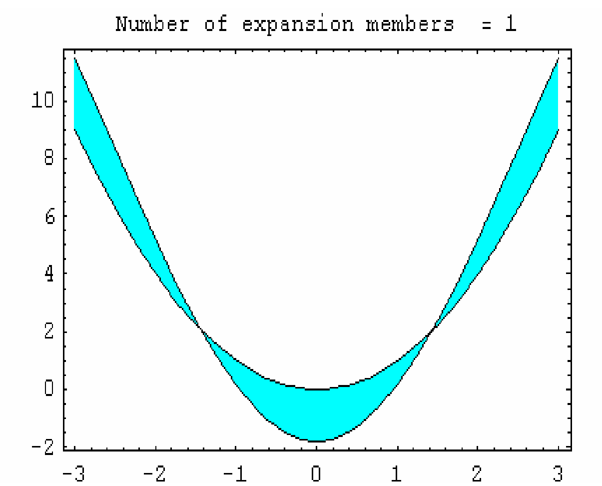

Fig. 1: Difference between original function and Fourier transform (Number of expansion members =1)

Fourier transform is an infinite series of expansion. Therefore it was necessary to find the first which could correspond with the origin function. In this case number of expansion members Fourier transform was equal to 7 (Fig. 2).

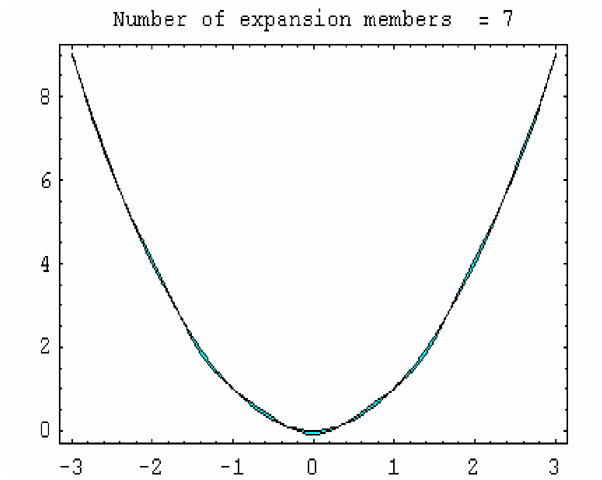

Fig. 2: Difference between original function and Fourier transform (Number of expansion members =7)

Fourier transform with coefficients for this case was found as an expression in the Eq. (5).

$$\begin{aligned}&8.33333 - 10.1321\,\mathrm{Cos}[0.628319\,t] + \\ &2.53303\,\mathrm{Cos}[1.25664\,t] - 1.12579\,\mathrm{Cos}[1.88496\,t] + \\ &0.633257\,\mathrm{Cos}[2.51327\,t] - 0.405285\,\mathrm{Cos}[3.14159\,t] + \\ &0.281448\,\mathrm{Cos}[3.76991\,t] - 0.206778\,\mathrm{Cos}[4.39823\,t]\end{aligned} \qquad (5)$$

Each member of expansion can be drawn as a single curve whose sum will give the origin function. Next two figures show curves of single components of Fourier transform. The Fig. 4 is detailed view of Fig. 3 to see the line with the biggest value of frequency. On both pictures this is drawn by dashed line. Detailed view can be seen in Fig. 5.

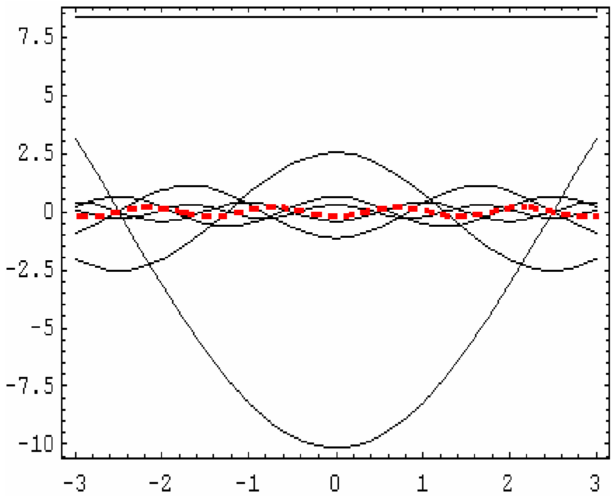

Fig. 3: Single components of Fourier transform for expansion members equal 7

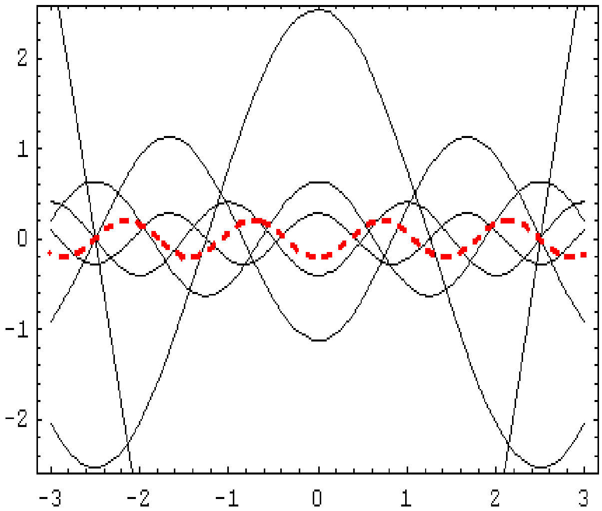

Fig. 4: Detailed view of Fig.3

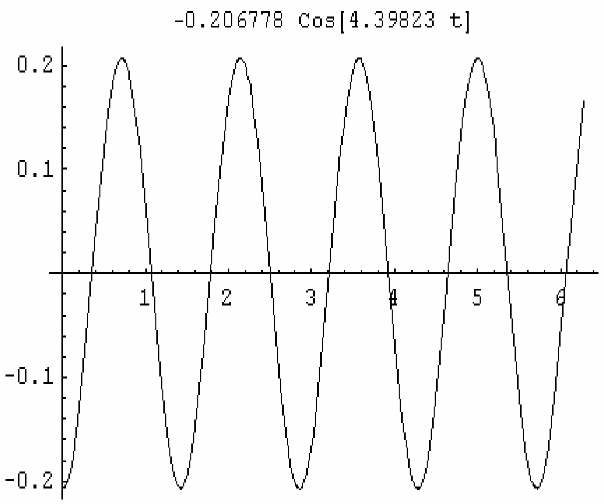

Fig. 5: Chart of the critical member of Fourier transform

This dashed drawn member interprets the critical one for the analysis of function from Shannon – Kotelnik theorem point of view. Its analytic transcription can be seen in Eq. (5) which determines the sampling frequency according to Shannon – Kotelnik theorem (Eq. (6)).

$$F_c = -0,206778 \text{ Cos } (4.39823 \text{ t}) \qquad (6)$$

Computation of critical frequency

The frequency can be calculated from this expression according to Eq. (7) as reciprocal value of period (Eq. (8)). In this case the result frequency is 0.7.

$$\text{frequency} = 1 / \text{period} \qquad (7)$$

$$\text{period} = (2\pi) / 4.3982 \qquad (8)$$

Then according to Eq. (2) the sampling frequency of SOMA should be twice more than the maximal frequency in the spectrum of function, i. e. The critical frequency, which a sampling of SOMA by means of Step should achieve, is equal to 1.4.

Analysis of parameter Step

Parameter Step as was described above can be explained as sampling parameter of SOMA. It does not work in absolutely values but in relative ones. Absolute value of the step is counted from length between Leader and active individual multiplied by value of parameter Step. It can be viewed as part of Eq. (1) – principle of SOMA run.
To compare values of critical frequency sampling of SOMA it was necessary to do reciprocal values of absolute step which can be regarded as a sampling period.
On the basis of carried simulations following charts were prepared. The simulation consists in comparing reciprocal values of absolute step with critical frequency. After that the critical Steps were determined. In other words, there were determined maximal values which can be set up in parameter Step satisfied the conditions of Shannon – Kotelnik theorem and thus fact that algorithm will use information obtained by stepping in an efficient way.
Next figures show the critical steps for each individual in the population during migrations. They are depicted without Leader because critical Steps of all other individuals are outspread to the position of Leader. Fig.6 shows first population which was generated randomly. The other population are generated during evolution process. It can be seen that in the first migration the critical Steps are small because individuals are spread on the surface of the function randomly and it is necessary to sample the function very precisely to get to exploit information about cost function. The individual which is very near to Leader is in the biggest peak. There is not important to make so small steps as in the case of others. The value of Critical Step is in the interval <0,3>. Where the value 3 means that the individual is on the same position as Leader and it is expected that they both could be in the extreme.
Fig. 7 shows the same case as the Fig. 6 but in the decreasing direction of values Steps. There can be better seen that not many individuals are near to extreme and that a lot need to sample the way to Leader in very small steps.
Fig. 8 and Fig. 9 are similarly generated as previous two but in selected migration during the run of SOMA. On these pictures can be seen that most of individuals achieved the same position as Leader. There is depicted agglomeration of individuals of typical run of SOMA in the global extreme as is described in (Zelinka 2002).

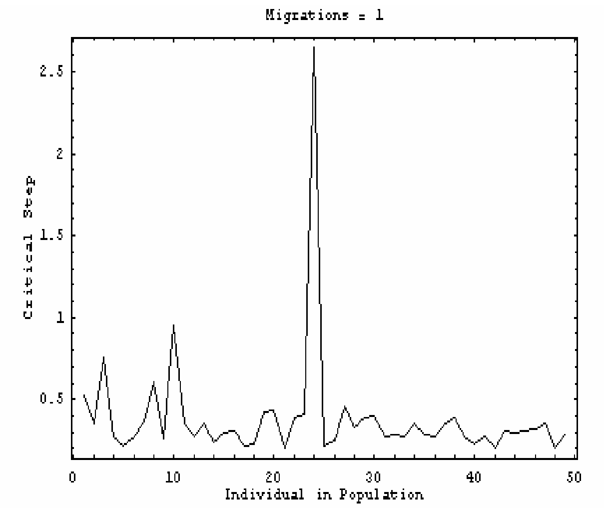

Fig. 6: Chart of the critical values of parameter Step in first migration

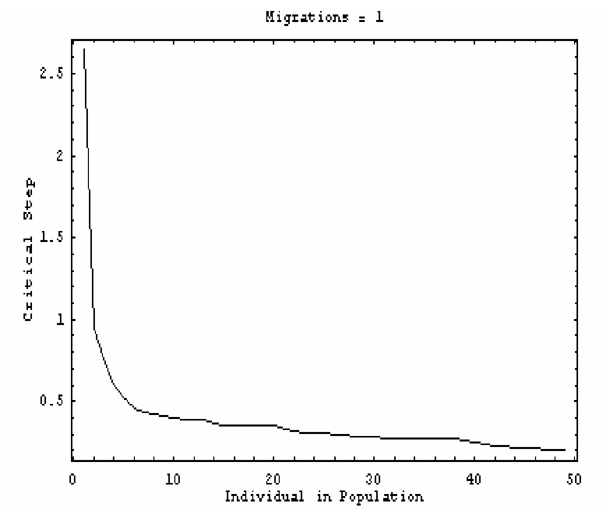

Fig. 7: Chart of the critical values of parameter Step in his decreasing direction in first migration

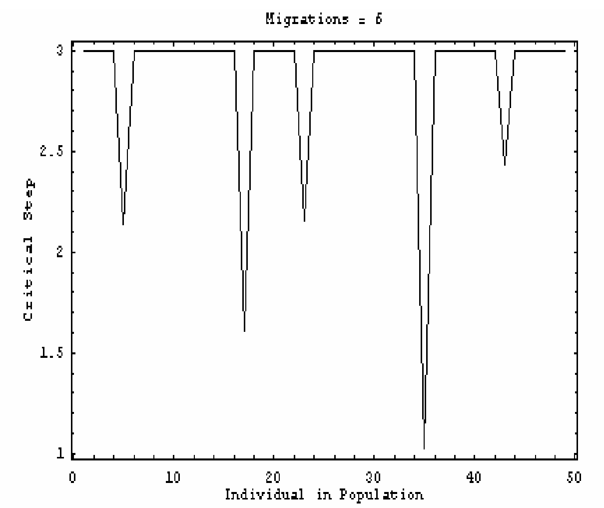

Fig. 8: Chart of the critical values of parameter Step during the run of SOMA

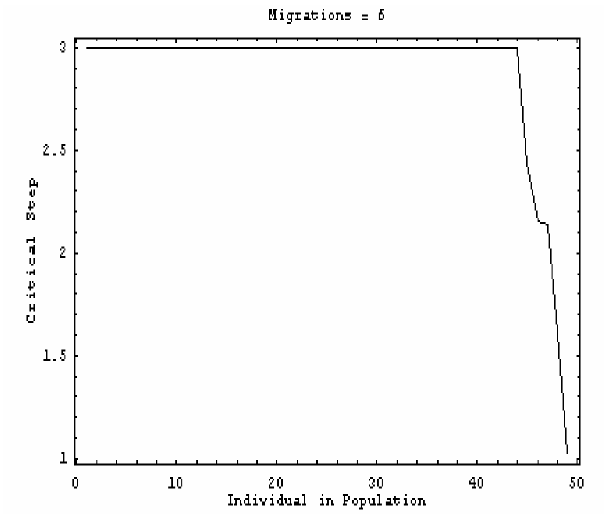

Fig. 8: Chart of the critical values of parameter Step in his decreasing direction during the run of SOMA

Fig. 9 shows the situation when all individuals are in the global extreme. The line is at the position of value 3 because such value of Step says that individual is in the same position as Leader. The aim of SOMA is to have all individual at same position in global extreme of cost function.

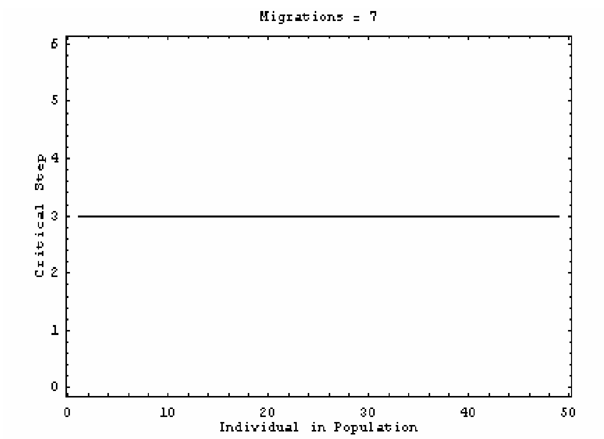

Fig. 9: Chart of the critical values of parameter Step in the last migration

CONCLUSION

This contribution deals with analysis of sampling method in the evolutionary algorithm SOMA. According to Shannon – Kotelnik theorem critical values of parameter Step were determined. They say which maximal value of Step has to be set up to keep conditions. All was well-arranged displayed in figures.
This study was preliminary in this field. Our future work will be concerned to make studies on more difficult cost functions (multimodal).
On the basis of the results from these studies is expected to be tried new version of SOMA which will use the method of this analysis to tune Step adaptively during the run of SOMA. It may improve to obtain high-quality

solution in shorter time. Then it is planned to make studies how much faster the new version of SOMA will be, compared to the current one.

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AUTHOR BIOGRAPHIES

ZUZANA OPLATKOVA was born in Czech Republic, and went to the Tomas Bata University in Zlin, where she studied technical cybernetics and obtained her degree in 2003. She is now Ph.D. student and lecturer (artificial intelligence) at the same university. Her e-mail address is: oplatkova@ft.utb.cz

IVAN ZELINKA was born in Czech Republic, and went to the Technical University of Brno, where he studied technical cybernetics and obtained his degree in 1995. He obtained Ph.D. degree in technical cybernetics in 2001 at Tomas Bata University in Zlin. Now he is associated professor (artificial intelligence, theory of information) and head of department. His e-mail address is: zelinka@ft.utb.cz and his Web-page can be found at http://www.ft.utb.cz/people/zelinka.