COMBINING ANALYTICAL AND SIMULATION APPROACHES TO QUANTIFICATION OF THE BULLWHIP EFFECT

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ABSTRACT

Nowadays, effective and competitive company operation can be achieved through incorporating the concept of supply chain operation into company management. Inventory control, as a critical part of the supply chain management, becomes the second most frequent application area for simulation technique in logistics (after manufacturing).

The dynamics of supply chain operation is characterised by the bullwhip effect that reflects an increase in demand variability while moving upwards the supply chain. This paper proposes an analytical model for the analysis and numerical evaluation of the bullwhip effect in supply chains. Simulation technique is used to validate the results obtained from the analytical model. Based on the validation results, the logic of the analytical model is examined, and some specifications of the analytical model are analysed and described.

INTRODUCTION

Supply chain management is the term used to describe the management of materials and information across the entire supply chain, from suppliers to component producers to final assemblers to distribution (warehouse and retailers), and ultimately to the consumer. Supply chain management has generated much interest in recent years because of the realisation that actions taken by one member of the chain can influence the profitability of all others in the chain (Silver and Peterson 1985). The bullwhip effect is considered as one of the main supply chain operation stability and efficiency measures. It characterises an increase in demand variability through the entire supply chain.

Many companies implement the supply chain concept to achieve efficiency in system operation; i.e., instead of responding to unknown and highly variable demand, they share information so that the variability of the demand they observe is significantly lower. The assumption that a new level of efficiency can be simply attained by sharing information and forming *strategic alliances* with firm supply chain partners is wrong. Knowing what to do with the data is as important as getting the data in the first place (Silver and Peterson 1985). Methods for coping with the bullwhip effect are discussed in (Simchi-Levi et al. 2000). They can significantly reduce, but will not eliminate, the bullwhip effect. It is important to investigate the magnitude of this effect to avoid holding an excessive inventory, insufficient capacities and high transportation costs. For better understanding and controlling the bullwhip effect it is useful to quantify it. Simchi-Levi et al. (2000) explain the increase in demand variability by the necessity for each supply chain stage to make orders based on the forecasted demand of the previous stage. They propose quantifying the magnitude of increase in variability between two neighbour supply chain stages by a function of the lead-time between the orders receipt and the number of demand observations on which a forecast is made. Disney and Towill (2002) developed an analytical expression for the bullwhip effect quantification from the control theory's point of view by using a z-transform model. Kelle and Milne (1999) suggest using approximations of the quantitative model, developed in accordance with asymptotic renewal theory, to evaluate a variance of placed orders (bullwhip effect) in inventory systems that implement the *S-s* inventory control policy.

This paper proposes a statistics-based analytical approach for evaluating the bullwhip effect in inventory systems. We focus on the supply chain from the perspective of inventory management. We consider the simplest of multi-echelon situations when the stocking points are serially connected.

The main cause of the bullwhip appearance in supply chains is uncertainty of demand inherent in supply chain operation environment. An analytical model for quantification of demand fluctuation magnification (the bullwhip effect) as orders move up the supply chain in case of stochastic demand is developed in this paper.

Simulation is a powerful tool for analysing inventory systems, because it is capable of capturing the uncertainty and complexity inherent in inventory systems. The ability to handle demand and lead time uncertainty is one of the main reasons why simulation is widely used for inventory systems (Bhaskaran 1998).

Banks and Malave (1984) identify inventory control problems as one of the most frequent areas of application for simulation methodology. They propose the following six categories of simulation techniques

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usage assignments in modelling and analysing inventory systems:

1. Analytic solution impossible or analytic solution extremely complex. An analytic solution to a problem may not be available because of stochastic operating environment, extremely complex problem or a very specific problem.
2. Comparison of model. It is one of the most frequently observed uses of simulation in inventory systems. Simulation is used to compare alternative inventory control policies.
3. Validation of analytical solution. Simulation is used to validate the results obtained from an analytic model.
4. Variance reduction techniques. Increasing the statistical efficiency of a simulation by reducing the variance of the output random variables.
5. Model validation and verification. It is the most important part of a simulation study and enables determining whether a model performs as intended and is an accurate representation of the real-world system under study.
6. Optimisation techniques. Considering optimisation techniques for inventory simulation two aspects should be determined: the length of simulation run and comparison method of different alternatives.

The developed simulation model of the considered inventory system validates the results produced by the analytical model. The simulation implementation in this case corresponds to the 3rd category of simulation techniques usage assignments in modelling and analysing inventory systems.

The rest of the paper is organised as follows. The analytical model for numerical evaluation of the bullwhip effect in inventory systems that control their inventories by the *S-s* ordering policy is elaborated in the next section. The section also presents a sample application of the described analytical model aimed to get numerical results. The following sections relate to analytical model validation performed by using an appropriate simulation model, analyse the accuracy of the obtained analytical solution, and discuss a combined analytical/simulation approach for evaluating the increase in variability of placed orders in supply chains. Conclusions are presented in the final section.

ANALYTICAL MODEL

A single-item, single-stage, multi-period inventory system is considered. The traditional *S-s* policy is used for inventory management. According to this policy, an order is placed when the stock declines to a lower control limit called the order point, *s*. The order quantity is the amount required to bring the inventory level to the order level, *S*. A more detailed description of the considered inventory control policy can be found in Merkuryev et al. (2004).

It is assumed that the demand $X_1, X_2, ..., X_i$ is a discrete random sample observed from some population. Accordingly, these data are independent and identically distributed (IID) observations on some underlying random variable X whose distribution governs the population. Values that numerically characterise the population/distribution, such as an expected value $E(X)$ and a variance $D(X)$ of the discrete random variable X are given. The order quantity Q_i is demanded when the on-hand inventory drops below the reorder point. It is equal to the sum of the demand quantities between the order placements:

$$Q_i = X_1 + X_i + ... + X_v, \qquad (1)$$

where

v – random variable, a period number when an order is placed.

Provided that the demand X is uncertain and the aforementioned inventory control method is employed, the placed order quantity Q is expected to be a random variable that depends on the demand quantities. The expected value $E(Q)$ and variance $D(Q)$ of the function $Q = \varphi(X)$ are estimated using the following formulas proposed by Feller (1967):

$$E(Q) = E(X) * E(v) \qquad (2)$$

and

$$D(Q) = E(v) * D(X) + D(v) * [E(X)]^2, \qquad (3)$$

where

$E(v)$ – expected value of a period number when an order is placed;

$D(v)$ – variance of a period number when an order is placed.

To investigate a probabilistic behaviour of the discrete random variable v it suffices to estimate its numerical characteristics (an expected value and its variance). The difference between the order level S and order point s has to be established to find a time period when an order should be placed:

$$\Delta = S - s \qquad (4)$$

The multi-experimental realisation of the following algorithm:

if $X_1 > \Delta$ THEN $v=1$ AND STOP
ELSE generate X_2
if $X_1 < \Delta$ and $X_1 + X_2 > \Delta$ THEN $v=2$ AND STOP
ELSE generate X_3
...
if $X_1 + X_2 + ... + X_{n-1} < \Delta$ and $X_1 + X_2 + ... + X_n > \Delta$ THEN $v=n$
STOP

allows one to collect statistics of v values (v_i, $i = \overline{1,n}$) and evaluate their probabilities p_i by relative frequencies $\hat{p}_i$ of their occurrences in the experiments performed.

The expected value of random variable is the weighted average of all possible values of the random variable, where the weights are the probabilities of the value occurrence. The expected value *E(v)* of the *v* value population is estimated by this formula:

$$\hat{E}(v) = \sum_{i=1}^{n} v_i * \hat{p}_i \qquad (5)$$

and its variance *D(v)* is estimated as follows:

$$\hat{D}(v) = \sum_{i=1}^{n} v_i^2 * \hat{p}_i - \hat{E}(v)^2 , \qquad (6)$$

where
$\hat{E}(v)$ and $\hat{D}(v)$ - experimental estimation of *E(v)* and *D(v)*, respectively.

A numerical example of the developed analytical model implementation for the bullwhip effect quantification is given in the next subsection.

Sample Application of Analytical Model

The performance of the inventory system is evaluated under various factors such as end customer mean demand *E(X)* and its standard deviation *STD(X)*, safety stock factor *z* and a lead time *LT*.

To collect a statistics of period numbers *v* when orders should be placed and experimentally estimate their expected value by formula (5) and variance by formula (6), it is supposed that the end customer demand is realised as a normal distribution, and 1000 experiments are performed. The minimal value of the observed *v* values for all alternatives is 1 time period and its relative frequency of occurrence is less than 0.007. The maximal value is 5 time periods and its relative frequency of occurrence does not exceed 0.004. Respectively, the most likely value is 2 time periods that can occur with the relative frequency greater than 0.5.

Experiments, when the standard deviation of the mean demand changes by the defined coefficient *Change Ratio* equal to 1.2 and remaining factors are considered to be constant numbers, are performed. The mean *E(X)* and the standard deviation *STD(X)* of the demand change proportionally, i.e. they are dependent through the *Signal To Noise* factor, equal to 5, that describes a variability of the demand:

$$STD(X) = \frac{E(X)}{Signal\ To\ Noise} \qquad (7)$$

The experimentally estimated probability of the period number when an order is placed will be the same for all alternatives because the *S-s* level will change in accordance with a new mean demand value. Based on the observed experimental results the following hypothesis could be built up – the relative frequency $\hat{p}$ of the random variable *v* occurrence corresponds to its probability *p* and its value depends only on the lead time length.

The estimation of orders variability *D(Q)* and its expected value *E(Q)* are calculated by formulas (3) and (2) respectively, while numerical results are given in Table 1.

By analysing the placed order variability for all the performed experiments we can conclude that even a small variation of the mean demand causes an increase in variability of the placed orders. The larger the initial value of the demand variation is, the more significant magnification of placed orders fluctuation will be observed.

VALIDATION OF ANALYTICAL MODEL

The considered inventory system has an explicitly dynamic character. Simulation is used to capture this behaviour of the system and to provide a more realistic representation of the inventory system operation, namely information about demand and order quantities collection over time.

The developed simulation model was used to validate the analytical solution presented in the previous section.

Conceptual Model of Inventory System

The structure of the considered inventory system corresponds to the analytical model described above.

It is assumed that end customer demands arrive with fixed time-intervals, and their quantity is variable and is derived from a normal distribution. A constant lead time between replenishment is considered. No order processing delay is taken into account, so all demand events are treated immediately by the inventory system. We will also assume no capacity constraints for supplier of the inventory system. In this case, stockouts will not lead to lost sales, but to backorders. We thus assume that we have loyal customers. It is worth noting that replenishment triggering will be based on the effective inventory level, which is the quantity on hand plus the quantity on order minus the unshipped backorders to customers.

Table 1: Placed Orders Variability Estimation by Analytical Model

Nr.	*E(X)*	*STD(X)*	*z*	*LT*	*s*	*S*	*S-s*	$\hat{E}(v)$	$\hat{D}(v)$	$D_{cal}(Q)$	$E_{cal}(Q)$
1	50	10	1.96	2	128	228	100	2.50	0.26	885.64	126.60
2	70	14	1.96	2	179	319	140	2.50	0.26	1743.30	178.15
3	90	18	1.96	2	230	410	180	2.50	0.26	2881.78	229.05
4	110	22	1.96	2	281	501	220	2.50	0.26	4304.88	279.95
5	130	26	1.96	2	332	592	260	2.50	0.26	6012.60	330.85

The objective of inventory management is to manage stable operation of the considered system, i.e., quantify and control the bullwhip effect.

Simulation Model of Inventory System

The simulation model was developed using the ARENA 5.0 simulation modelling environment. The described conceptual model is converted into a computer model. Simulation is used to analyse and evaluate the increase in variability of placed orders in the described inventory system.

Simulation Experiments

The objective of experimental studies is to determine the bullwhip effect magnitude in the inventory system that implements the *S-s* inventory control policy and validate the results produced by the analytical model of the same inventory system. For that purpose, a set of experiments with the simulation model is performed. The performance of the inventory system is evaluated under various factors similar to the sample application of the analytical model for the quantification of the bullwhip effect (see Table 1).

The model was run for 5 replications. Each replication length is defined as 5000 time periods. The warm-up period is avoided by setting the initial inventory level equal to the order level *S* at the beginning of each replication.

The mean value and variance of placed orders during simulation are shown in Table 2.

Considering the first row in Table 2, one could observe that in this case 95% confidential intervals for the expected average order quantity $E_{sim}(Q)$ and its variation $D_{sim}(Q)$ are [125,78±0,9] and [311,77±4,5], respectively. Generally speaking, observations within all 5 series of simulation experiments vary non-essentially. Namely, for each row in Table 2 a 95% confidential interval for variation of placed orders lies within 1.5% from its average value.

Table 2: Placed Orders Variability Estimation by Simulation Model

Nr.	*E(X)*	*STD(X)*	*z*	*LT*	*s*	*S*	*S-s*	$D_{sim}(Q)$	$E_{sim}(Q)$
1	50	10	1.96	2	128	228	100	311.77	125.78
2	70	14	1.96	2	179	319	140	603.03	176.53
3	90	18	1.96	2	230	410	180	996.55	226.75
4	110	22	1.96	2	281	501	220	1501.83	277.63
5	130	26	1.96	2	332	592	260	2082.93	328.10

ANALYSIS OF SIMULATION RESULTS

The results given by the analytical model proved to be in disagreement with those given by the simulation model. The variance of placed orders calculated by analytical model (see Table 1) is approximately 3 times greater in all experiments than actual variance of placed orders derived from the simulation model (see Table 2). The reason for the inadequate bullwhip effect quantification by the analytical model is an existing dependence between a period number when an order is placed *v* and realisations of the end demand X_i. In other words, the proposed formula (3) assumes *v* and *X* independence, but in the described inventory control system they are dependent in the way of conditional probability of *v* occurrence p_v=P($X_1+X_2+\ldots+X_v>S-s/X_1+X_2+\ldots+X_v-1<S-s$). The period number when an order is placed directly depends on the demand quantity (the larger the demand quantity during the order cycle is, the faster inventory level reaches the order point *s* and frequency of orders increases; i.e, *v* decreases). From this it follows that random variables *v* and $Q=\sum_{i=1}^{v} X_i$ are correlated. Random variables *Q* and *v* that are denoted by the expected values and standard deviations M_Q, σ_Q and M_v, σ_v correspondingly, are dependent random variables.

In order to establish a statistical dependence between the placed order quantity and a period number when it is placed, *Q* and *v* are represented as a system of the two dependent normally distributed variables that have the following joint probability density function:

$$W(Qv) = \frac{1}{2\pi\,\sigma_Q\,\sigma_v\sqrt{1-r^2}}\,exp\left\{-\frac{1}{2(1-r^2)}*\right.$$
$$\left.*\left[\frac{(Q-M_Q)^2}{\sigma_Q^2} - 2r\frac{(Q-M_Q)(v-M_v)}{\sigma_Q\,\sigma_v} + \frac{(v-M_v)^2}{\sigma_v^2}\right]\right\}$$

where
r – correlation coefficient between *Q* и *v*, -1≤ *r* ≤1.

It should be noted that for jointly distributed normal random variables concepts of independence and uncorrelation are the same. That is, if random variables are independent, they are uncorrelated and vice versa.

If the value of the random variable v is known, then the value of the random variable Q is conditional. In this case, it has a conditional probability density function:

$$W(Q/v) = \frac{W(Qv)}{W(v)} = \frac{1}{\sqrt{2\pi}\,\sigma_Q\sqrt{1-r^2}} *$$

$$* \exp\left\{-\frac{1}{2(1-r^2)}\left[\frac{(v-M_v)r}{\sigma_v} - \frac{Q-M_Q}{\sigma_Q}\right]^2\right\}$$

The conditional random variable has a conditional expected value:

$$M(Q/v) = \int_{-\infty}^{\infty} QW(Q/v)dQ = M_Q + r\frac{\sigma_Q}{\sigma_v}[v - M_v]$$

and a conditional variance:

$$\sigma^2(Q/v) = \int_{-\infty}^{\infty}[Q - M(Q/v)]^2\, W(Q/v)dQ = \sigma_Q^2(1-r^2)$$

Thus, the conditional variance of the random variables Q is independent of the v value. It is estimated by their own unconditional variance σ_Q^2 and the correlation coefficient r between Q and v.

Analytical model implementation gives an unconditional variance of the placed orders σ_Q^2, as it is calculated for unknown period number v when the order should be placed for each order cycle.

Simulation model allows one to estimate a conditional variance of the placed orders $\sigma^2(Q/v)$. Based on the results obtained, it is possible to calculate the correlation coefficient r between Q and v, using this formula:

$$D(Q)_{sim} = D(Q)_{cal} * (1-r^2) \Rightarrow r = \sqrt{1 - \frac{D(Q)_{sim}}{D(Q)_{cal}}},$$

where

$D(Q)_{sim} = \sigma^2(Q/v)$ – variance of placed orders estimated by the simulation model with known v (conditional variance);

$D(Q)_{cal} = \sigma_Q^2$ – variance of placed orders estimated by the analytical model with unknown v (unconditional variance).

Table 3: Coefficient of Correlation between Q and v

Nr.	*D(X)*	$D_{cal}(Q)$	$D_{sim}(Q)$	*r*
1	100	886	312	0.8
2	196	1743	603	0.8
3	324	2882	997	0.8
4	484	4305	1502	0.8
5	676	6013	2083	0.8

The calculated correlation coefficient between Q and v (see Table 3) in the inventory system that implements the *S-s* inventory control policy when end customer mean demand and its standard deviation change proportionally, i.e. they are dependent through the *Signal To Noise* factor, is the same for all 5 experiments. It is supposed that the correlation coefficient depends only on the lead time length. A set of corresponding correlation coefficients for various lengths of the lead time could be estimated by the elaborated combined analytical/simulation approach. As soon as the dependence between the placed order quantity and a period number when it is placed is found, the described analytical model can be used for numerical evaluation of the bullwhip effect.

CONCLUSIONS

The analytical model for the quantification of demand fluctuation magnification (the bullwhip effect) as orders move up the supply chain in case of stochastic demand is elaborated. A combined analytical/simulation approach is used to estimate the dependence between a period number when an order is placed and end customer demand quantities with a view to make the analytical solution more accurate.

The effect of different ordering policies and inventory system parameters on the above-mentioned dependence is a subject of future research.

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