

# MODELLING PLANT SPACING AND YIELDS OF CROPS BY SOWING SEEDS AT EXACT INTERVALS

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## KEYWORDS

Modelling plant spacing, plant density, yields, mathematical coherences.

## ABSTRACT

A study of probable plant spacing in field crop and vegetable plantations and their corresponding yields by sowing seeds at exact intervals and growing without thinning have been carried out. By modelling relationships of the plant distribution, spacing density and yields mathematical coherences are obtained. It is stated that the plant spacing density is a function of the seed germinating power in the field. The lower is the germinating power, the higher is the irregularity of the seedlings, i.e. the number of the longer spacing increases. The obtained coherences allow to prognosticate the irregularity of the seedlings depending on the expected seed germinating power and to specify the standard quantity of seeds per hectare. A formula is obtained for the estimation of field crop and vegetable yields depending on plant density.

## INTRODUCTION

To obtain high and qualitative yields in growing field crops and vegetables, it is of great importance to achieve the required density and their uniform spacing. This problem has become particularly urgent due to the transition from growing crops (sugar and fodder beet) and vegetables (cucumbers, red beet) with their thinning to growing them without thinning, by sowing the seeds at extreme intervals.

The purpose of this study is to clarify mathematical coherences between the plant spacing density, their distribution and the yields to be reached.

## APPROACH

Theoretical and experimental research has been carried out to obtain the relationships of plant density, their spacing and yields of crops obtained. Theories of probability and mathematical statistics are used in the investigations. The results of theoretical research carried out to clear up the relationships of plant density and their spacing has been affirmed with experiment data.

## RESULTS

### Plant spacing relationships

It is found out in the previous investigations that at a

great sowing ratio (more than 20 seeds per metre of the row) the plant spacing in the row approaches the binomial distribution, but at thin sowing of the seeds at certain intervals it forms a series of normal spacing with a decreasing mode frequency. A similar picture is observed in the later investigations too (Vilde and Cesnieks 1999).

To reach the desired plants distribution density  $N_{opt}$ , the number of the seeds  $N_s$  to be sown per unit of the area (ha) is determined when plant distribution density is divided by the germinating power  $q$  of the seeds:

$$N_s = N_{opt} q^{-1} \quad (1)$$

The number of the seeds  $n_s$  to be sown out per metre of the row at the distance  $b$  between the rows:

$$n_s = 10^4 N_s b \quad (2)$$

and the distance  $l_s$  between the seeds sown:

$$l_s = n_s^{-1} \quad (3)$$

The plant spacing uniformity in the plantations can be characterised by the frequency of intervals (distances) between the plants.

If each seed has sprouted, the distance between the plants corresponds to the distance between the seeds sown  $l_s$ . But there is a probability that one, two, three, etc. seeds that follow each other have not sprouted. In this case the intervals between the plants will increase correspondingly two, three etc. times (see Fig.1).

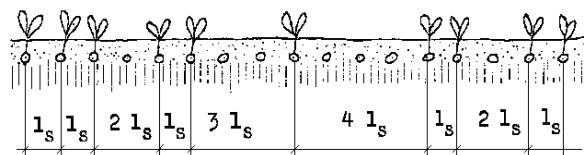


Fig.1. Spacing of seeds and seedlings in the row.  
 $l_s$  - the interval between the seeds

By applying: the probability theory and the methods of mathematical statistics a mathematical coherence is obtained to be used to determine the distribution of intervals between the plants and their spacing. It is found that the intervals between the seedlings and their frequency are dependent on the distance between the seeds sown at the interval  $l_s$  and their germinating power in the field. With the decrease of the germinating power the irregularity of the sprout spacing increases, that is, the number of the greater intervals increases.

The expected spacing frequency  $p(l_s)$  between the

seedlings corresponding to the distance between the seeds  $l_s$  is equal to the germinating power of the seeds in the field  $q$ . For double distance  $2 l_s$  the distribution of the corresponding intervals  $p(2 l_s)$  will correspond to  $(1-q)q$ , and so on.

As a result, a series of formulae is obtained for the calculation of the interval frequency between the crop seedlings depending on their germinating power  $q$  in the field:

$$\begin{aligned}
 p(l_s) &= q \\
 p(2 l_s) &= q(1-q) \\
 p(3 l_s) &= q(1-q)^2 \\
 &\dots\dots\dots \\
 p(n l_s) &= q(1-q)^{n-1}
 \end{aligned}
 \tag{4}$$

In the set of formulae (4) the given coherence is in force if the seeds are sown out one by one, the deviation of the seeds and the sprouts from the pre-set sowing interval will not exceed its half and in the given part of the field the germinating power of the seeds is approximately the same.

The interval frequency between the seedlings at various germinating power is presented in the Table 1 and the diagram (Figure 2).

It is obvious from Table 1 and Figure 2 that the decrease in the germinating power of the seeds is followed by increased irregularity of spacings between the seedlings. Therefore, in order to obtain uniform plant distribution, not only exact spacing of the seeds is important but also their high germinating power in the field, which can be achieved by using high-quality seeds and ensuring optimum conditions for their germination (qualitative soil preparation, sowing at optimum depth with shares of a correct shape, protection against diseases and pests etc.).

### Correlations of plant distribution on non-homogeneous fields.

Experience shows that the large fields formed as a result of joining smaller fields together often have non-uniform physical and mechanical soil composition, which considerably affects the germinating power of the seeds on the field and the density of sprouts. This circumstance should be taken into particular consideration when the plants are grown without thinning by sowing them out in rows at extreme intervals. At such a technology, areas (spots) often appear on the fields with insufficient plant density, which has an adverse effect on the yield obtained. In this connection theoretical studies have been carried out and mathematical coherences are derived on the plant distribution on such non-homogeneous fields.

On non-homogeneous fields the medium interval frequency between the plants is determined on areas with different germinating power of the seeds on the field.

Table 1. The interval frequency between the seedlings, %

The interval		The germinating power in the field, %					
General case	Particular case, cm	100	80	60	40	20	10
$1 l_s$	16.7	100	80	60	40	20	10
$2 l_s$	33.4	0	16	24	24	16	9.0
$3 l_s$	50.1		3.6	14	14	13	8.1
$4 l_s$	66.8		0.2	3.8	8.6	10	7.3
$5 l_s$	83.5			1.5	5.2	8.2	6.6
$6 l_s$	100.2			0.6	3.1	6.6	5.9
$7 l_s$	116.9			0.2	1.9	5.2	5.3
$8 l_s$	133.6			0.1	1.1	4.2	4.8
$9 l_s$	150.3				0.7	3.4	4.3
$10 l_s$	167				0.4	2.7	3.9
$11 l_s$	183.7				0.2	2.1	3.5
$12 l_s$	200.4				0.1	1.7	3.1
$13 l_s$	217.1					1.4	2.8
$14 l_s$	233.8					1.1	2.5
$15 l_s$	250.5					0.1	2.3
$16 l_s$	267.2						2.1
$17 l_s$	283.9						1.9
$18 l_s$	300.6						1.7
$19 l_s$	317.3						1.5
$20 l_s$	334						1.4

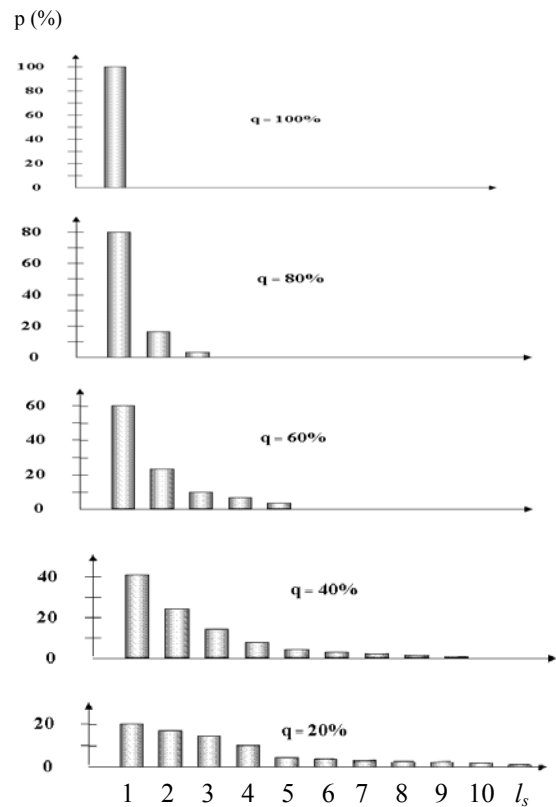


Fig. 2. Frequency  $p$  (in %) of intervals between the seedlings for different germinating power  $q$  of the seeds in the field.

$l_s$  - the interval between the sowed seeds.

The expected interval frequency between the plants  $p(l_s)$  corresponding to the interval between the seeds  $l_s$  is dependent on the interval frequency on these areas:

$$p(l_s) = (c_1 q_1 + c_2 q_2 + \dots + c_m q_m), \tag{5}$$

where  $q_1, q_2 \dots q_m$  - the germinating power of seeds on the corresponding area of the field;

$c_1, c_2 \dots c_m$  - the ratio of the area (spot) in the total area of the field.

The frequency corresponding to a double distance  $2 l_s$ :

$$p(2l_s) = [c_1 q_1(1-q_1) + c_2 q_2(1-q_2) + \dots + c_m q_m(1-q_m)] \quad (6)$$

A series of formulae is obtained in a similar way for the calculation of frequencies for the intervals that are three, four and more times larger.

### Yields.

The plant distribution density and its non-uniformity affect the expected yield of the crops (beets) (Figure 3).

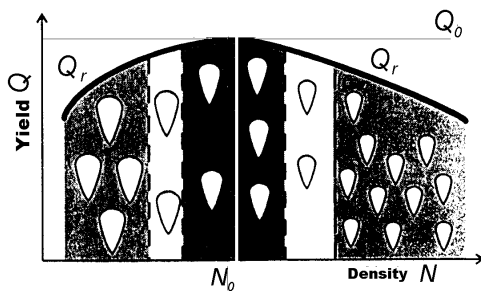


Fig.3. Variations in the crop productivity  $Q$  depending on plants distribution density  $N$ .

- desired density
- acceptable density
- insufficient density

A formula is obtained for the estimation of yields  $Q$  that are dependent on the plant distribution density  $N$ :

$$Q = Q_0 e^{-c(N_0 - N)^2} \quad (7)$$

and the fall in the yield  $Q_r$  is determined by the deviation from the optimum density  $N_0$  and the corresponding yield  $Q_0$ :

$$Q_r = Q_0 [1 - e^{-c(N_0 - N)^2}] \quad (8)$$

where  $c$  – a coefficient that characterises the rate of the yield decrease due the deviation of the plant distribution density from their optimum. The value of coefficient  $c$  depends mainly on soil fertility.

For sugar beet:

$N_0 = 80\ 000 \dots 90\ 000$  plants at the harvesting time;

$c = 2 \cdot 10^{-10} \dots 3 \cdot 10^{-10}$  at the density of 40...80 thousand plants per hectare;

$c = 1.1 \cdot 10^{-10} \dots 1.4 \cdot 10^{-10}$  at the density of 80...120 thousand

In determining the productivity of plants when their density varies from one area of the field to another, one cannot be guided by the average density indices. Their productivity should be evaluated for each area of the field having a particular plant distribution density and the calculated average crop productivity  $Q_{av}$ :

$$Q_{av} = \sum c_i Q_i \quad (9)$$

where:  $Q_i$  - the yield of crops on an area of the field;  
 $c_i$  - the ratio of the area (spot) in the total area of the field.

### Data of experimental studies.

As an example, the germinating power of sugar beet, their spacing, beet and sugar yields of more than 20 various sorts were studied for nine years. The results show that, when the seeds are sown at extreme distances (16-20cm) to avoid plant thinning, the field germination by years, as well as of separate sorts or seed batches vary within a very wide range. The average field germination is 40-70% but under bad conditions it fell to 20%, while under favourable conditions it reached 85% and, in individual cases, even 95%.

At the corresponding germinating power the plant interval distribution is, on the average, adequate to that calculated according to formulas (4). The lower is the seed germinating power, the less uniform is the plant spacing (the frequency of the longer intervals is greater). For instance, when the field germination is 20%, some plant intervals may surpass 2 metres.

Increasing the sowing rates does not improve the uniformity of plant spacing. It only decreases the frequency of the long intervals with a simultaneous increase in the frequency of the short intervals.

The fall in the sugar beet yield at low seed germination in the field is caused not only by a decreased plant distribution density but also by their lesser uniformity and weeds on the area not covered by the plants.

### CONCLUSIONS

1. By modelling of seed germination process mathematical coherences are obtained for the determination of plant spacing density, their distribution as well as yields. They enable to prognosticate plant spacing irregularity in the rows depending on the expected seed germinating power and to specify their sowing ratios.
2. Plant spacing frequency is a function of the seed germinating power in the field. The lower is the germinating power of seeds, the lower is the regularity of sprouts, i.e., the number of the longer intervals increases.
3. The desired plant distribution density and yield is ensured by quality soil tillage, precise sowing and embedding of high-quality seeds, good plant protection from pests and diseases.

### REFERENCES

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## AUTHOR BIOGRAPHIES



**ARVIDS A. VILDE** was born in 1929 in Bauska, Latvia, in a farmer's family. After finishing a secondary school he entered the Latvia Academy of Agriculture, where he studied Agricultural Machinery and obtained the degree of an engineer in 1954. Subsequently he obtained the following degrees: Candidate of Technical Sciences (SU) in 1965, Doctor of Technical Sciences (SU) in 1986, Doctor Habil. Sc. Ing. in 1992. He worked for 5 years as the chief engineer in the field of agricultural machinery and production. Since 1960 up to this time he is engaged in the leading research groups in Latvia Research Institute of Agricultural Machinery. The research branches: soil dynamics in tillage, including the dynamic properties of soil; technology, machines and units for soil tillage and sugar beet growing, their rationalization and perfection; the use of big high-speed tractors and machines; energy requirements for field crop production and ways of its reduction; normative requirements for tractors and machines, their working load and fuel consumption. Now he is leading the research in Precision Agriculture using the GPS and in the plant spacing simulation by sowing seeds at exact intervals, as well as in the simulation of forces acting on the plough body in order to determine its draft resistance and optimal parameters. A. Vilde has received several medals and diplomas at the Exhibition of Economic Achievement in Moscow. In 1985 he received the Latvian State Prize. He was named a Merited Inventor of Latvia in 1990, International Man of the Year for 2000-01 and Latvia State Emeritus Scientist 2001. He is an expert of promotion councils and a publicist who has written more than 700 publications including eighteen monographs. He enjoys orchards and stenography. With his late wife Velta, he has four children and eleven grandchildren. His e-mail address is [vilde@delfi.lv](mailto:vilde@delfi.lv)



**AIVARS H. CESNIEKS** was born in 1938 in Jelgava, Latvia in a farmer's family. After finishing a secondary school he entered the Latvia Academy of Agriculture, where he studied Agricultural Machinery and obtained the degree of an engineer in 1966. Subsequently he obtained the following degrees: Candidate of Technical Science (SU) in 1983, Doctor Sc. Ing. in 1992. He worked for 3 years as an engineer-technologist in repair shop. From 1969 till 2002 he has been working in Latvia Research Institute of Agricultural Machinery as a leading researcher and department leader. Now he is a farmer in Dobeles District, simultaneous taking part in the research activities of the Institute. The research branches: soil tillage technology, machines and units, their rationalization and perfection; the use of big high-speed tractors and machines; technologies and machinery for field crop growing, grain drying and conditioning, energy requirements for the field crop production and ways of its reduction, simulation plant spacing by sowing seeds at exact intervals. A Cesnieks has received medals and diplomas at the Exhibition of Economic Achievement in Moscow. In 1985 he received the Latvian State Prize. He has also written numerous publications. He enjoys bee-keeping and justice. With his wife Ilga he has two sons and two grandchildren.