SIMULATION-BASED STATISTICAL ANALYSIS OF THE BULLWHIP EFFECT IN SUPPLY CHAINS

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ABSTRACT

The paper proposes both statistical and simulation-based analysis and evaluation of the bullwhip effect in supply chains. The demand distortion, called the bullwhip effect, is considered as an important characteristic of supply chain operation stability. A mathematical justification of the stochastic demand as a cause of the bullwhip effect is discussed. Results of simulation studies to analyse the impact of information sharing strategies on the magnification of demand fluctuations as orders move up the supply chain are presented. An approach to measuring the bullwhip effect for the entire supply chain is proposed and practically applied for comparison of different supply chain's configurations.

INTRODUCTION

The bullwhip effect that describes the increase in variability of a demand through the entire supply chain is used, as a supply chain operation stability measurement. The fluctuation of orders across the supply chain is mainly caused by the uncertainty inherent in the system operation environment, such as customers demand and lead times. This means that even small disturbances in demand at the customer level causes the demand amplification for the next supply chain member. It is important to investigate the nature of this effect to avoid holding an excessive inventory, insufficient capacities and high transportation costs. The bullwhip effect characterises the behaviour of the entire supply chain and is used to measure operation effectiveness of the system (e.g., asset/inventory, financial metrics), but not service metrics (e.g. customer response to service, required service level).

The research is aimed to justify the increase in variability in placed orders under the stochastic customer demand using standard statistical tools as well as simulation technique and to propose a measure of the bullwhip effect magnitude for the entire supply chain.

BACKGROUND

Banks and Malave (1984) identify inventory control problems as one of the most frequent area of application for simulation methodology. They propose six categories of simulation techniques usage assignments in modelling and analysing inventory systems: (1) analytic solution impossible or analytic solution extremely complex, (2) comparison of models, (3) verification of analytic solutions, (4) variance reduction, (5) model validation and verification, and (6) optimisation. A lot of simulation studies are performed in order to handle uncertainty inherent in system operation environment. For instance, simulation is used to enhance operational system decision making in an uncertain environment (Petrovic et al., 1998). Chandra et al. (2001) investigate information coordination influence on the demand forecast accuracy in a supply chain through simulation. Landeghem and Vanmaele (2001) study the behaviour of the supply chain under different sources of uncertainty using the Monte Carlo simulation approach.

In this paper simulation is used to analyse multi-stage, single-item, multi-period inventory system, called a supply chain, from the operation stability point of view. The bullwhip effect that describes the magnification of demand fluctuations as orders move up the supply chain is used as a supply chain operation stability measurement. In this research a background of the bullwhip effect occurrence as a result of the stochastic nature of the customer demand is considered. Simchi-Levi et al. (2000) explains that the increase in demand variability with the necessity for each supply chain stage makes orders based on the forecasted demand of the previous stage. Since variability in placed orders is significantly higher than variability in customer demand the supply chain stage is forced to carry more safety stock in order to meet the same service level. Proposed quantifying the magnitude of increase in variability between two neighbour supply chain stages is expressed as a function of a lead time between the orders receiving and a number of demand observation on which forecast is made:

$$\frac{Var(Q)}{Var(D)} \geq 1 + \frac{2L}{p} + \frac{2L^2}{p^2},$$

where

Var(Q) – the variance of the orders placed by the supply chain stage;

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$Var(D)$ – the variance of the demand seen by this supply chain stage;
L – lead time between the orders receiving;
p – number of observation on which further demand forecast is based.

As a result the bullwhip effect is magnified with increasing the lead time and decreasing the observations number. Chen et al. (1998) describe the causes of the bullwhip effect and show the relationship between the increase in variability and forecasting techniques. Metters (1997) concludes that the bullwhip effect affects many businesses in supply chains across a variety of industries. A method of optimal inventory policy calculation in case of stochastic and seasonal demand is proposed and found by a dynamic programming. To aid in the justification of determining causes and remedies, expressing the significance of the bullwhip effect in monetary terms is described. Lee and Padmanabhan (1997) identify ways to control and counteract the bullwhip effect using different information and management technologies, such as electronic data interchange (EDI), computer-assisted ordering (CAO), vendor-managed inventory (VMI), point-of-sale information (POS), etc. The main conclusion of the research is that methods for coping with the bullwhip effect can significantly reduce, but not eliminate it.

In this paper a statistical justification of the bullwhip effect occurrence in the inventory systems with stochastic demand is given. The proposed method allows statistically justify the increase in placed orders variability for the different types of inventory control policies. Simulation offers possibility to model and analyse system dynamic processes under a stochastic demand. Using simulation technique, the bullwhip effect characterised metrics (standard deviation of placed orders for each supply chain stage) is obtained. Experimental studies are performed with the simulation models, and a measure of the bullwhip effect for the entire supply chain is proposed to compare different supply chain's configurations.

In the next section, implementation of standard statistical tools to justify the bullwhip effect occurrence in the inventory systems is described. In further sections the four-stage inventory system is simulated under two information sharing strategies (centralised and desterilised), experimental results are described and an overall bullwhip effect measure for the entire supply chain is proposed. Conclusions follow in the last section.

STATISTICAL ANALYSIS OF THE BULLWHIP EFFECT IN INVENTORY SYSTEMS

Regular or cyclical in nature inventories with additional safety stock are considered. These are the inventories necessary to meet the average demand during the time between successive replenishments and safety stock inventories are created as a hedge against the variability in demand for the inventory and in replenishment lead time. A method to control such inventories assume that the conditions of demand level, its variability and lead time are known and involves the following main steps:

1. find the current on-hand quantities at the stocking point;
2. establish the stock availability level at the stocking point after the demand satisfaction;
3. calculate total requirements that is the amount of cycle stock plus additional quantities needed to cover the uncertainty in demand;
4. determine an order quantity as the difference between the total requirements and the quantity on hand in case if the on-hand inventory drops below the allowed level when a replacement order should be placed.

The graphical representation of the above mentioned inventory control method is depicted in Figure 1.

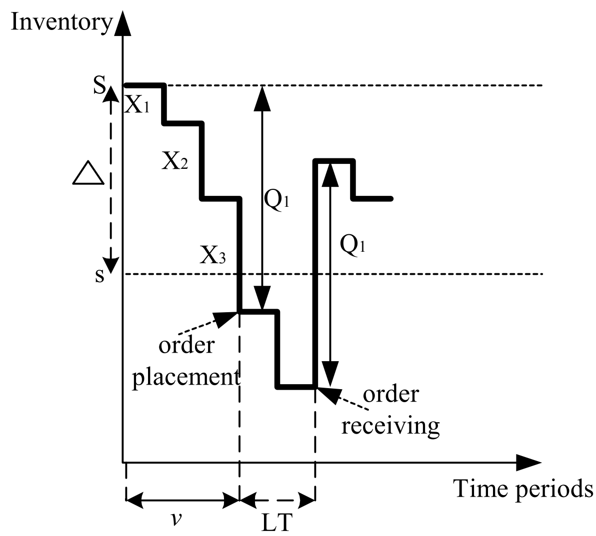

Figure 1: Inventory Control Method

It is assumed that the demand $X_1, X_2, \ldots, X_i$ is a discrete random sample observed from some population. Equivalently, these data are independent and identically distributed (IID) observations on some underlying random variable X whose distribution governs the population. Values that numerically characterise the population/distribution, such as an expected value $E(X)$ and a variance $D(X)$ of the discrete random variable X are given.

The inventory level to which inventory is allowed to drop before a replacement order is placed (reorder point level) is found by a formula:

$$s = E(X) * LT + STD(X) * \sqrt{LT} * z \, , \qquad (1)$$

where
LT – constant lead time between replenishments;

$STD(X) = \sqrt{D(X)}$ - standard deviation of the mean demand;
z - the safety stock factor, based on a defined in-stock probability during the lead time.

The total requirements for the stock amount or goal stock level S is calculated as a sum of the reorder point level and a demand during the lead time quantity:

$$S = s + E(X) * LT \quad (2)$$

The order quantity Q_i is demanded when the on-hand inventory drops below the reorder point is equal to the sum of the demand quantities between the order placements:

$$Q_i = X_1 + X_i + ... + X_v, \quad (3)$$

where
v – random variable, a number of period in which order is placed.

While the demand X is uncertain and implementing such a type of inventory control method, placed order quantity Q is expected to be a random variable that depends on the demand quantities. An expected value $E(Q)$ and a variance $D(Q)$ of the function $Q = \varphi(X)$ are estimated using the following formulas proposed by Feller (1967):

$$E(Q) = E(X) * E(v) \quad (4)$$

and

$$D(Q) = E(v) * D(X) + D(v) * [E(X)]^2, \quad (5)$$

where
$E(v)$ – expected value of a time period in which an order is placed;
$D(v)$ – variance of a time period in which an order is placed.

In described above inventory control method order placement frequency mainly depends on the replenishment lead time and the variance of the demand, since the organisation of the regular inventory stock should cover the demand during the lead time. In suchlike systems the order placement occurs approximately once in lead time period or even infrequently, therefore $E(v) \geq 1$. In case orders are placed once in each time period the variance of placed orders will be equal, but not smaller, than the variance of the demand. However, considering a stochastic nature of the demand, the variation of the order placement frequency depends on the demand variance, the smaller is this variance the more stable is the order placement process. That's why discrete random variable v that determines a period in which an order is placed could not be smaller than one and more likely will be equal to the lead time or even bigger. So far we can conclude that the statistical estimation of the placed orders variance justify that the variability of placed order $D(Q)$ will be bigger than the variability of demand $D(X)$.
To investigate a probabilistic behaviour of the discrete random variable v is enough to estimate its numerical characteristics (an expected value and its variance). The difference between the goal stock S and reorder point s levels should be established to find a time period when an order should be placed:

$$\Delta = S - s \quad (6)$$

Multi-experimental realisation of the following algorithm:

if $X_1 > \Delta$ THEN $v=1$ AND STOP
ELSE generate X_2
if $X_1 < \Delta$ and $X_1 + X_2 > \Delta$ THEN $v=2$ AND STOP
ELSE generate X_3
...
if $X_1 + X_2 + ... + X_{n-1} < \Delta$ and $X_1 + X_2 + ... + Xn > \Delta$ THEN $v=n$
STOP

allows to collect statistics of v values (v_i, $i = \overline{1,n}$) and evaluate their probabilities p_i by relative frequencies $\hat{p}_i$ of their occurrences in performed experiments.

Expected value of random variable is the weighted average of all possible values of the random variable, where the weights are the probabilities of the values occurring. The expected value $E(v)$ of the v value population is estimated by a formula:

$$\hat{E}(v) = \sum_{i=1}^{n} v_i * \hat{p}_i \quad (7)$$

and its variance $D(v)$ is estimated by a formula:

$$\hat{D}(v) = \sum_{i=1}^{n} v_i^2 * \hat{p}_i - \hat{E}(v)^2, \quad (8)$$

where
$\hat{E}(v)$ and $\hat{D}(v)$ - experimental estimation of $E(v)$ and $D(v)$ correspondingly.

Described statistical analysis could be implemented in inventory systems that control inventories by the two major policies – reorder point and periodic review methods, and allows to justify demand fluctuation magnification (the bullwhip effect) as orders move up the supply chain in case of stochastic demand.
In the periodic review method, a time period v in which an order is placed could be estimated with more certainty because an order could be placed only in the predefined review periods t. While in reorder point method an order placement could occur in any period.

The proposed statistical approach could also be used to evaluate the bullwhip effect numerically. For this aim a dependence between a number of period in which order is placed v and realizations of the end demand X_i should be taken into consideration. That is, proposed formula (5) assumes v and X independence, but in the described inventory control system they are dependent in the way of conditional probability of v occurrence $p_v=P(X_1+X_2+\ldots+X_v>S-s/X_1+X_2+\ldots+X_v-1<S-s)$. The direction of the dependence (positive or negative) should be studied as well.

SIMULATION MODEL OF FOUR-STAGE INVENTORY SYSTEM

Conceptual Model

A four-stage, single-item, multi-period supply chain is considered. The structure of the considered supply chain corresponds to the well known "beer distribution game" where a supply chain consisting of a beer retailer, wholesaler, distributor and factory is simulated (Simchi-Levi et al., 2000). The practical re-order point method – min-max is used for the inventory management. In the min-max inventory control method, a replenishment order will be placed as soon as the inventory level drops below the reorder point (1). The order size is the difference between a target level (2), and the effective inventory level. It is important to remark that replenishment triggering will be based on the effective inventory level, which is the quantity on hand plus the quantity on order minus the unshipped backorders to customers or the quantity allocated to production.

The amount of information that is passed on between stages will determine the information sharing strategy. Two types of the information sharing strategies are modelled: decentralised and centralised information.

In the supply chain with decentralised information, each stage forecasts a demand based on the orders it gets from the previous stage. Except for the first stage in the supply chain, there are no stages having access to the end customer demand data. In the second type of the supply chain, the first stage observes the end customer demand and shares all information about its size with the other stages in the supply chain. Other stages in the supply chain can use this end customer demand data to forecast the demand, instead of using the orders they get from the previous stage. All stages use the same forecasting technique – moving average of the demand during the last ten periods.

The inventory management objective is to manage stable operation of the considered supply chain, that is decrease the bullwhip effect. As alternative system configurations supply chain with different information sharing strategies are analysed (Figure 2).

Simulation Model

The system described above has an explicitly dynamic character and simulation is used to capture this behaviour of the system.

It is assumed that end customer demands arrive with fixed time-intervals and their size is variable and is derived from a normal distribution. A constant lead time between all stages is considered. No order processing delay is taken into account, so all demand events are treated immediately by the upstream stage. We also will assume no capacity constraints for the last stage of the supply chain. In this case, stockouts will not lead to lost sales, but to backorders. We thus assume that we have loyal customers.

The simulation model was developed using the ARENA 5.0 simulation modelling environment (Kelton et al., 2002). Evaluation of the inventory control parameters, as well as forecasting procedure was implemented using Visual Basic.

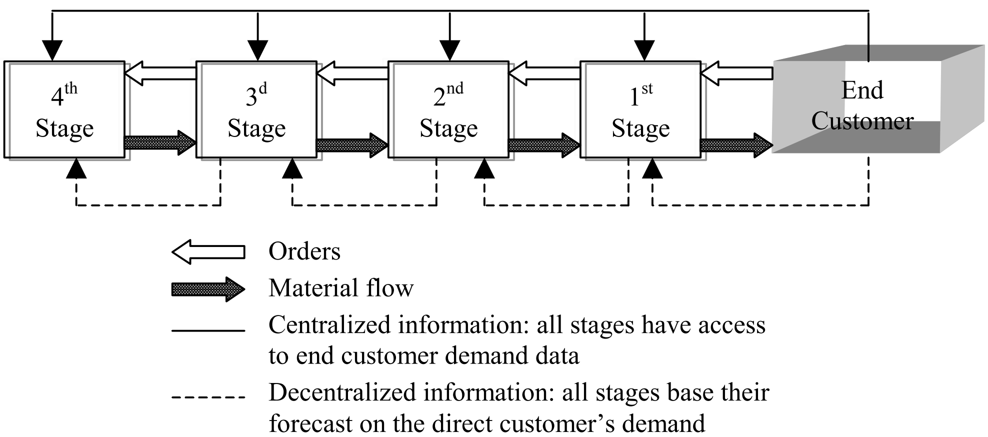

Figure 2: Conceptual Model

EXPERIMENTAL RESULTS

Objective of the experimental studies is to determine the demand variability that occurs at every stage of the supply chain. The variability in the supply chain can be measured taking into account demand of the previous supply chain stage and orders placed to the next stage of the supply chain. All supply chain stages recalculate and if necessary place an order in every period and this order, actually, is a demand for the next supply chain member. Since the demand changes every period, the mean and standard deviation can be calculated, on the basis of last p observations.

Performance of the supply chain is evaluated under various factors such as end customer mean demand *E(X)* and its standard deviation *STD(X)*, lead time *LT*, safety stock factor *z* and number of observation on which further demand forecast is based *p* (Table 3) implementing both decentralized and centralized information sharing strategies.

Table 3: Experimental Design

Factors	$E(X)$	$STD(X)$	z	LT	p
Values	100	30	1.96	2	10

After the determination of the warm-up period, models were run for three replications, each replication lasting for 1,000 periods. As a measure of performance for these experiments the standard deviation of demand at each supply chain stage is calculated and results are presented in Figure 3.

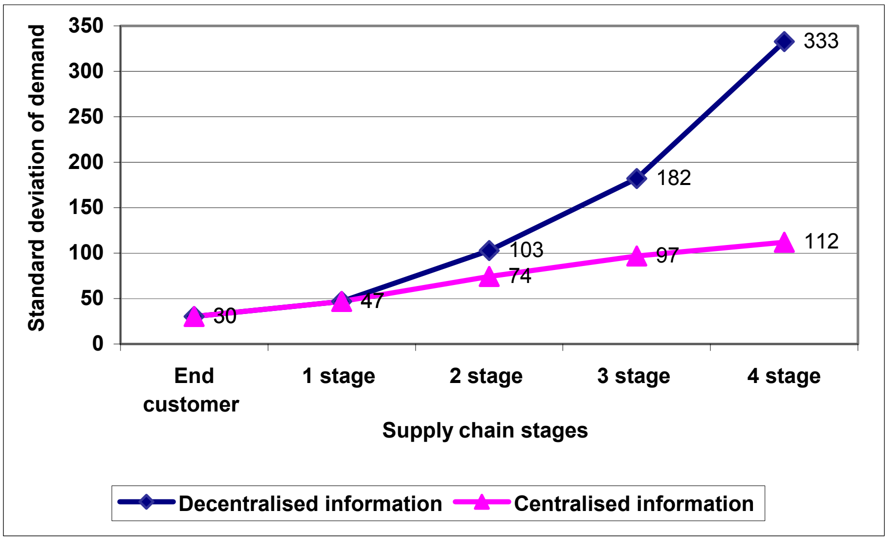

Figure 3: Standard Deviation at each Supply Chain Stage

Experimental results show that the bullwhip effect is present in both supply chain configurations. For the supply chain with centralised information sharing, the variation of placed orders is visibly smaller, but to give an indication of the seriousness of the bullwhip effect an overall bullwhip effect measure should be considered.

OVERALL BULLWHIP EFFECT MEASURE

The propagation of instability up the supply chain is proved by analytical study, the same is by simulation study. Therefore, it makes sense to quantify the bullwhip effect; that is to quantify the increase in demand variability that occurs at all stages of the supply chain. Bullwhip effect measure over the entire supply chain allows compare different system configurations from the stability point of view.

To identify the bullwhip occurrence at each stage of the supply chain it is proposed to compare a standard deviation of demand faced by the neighbour supply chain stages by calculating a ratio BE_i:

$$BEi = \frac{STD(Q_i)}{STD(Q_{i-1})} \in (0,\infty), i = \overline{1,n}: \tag{10}$$

- if $BEi > 1$ then the bullwhip effect exists;
- if $BEi \le 1$ then the bullwhip effect doesn't exist;

where

n – number of supply chain stages;

$STD(Q_i)$ - standard deviation of orders placed by stage i to its supplier;

$STD(Q_{i-1})$ - standard deviation of demand received by supply chain stage i.

In case of supply chain stages do not perform the same operation strategy bullwhip effect between some stages could be eliminated. Therefore to evaluate a magnitude of the demand variability increasing over the entire supply chain two approaches could be offered. The first approach is to calculate overall increasing in demand variability taking into consideration ratios BE_i of each supply chain stage in respect of bullwhip existence. The second approach proposes that only the operation performance measures (BE_i ratios) of stages where the bullwhip effect is present should be taken into account. In this paper the second approach will be considered in details.

To calculate the overall measure of the bullwhip effect for the entire supply chain the average increase in variability for each supply chain stage should be determined. There are two the most frequently used ways to manage it (1) to calculate a geometrical mean or (2) to calculate an arithmetical mean of all observed BE_i

ratios that correspond to increase in variability between stages.

Calculating the geometrical mean of the BE_i ratios by a formula (11) determines the existence of the bullwhip effect between first and last supply chain stages. Values of orders placed by stages $\overline{i,n-1}$ are the demand received by stages $\overline{i+1,n}$ and they are cancelled performing multiplication operation. As a result the increase in variability of demand between the first, i.e. end customer and the last supply chain stages is found:

$$\overline{BE_g} = \sqrt[n]{BE_1 * BE_2 * ... * BE_n} = \\ = \sqrt[n]{\frac{STD(Q_1)}{STD(Q_0)} * \frac{STD(Q_2)}{STD(Q_1)} * ... * \frac{STD(Q_n)}{STD(Q_{n-1})}} = \\ = \sqrt[n]{\frac{STD(Q_n)}{STD(Q_0)}}, \quad (11)$$

where

$STD(Q_0)$ – standard deviation of placed orders by the end customer;

$STD(Q_n)$ – standard deviation of placed orders by the last stage of the supply chain.

The bullwhip effect existence in intermediate stages of the supply chain is not taken into account.

The above described property could be eliminated by performing addition operation, so all BE_i ratios, which determine the existence of the bullwhip effect will be taken into account. Calculating an average increase in variability demand of all supply chain stages, i.e. arithmetical mean of the increase in variability between all stages could be found by a formula:

$$\overline{BE_a} = \frac{\sum_{i=1}^{n} BE_i}{n}, \text{ for } BE_i > 1 \quad (12)$$

The proposed bullwhip effect overall measure allows adequately determine the stability of the entire supply chain considering only the situations when variability of the demand increases. The proportional splitting of the increase in variability between all supply chain stages makes possible to analyse different supply chain structures and configurations.

The standard deviation of the demand at each supply chain stage for the both supply chain configuration is presented in Table 4.

Calculated values of the ratio BE_i (10) for all four supply chain stages identify that the increase in demand variability is present in all stages (Table 5). The measure of the bullwhip effect over the entire supply chain is calculated as proposed in (11) and (12).

The supply chain with centralised information sharing strategy is more stable, a destabilisation effect of increasing the volatility of demand as it passes through the chain is smaller than in the supply chain with the decentralised information. This result is derived from comparison of the both overall bullwhip effect measures $\overline{BE}_a$ and $\overline{BE}_g$.

The smaller is the value the less significant is the increase in variability as we travel up in the supply chain. Since there is a measure for the bullwhip effect over the entire supply chain the difference between the stability ($\overline{BE}_a$) for both supply chain alternatives could be expressed in percentages – variation of demand in the supply chain with centralised information is by 23% smaller than in the supply chain with decentralised information.

Table 4: Simulation Results

Supply chain alternatives	**Standard deviation of the**				
	End customer demand	**1 stage demand**	**2 stage demand**	**3 stage demand**	**4 stage demand**
Decentralised information	30	47	103	182	333
Centralised information	30	47	74	97	112

Table 5: Overall Bullwhip Effect Measure

Supply chain alternatives	BE_1	BE_2	BE_3	BE_4	$\overline{BE}_g$	$\overline{BE}_a$
Decentralised information	1.55	2.20	1.77	1.83	**1.82**	**1.84**
Centralised information	1.55	1.59	1.31	1.16	**1.39**	**1.40**

CONCLUSIONS

The statistical analysis of the demand distortion phenomenon in inventory systems with a stochastic demand has been performed. Performed statistical analysis allows analytical justification of the increase in variability between received demand and placed orders in inventory systems. Experimental studies show that the probability of the time period in which an order is placed is depended from the customer demand.

Analysis of the placed order variability for the all considered situations shows that even a small variation of the mean demand cause an increase in variability of the placed orders. The bigger is the initial value of the demand variation the more significant magnification of placed orders fluctuation will be observed.

The appropriate simulation model of single-item, four-stage, multi-period inventory system is developed to determine the demand variability that occurs at every stage of this supply chain. An impact of information sharing strategies on the bullwhip effect magnitude is evaluated through simulation.

To evaluate a magnitude of the demand variability increasing over the entire supply chain the bullwhip effect overall measures is proposed. The proposed bullwhip effect overall measures could be used to compare the operation stability of supply chains with different supply chain configurations, under different inventory control policies, etc.

Simulation studies indicate that a demand distortion as we travel up the supply chain with centralised information is less significant than in the supply chain with decentralised information, because of for a decentralised supply chain takes much longer time to react to the changing demand. However, different other factors should be taken into account as well for coping with the bullwhip effect, such as lead times, forecasting techniques, etc. in order to significantly reduce it in the supply chain.

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