FORMAL MODELLING OF SUPPLY CHAIN: AN INCREMENTAL APPROACH USING PETRI NETS

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ABSTRACT

In this paper the authors propose an implementation of an incremental approach to modelling discrete event systems at the structural level of systems specification. Using the well-known example of the Beer Game, a systematic method supporting the bottom-up construction of re-usable models of supply chains in the Petri nets domain together with their associated experimental frames is presented. The construction adheres to well-defined rules, which would enable computer-based model generation.

1. INTRODUCTION

The concepts of Supply Chain Management (SCM) and its extension Demand Chain Management have been at the center of much recent research. The increasing interest in this area has led to the development of various models and tools aimed at supporting the design and analysis of supply chains (SCs). However, as remarked in [SKS00], "the first generation of this technology was not robust and flexible enough to allow industry to use it effectively". Although analysis and insight have improved in the last few years, we still feel that industry is not familiar with the recent developed models and decision-support systems.
Looking for an appropriate tool for the representation and analysis of the SC, Petri nets (PN) seem to be very attractive because they provide a suitable framework for representing reasoning on concurrent active objects which share resources and their changing states. Due to their graphical nature and easy-to-validate specifications by analyzing the net structure, PN have proved helpful in rendering modelling simple and readable for complex problems. Moreover, as noticed by Valette [Val97], the above working paradigm is not far from the industrial engineers' habitual notions of systems design and operation.
This paper demonstrates the application of a systematic method for developing PN-based simulation models to the SC. Several ideas of the authors' previous developments addressing the modular synthesis of PN-based simulation models of complex discrete event systems (DES) are combined with new concepts. The goal is to develop techniques allowing for a natural, simple and powerful method for describing and analysing the flow of information and material in SCs in terms of PN. The research considered in this presentation specifically addresses issues related to the representation of the co-ordination mechanism of a SC.
The application of the proposed approach is explained using the well-known example of the Beer Game [Ste89]. Experiments with the resulting structures are beyond the scope of this paper.
The paper is organized as follows: The motivation of the authors' research is made in **Section 2**. **Section 3** revisits the authors' previous work [KAB98], [BK99], [KB99], [Bob00], [KB01], [BK01] and integrates into a new setting results addressing a systematic method supporting the modular synthesis of PN-based simulation models of DES with application to the SCs. The characteristics and results of the proposed approach are illustrated in **Section 4** using the Beer Game.

2. SYSTEMS CO-ORDINATION - A KEY ISSUE IN SUPPLY CHAIN MANAGEMENT

The Supply Chain concept [Chr92] brings forth the value of integration to business partners, which applies both to the material flow (from raw material supplier to finished product delivery) and to the information flow (from the market back to the SC partners). Therefore, trying to find the best set of trade-offs involved by the management of its tightly coupled component systems (including various manufacturing, storage, transportation, and retail systems) is not sufficient. We need to consider the entire system and co-ordinate decisions at each stage of the SC.
In this paper a **disciplined and systematic approach to the modelling and simulation of the SC** is pleaded for. It is the authors' belief that the above method holds the promise of better linkage to the applications context, greater control of the modelling and simulation development process, greater reuse of previous development results and longer model application life [Van98].

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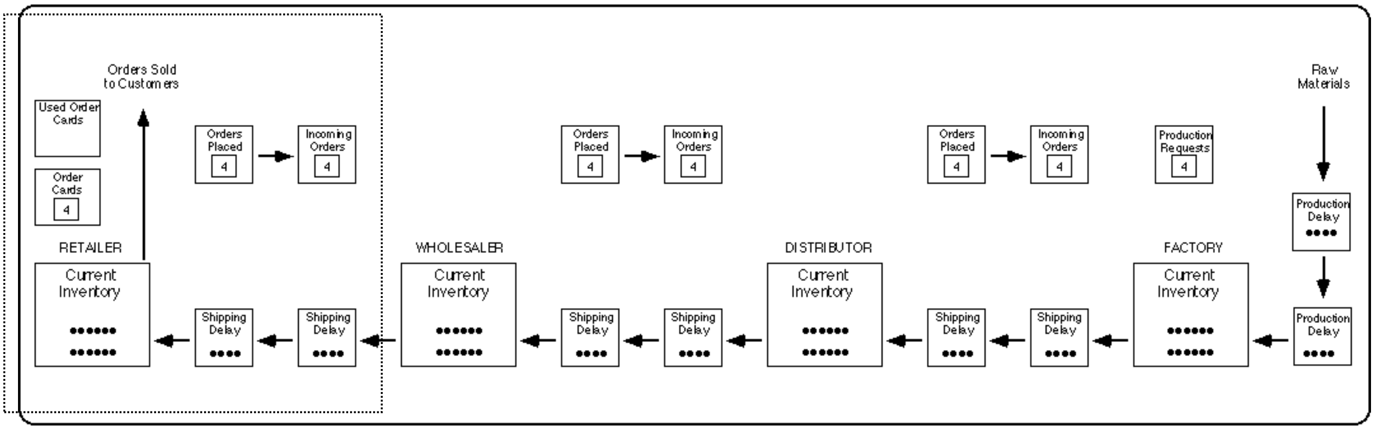

Figure 1. The Beer Game Board with Initial Conditions [Ste92]

3. THE SYSTEMATIC CONSTRUCTION OF MULTICOMPONENT PN-MODELS OF THE SUPPLY CHAIN

We reconsider the systematic construction of compound PN-models and adjoint experimental frames (EFs) proposed in the authors' previous work [KAB98], [BK99], [KB99], [Bob00], [KB01], [BK01]. We insist on the main characteristic induced by the addressed research:

The authors' endeavour is towards perfecting the design of re-usable Petri nets-based models by handling building block components with a well-defined interface, and using coupling templates as standardised means to link them.

When starting with the key structures and techniques of the suggested incremental approach for the representation and analysis of the flow of entities in real-world systems, further refinements have been introduced in order to anticipate their application to the SC.
One of the new ideas considered in this paper is that the development of a well-grounded set of coupling rules forces one to partition the entity flows involved in a SC into **three** distinct **types**:

1. orders placed by the customer (denoted in the sequel as *Ea*);
2. internal orders (denoted as *Eb*);
3. products (denoted as *Ec*).

The formal developments proposed at this stage as a result of this refinement broaden the applicability of the authors' contribution to also address the interfacing questions of different flows of entities. In this perspective the following rules are introduced:

I. The **internal coupling templates** of two **sequential models** *A* and *B* will allow a fusion to be applied to a pair of transitions (T_o^a, T_i^b), with T_o^a the output transition of the model A and T_i^b the input transition of the model B, provided the above transitions are enabled for the same type of entities.

II. A **representative system of the input/ output transitions sets** of a coupled model to be used by an internal coupling, will be structured according to the types of entities (or classes identified within the scope of a given type) enabling the addressed input/ output ports.

This obviously asks for corresponding modifications of the rules underlying the handling of **routing constructs** (RCs) identified in a real system each time when a sequential routing is addressed or a representative system of a transitions set should be defined. The current approach integrates into a new setting the rules applied for six (sequence, waterfall, conflict, accumulation, parallelism and synchronization) of the seven RCs identified in the authors' previous work.
On the abstract level, starting with the components of an EF proposed in [BK98] we further refine the key specifications provided by the transducer component as follows:

- *current marking*, which will further allow dynamics to be expressed by the "token game";
- *history of marking evolution*, supporting penalty costs and holding costs assessment;
- *priorities* associated on the input places of transition components, enabling the implementation of the **tie-breaking selector,** which selects one event (transition) to process (fire) out of a set of contending simultaneous events.

4. THE SYNTHESIS OF PETRI NETS-BASED MODELS IN A WELL-KNOWN EXAMPLE: THE BEER GAME

The characteristics and results of the proposed approach are illustrated in this Section using the Beer Game (Figure 1). The Beer Game is an interesting exercise to formulate a replenishment strategy, which enables the player to explore a variety of SCM concepts (e.g., the value of the information sharing, the impact of long and short lead times, the difference between centralized and decentralized decision making on SC performance).
The above-mentioned "management flight simulator" [Ste92] models the material and information flows in a production-distribution channel serving a stationary market where the customer demands in different periods are independent and stochastic. The linear distribution chain addressed by the game consists of four echelons: a retailer, a wholesaler, a distributor, and a factory. They share a

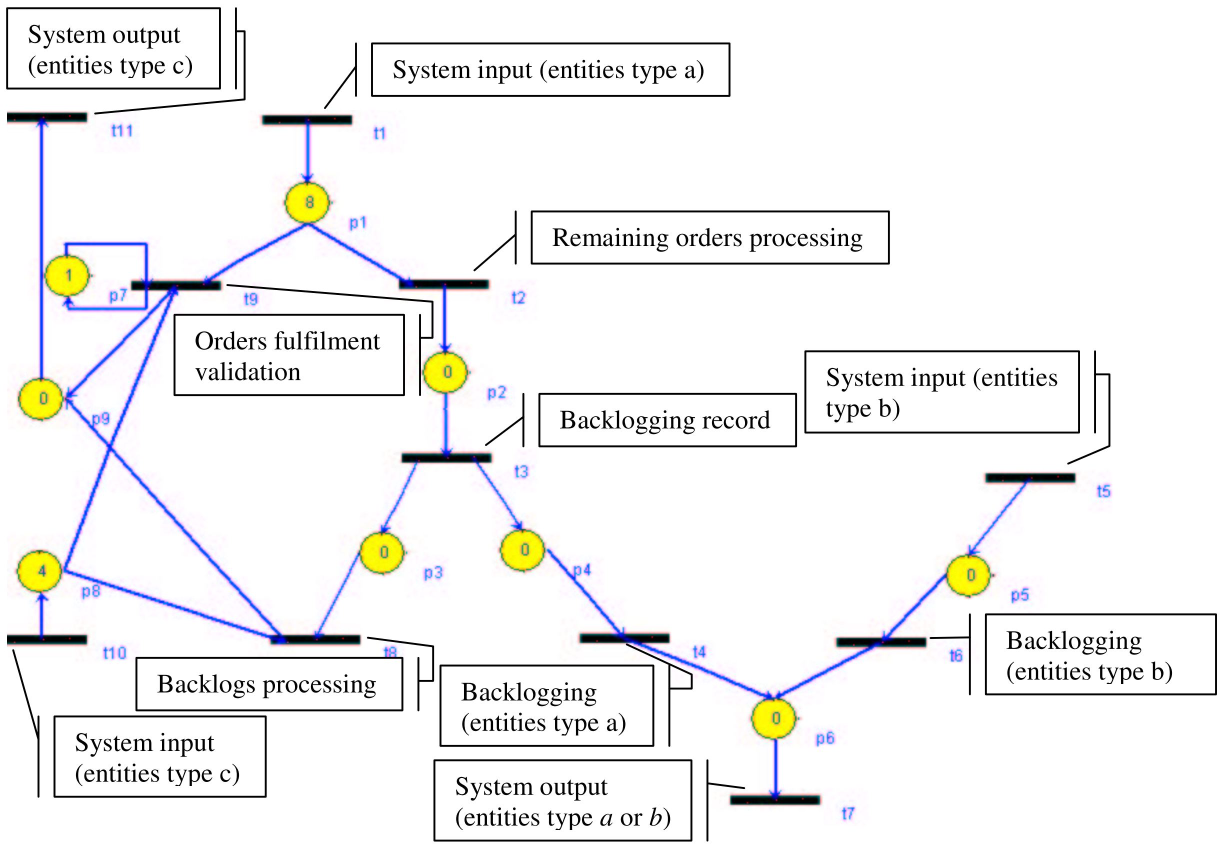

Figure 2. Figure 2. Petri nets-based model of the retail store

common objective to optimize system-wide performance. While material flows from upstream to downstream, information flows in the opposite direction through order placements. At each stage there are shipping delays and order delays.

We emphasize on the advantages of using PN for getting a better insight into the mechanism of the replenishment procedure. The intuitively appealing graphical representation of the PN model helps the user to better appreciate the multiple feedback loops and nonlinearities in the SC (as revealed by Figure 2 in comparison with the marked area of Figure 1).

We illustrate in the sequel how the proposed systematic step-by-step approach should apply to modelling the Beer Game in the PN domain together with its EF.

One of the key remarks is the similar structure as well as the similar principles underlying the behaviour of each echelon of the Beer Game. As this will of course result in similar constructions of the models associated to each component of the SC, the presentation will only refer to the PN-based model of the retail store (Figure 2).

The modelling of the retail store implies the following step-by-step procedure:

Step 1. 1a) Nine **basic system components** (BSCs) are revealed by the analysis of the retailer component:

- Entity *Ex* arrival, with $x \in \{a,b\}$;
- Orders fulfilment;
- Remaining orders processing;
- Backlogs record;
- Backlogs accumulation;
- Local replenishment decision process;
- Storing entity *Ec* in the local stock;
- Orders shipping.

Their corresponding **basic model components** (BMCs) are further represented using the refined PN models proposed by the authors. In order to illustrate the result of applying the above step, we refer to, for instance, the BSC representing the “Remaining orders processing”. The PN representation of this BMC is represented by the place *p2*, the transitions *t2* and *t3* (with *t2* the input transition of the BMC and *t3* the output transition), and the arcs (*t2*, *p2*) and (*p2*, *t3*) in Figure 2. Similarly, we can further model all the above-identified BSCs.

1b) Then the construction moves on to the abstract level and relates an **experimental frame** (EF) to every BMC constructed at this Step. In order to illustrate the results of applying this procedure, we extend, for instance, the above-constructed BMC representing “Remaining orders processing”. We relate an empty structure as the generator, G^2, defining arrival processes of entities of type *a*, to the input transition *t2*. An empty structure is assigned as a transducer, T^2, supporting statistics gathering as well as a tautology as an acceptor A^2, to the production place *p2*.

Step 2. 2a) Five system’s **routing constructs** (RCs) are then identified following the flow of entities in the retailer system. They are listed in Table 1 together with their associated priorities according to logical inter-relationships. In the sequel we will explain how to handle the above-identified RCs and their associated PN model components resulting in new model components. As recommended by our method, we may start with the construction of the compound model (CM) associated to “Backlogs record and

Table 1. RCs involved in the entities flow at the retail store

ROUTING CONSTRUCT	THE REAL-SYSTEM COMPONENT ADDRESSED BY THE ENVISAGED ROUTING CONSTRUCT	PRIORITY
Sequence	Orders fulfilment activation process.	1
Conflict	Entity *Ea* arrival followed by orders fulfilment and remaining orders processing.	3
Accumulation	Backlogs record and entity *Eb* arrival, followed by local replenishment decision process.	1
Parallelism	Remaining orders processing followed by backlogs accumulation and backlogs record executed in parallel.	2
Synchronization	The conflict induced by orders fulfilment and backlogs processing, and storing entity *Ec* in the local stock, followed by orders shipping.	4

entity *Eb* arrival, followed by local replenishment decision process" (priority 1), and follow the construction of the overall model according to the proposed method, and the priorities indicated in Table 1.

Following the flow between the underlying BSCs revealed by the analysis of "Backlogs record and entity *Eb* arrival, followed by local replenishment decision process", it is easy to see that the case "accumulation" applies to the BMCs addressed by this RC. The PN-based representation of the corresponding CM is represented by the places *p4*, *p5* and *p6*, the transitions *t3* (with *t3* the input transition of the CM, supplying entities of type *a*), *t4*, *t5* (with *t5* the input transition of the CM, supplying entities of type *b*), *t6*, *t7* (with *t7* the output transition of the CM, evacuating entities of type *a* or *b*), and the arcs (*t3*, *p4*), (*p4*, *t4*), (*t4*, *p6*), (*t5*, *p5*), (*p5*, *t6*), (*t6*, *p6*) and (*p6*, *t7*) in Figure 2.

A special note should be made on the implementation of the formal developments presented in Section 3:

I. The refined internal coupling rule of two sequential models is used in handling the RC "Remaining orders processing followed by backlogs accumulation and backlogs record executed in parallel" (which normally should have asked for a fusion to be applied to the pair of transitions (*t3*, *t5*)).

II. A structured representative system of the output transitions set of a coupled model to be used by an internal coupling, is illustrated by the pair of transitions (*t8*, *t9*), used in the representation of the RC "The conflict induced by orders fulfilment and backlogs processing, and storing entity *Ec* in the local stock, followed by orders shipping". This is induced by the classes identified within the scope of the entity type *a*: fulfilled orders and backorders.

2b) Following each iteration undertaken at the structural level, the construction moves on to the abstract level. For each model component generated in this Step, an overall EF is assembled from the EFs related to the model components underlying the RC concerned, according to the rules proposed by the authors ([KB99], [KB01]). For example, the coupled model associated with "Backlogs record and entity *Eb* arrival, followed by local replenishment decision process" accommodates an EF with a generator, $\{G^3, G^5\}$, defining arrival processes of entities of type *a*, to the input transition *t3* and of entities of type *b*, to the input transition *t5*. An empty structure is the result of the modular synthesis of the transducer component, and a tautology makes the acceptor component of the above EF.

The overall system description in terms of PN achieved by applying the proposed method is shown in Figure 2. The interpretation of places in Figure 2 is provided in Table 2.

It is worth to notice that the design of the overall model supports re-usability through self-containedness with input/output ports.

Table 2. Interpretation of Places in Figure 2

Notation	Interpretation
p1	Clients' orders counter
p2	Remaining orders counter
p3	Backlogs accumulator
p4	(Recent) orders to be placed upstream
p5	Local orders to be placed upstream
p6	Accumulated orders to be placed upstream
p7	Orders fulfilment activation precondition
p8	Inventory
p9	Products to be delivered

The overall CM accommodates the EF depicted in Figure 3, where the arrows originating in the Generator component point to the input ports where the demand and service characteristics are specified. The dashed arrows originating in the Transducer component point to the places where the statistics used for the assessment of the value of the demand as well as the penalty costs and holding costs, are gathered.

Similarly, the models of the warehouse, the distribution center and the factory are generated. Finally, the overall model of the Beer Game is easily obtained by assembling the four above-mentioned PN model components.

Given a suitable simulation environment, the design and performance of simulation experiments using the above constructions should be straightforward.

5. CONCLUSIONS

In this paper a systematic approach of the structural modelling of SCs is proposed, with a final aim to encourage the use of simulation-based methods as a key factor which supports the representation and analysis of (complex) SCs. The concepts and the generic method supporting the bottom -up construction of the PN models of SCs can provide the

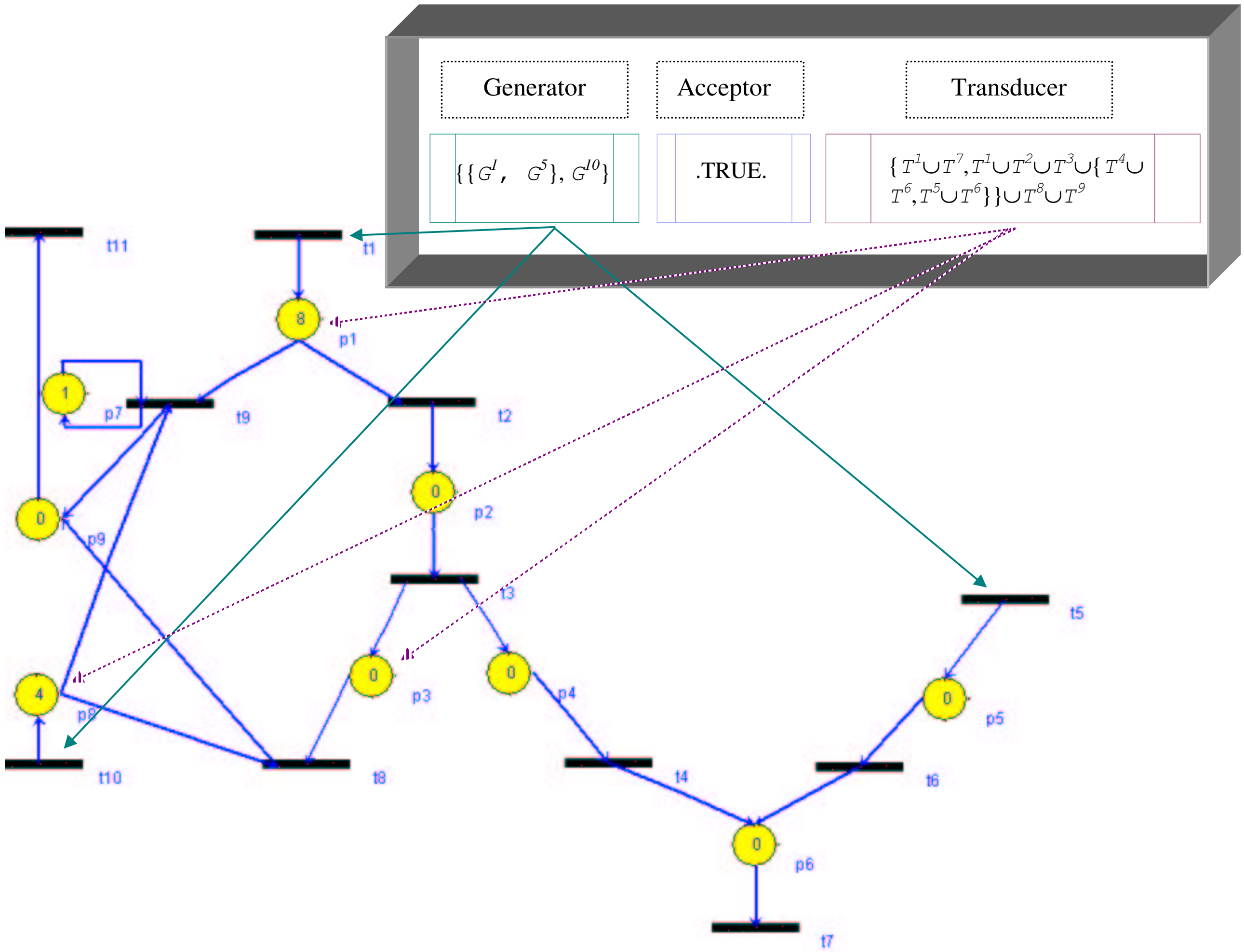

Figure 3. The EF applied to the PN model of Figure 2

basis for further automated generation of PN models. For instance, using the step-by-step approach and the well-defined coupling rules, an expert system could be developed supporting human modellers in the construction of (complex) PN-based models of SC.

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