STRUCTURAL IDENTIFIABILITY OF BOND GRAPH MODELS

B. Decotte*,, G.Dauphin-Tanguy***

benjamindecotte@hotmail.com, gdt@ec-lille.fr

* : LAIL, Ecole Centrale de Lille, BP48, 59651 Villeneuve d'Ascq ;

** : VIRTUAL DYNAMICS, 69 rue de Paris 94000 Orsay

ABSTRACT

The global identifiability is a structural property of models, which associates a unique set of parameters with given input/output response. The translation of this property into bond graph modelling language allows the combination of the physically meaningful language of bond graph methodology and the numerical accuracy of identified transfer function models.

Based on the building mechanisms of a transfer function from a bond graph model, the paper develops and explains why a bond graph can be not identifiable. Both internal and input/output dynamics can be written with the Mason's rule, using causal loops and action chains. Then the way the combination of causal loops and action chains influences the identifiability of models is discussed.

As a result a criterion is given, which decides whether a bond graph model is structurally globally identifiable or not. This is a crucial issue in order to guarantee the reliability of identification processes.

KEY WORDS : bond graph, identifiability, identification

INTRODUCTION

Any engineer has once wondered what is the best numerical value of a specific parameter (most of the time it is about heat dissipative phenomena) for his model to suit the input/output data. The most common solution to this problem is the parameter identification, a priori whatever the kind of model.

Many identification methods are available, from the Strejc method to the neural network one. Each method deals with a specific model structure and with a specific optimization algorithm. But in every case the results may be not reliable if the model is structurally locally identifiable or not identifiable. This is the reason why there may exist several minima of an error function, which could explain the failure or unreliability of the optimization algorithms.

The method to test the structural identifiability of a model depends on the structure of the model. The choice of bond graphs to model physical systems is ground on several facts. Bond graphs are highly suitable to model physical systems. They belong to the class of structural models in opposition to transfer functions, which are behavioral models. Bond graphs allow the translation of human prior knowledge into a language understandable by computer in order to be simulated. Some structural properties of models have already been successfully applied on bond graphs, as the structural controllability and the structural observability [Sueur-91], this paper presents the structural identifiability of the bond graph element parameters.

We are going to establish a bond graph identifiability criterion in two phases. In the first one we discuss about the structural identifiability applied on transfer functions. In the second one the internal structure of bond graphs is developed in causal loops and action chains in order to apply to concepts of the first part.

1 STRUCTURAL IDENTIFIABILITY OF TRANSFER FUNCTIONS

Be $\boldsymbol{u}$ the input of a SISO model $\boldsymbol{M}$, $\boldsymbol{y}$ its output, v_p and v_p^* some sets of parameters associated with its structure. The structural identifiability of a model is generally defined according to the solutions of equation [1]:

$$M(v_p) = M(v_p^*) \qquad [1]$$

If equation [1] implies $v_p = v_p^*$, then [1] has a unique solution. The model is said structurally globally identifiable. If [1] has several solutions, but in finite number, the model is structurally locally identifiable around each solution in v_p. If the equation has uncountable number of solutions or no solution, the model is structurally not identifiable. [WALTER-87]

Equation [1] may be rewritten in various ways. It depends on the structure of the model. If the model is a transfer function, the coefficients of the polynomial numerator $\alpha_j(v_p)$, and denominator $\beta_j(v_p)$, of $M(v_p)$ and the coefficients of the polynomial numerator, $\alpha_j(v_p^*)$, and denominator, $\beta_j(v_p^*)$, of $M(v_p^*)$ can be equalized. Then [1] is equivalent to:

$$\begin{pmatrix} \alpha_j(v_p) \\ \vdots \\ \beta_j(v_p) \end{pmatrix} = \begin{pmatrix} \alpha_j(v_p^*) \\ \vdots \\ \beta_j(v_p^*) \end{pmatrix} \qquad [2]$$

Until now the structural identifiability test was done afterward. It was an attempt of solving a non-linear system of equations involving the parameters of the model. The aim of this paper is to substitute the need of solving any equation by an analysis of the construction of the transfer function.

2 TRANSFER FUNCTIONS FROM A BOND GRAPH

According to the Mason's rule, a transfer function can be deduced from every bond graph involving linear constitutive component laws, [MASON-53] and [BROWN-72]. A short recall on bond graph definitions is given in annexe. Let the transfer function be defined by:

$$\frac{Y}{U}(s) = \frac{\alpha_0 + \alpha_1 \frac{1}{s} + \cdots + \alpha_n \left(\frac{1}{s}\right)^n}{\beta_0 + \beta_1 \frac{1}{s} + \cdots + \beta_n \left(\frac{1}{s}\right)^n} \qquad [3]$$

The Mason's rule gives for the transfer function :

$$\frac{Y}{U}(s) = \frac{\sum_k T_k(s)\Delta_k(s)}{\Delta(s)} \qquad [4]$$

Δ is the bond graph determinant, calculated with $\boldsymbol{B_i}$ the gains of the causal loops as

$$\Delta = 1 - \sum_i B_i + \sum_{i,j} B_i B_j - \sum_{i,j,k} B_i B_j B_k + \cdots \qquad [5]$$

T_k is the gain of the k^{th} action chain. Δ_k is computed like Δ, the causal loops taken into account are the ones separated from the action chain T_k .

Then we study the application, which transforms the parameters into gains of causal loops and action chains, and the one, which transforms gains into transfer function coefficients.

3 STRUCTURAL IDENTIFIABILITY OF BOND GRAPH MODELS

The non-linear application that transforms the set of parameters into the set of transfer function coefficients has to be invertible in order to ensure the existence of solutions to equation [1]. This application can be analysed as a composition of two non-linear applications. The first one, $\boldsymbol{T_1}$, transforms the parameters into the set of causal loop and action chain gains; the second one, $\boldsymbol{T_2}$, transforms this set into numerator and denominator coefficients.

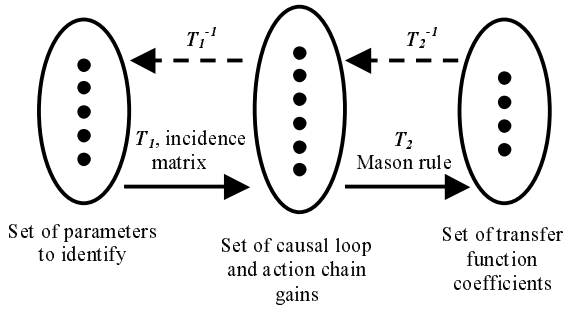

Figure 1: transformations T_1 and T_2

Transformation T_1, incidence matrix

We study the invertibility of $\boldsymbol{T_1}$ with a special incidence matrix.

Definition : Be a bond graph **BG** composed with $\mathbf{n_p}$ parameters to identify and $\mathbf{n_g}$ gains of causal paths and action chains. We introduce M_i, the incidence matrix of **BG**. M_i is a $\mathbf{n_g} \times \mathbf{n_p}$ matrix, where each row is associated with a gain of a causal loop or an action chain, and each column is associated with a parameter. The coefficients of the incidence matrix are the power of every parameter that appears in the gains, deprived of their sign and of their symbolic Laplace operator s.

This definition of incidence matrices is based on a specific space where internal and external composition laws exist. This justifies the use of the invertibility of the matrix to prove the bijectivity of the associated application. The inverse transformation T_1^{-1} is also directly deduced from the inverse of M_i.

Transformation T_2, partial solve of coefficients

To study the invertibility of T_2, we split the application T_2^{-1} into two transformations : PST_2^{-1} and CT_2^{-1}. These transformations are described on figure 2. The first one transforms the set of transfer function coefficients into PSC (Partially Solved Coefficients). Since PST_2^{-1} is injective, the property of the second transformation CT_2^{-1} is conclusive for the invertibility of T_2 .

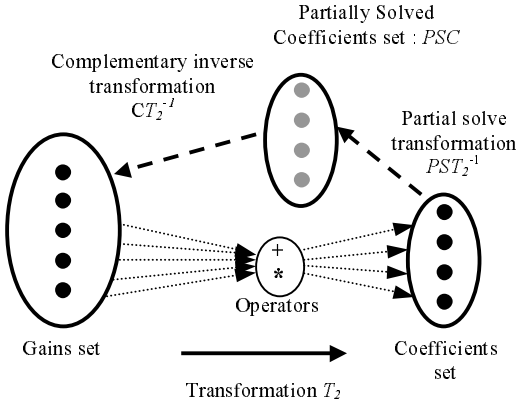

Figure 2: invertibility of T_2

The application PST_2^{-1} transforms the set of coefficients into PSC by recursively applying some substitution rules, figure 3 : from the set of 2 values α and $\alpha+\beta$ (or $\alpha\beta$), can be derived the terms α and β separately.

$$\{\alpha, \alpha+\beta\} \rightarrow \{\alpha, \beta\}$$
$$\{\alpha, \alpha\ \beta\} \rightarrow \{\alpha, \beta\}$$

Figure 3: transformation rules

The PSC are divided into three main sets. The PSC type is numbered I if the results of PST_2^{-1} are all different and if they are equal to causal loop or action chain gains. Their type is numbered II if they are different and if some pairs of them are the sum and the product of two gains. Otherwise some of them are similar or a combination of several gains, then the PSC type is III.

Theorem : *a model is structurally globally identifiable if and only if the three following conditions are satisfied :*

- *The number of PSC is equal to the number of parameters to be identified,*
- *the PSC type is I,*
- *the incidence matrix* $\boldsymbol{M_i}$ *is invertible.*

Proof: The first part of the proof deals with the necessary condition of the theorem: if the incidence matrix is not invertible, then the inverse transformation T_1^{-1} is not defined, then the equation [1] has no unique solution. The model is not structurally identifiable. If the PSC type is not I or if there are less PSC than the number of the parameter to identify, then the inverse transformation T_2^{-1} is not uniquely defined. The model is not structurally identifiable.

If both conditions, regarding T_1^{-1} and T_2^{-1}, are satisfied, both transformations have a unique inverse transformation. Then equation [1] has a unique solution, and the model is structurally globally identifiable.

The structural global identifiability is a very binding characteristic for a model. But this is the only one that assures a reliable and convenient identification process.

4 APPLICATION

We study the influence of the place of a sensor, in the structure of a bond graph, on the identifiability of its parameters. We model the very simple mechanical system of figure 4 by the bond graph figure 5.

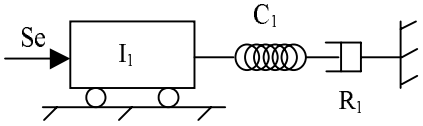

Figure 4 : mechanical system

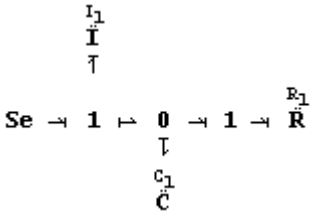

Figure 5 : bond graph model

First case : we put a speed sensor (Df) on mass I_1, the model and the associated transfer function are represented on figure 6.

The incidence matrix, as defined previously, is constructed from the gains of the causal loops and the action chain. B_1 is the gain of the causal loop between I_1 and C_1, B_2 is the gain of the causal loop between R_1 and C_1, and Ca_1 is the gain of the action chain.

$$B_1 = \frac{-1}{I_1 C_1 s^2} \quad B_2 = \frac{-1}{R_1 C_1 s} \quad Ca_1 = \frac{1}{I_1 s} \qquad [6]$$

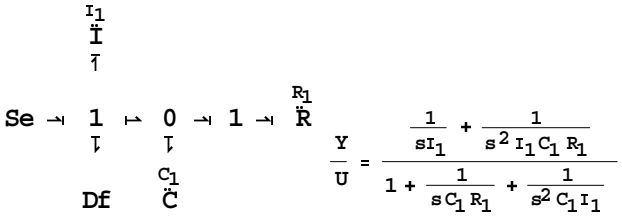

Figure 6: flow sensor on I element

We deduce the transfer function (figure 6) from the causal loops and the action chain gains, according to the Mason's rule described in paragraph 2.

The figure 7 gives the frame of the incidence matrix build from the bond graph figure 5, with :

$$\widetilde{B}_1 = \frac{1}{I_1 C_1} \quad \widetilde{B}_2 = \frac{1}{R_1 C_1} \quad \widetilde{Ca}_1 = \frac{1}{I_1} \qquad [7]$$

The "~" on the gains means that we eliminate the sign and the Laplace operator ***s***.

$$\begin{array}{c} \\ \widetilde{B}_1 \\ \widetilde{B}_2 \\ \widetilde{Ca}_1 \end{array} \begin{array}{c} \begin{array}{ccc} I_1 & C_1 & R_1 \end{array} \\ \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \end{array}$$

Figure 7: frame of incidence matrices

In this first case the incidence matrix of the bond graph is invertible, equation [8].

$$\mathrm{Det}\left[\begin{pmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{pmatrix}\right] = -1 \qquad [8]$$

Then the first transformation T_1 described in figure 1 is invertible. We still have to test the invertibility of the second transformation. As shown figure 2, we compute the PSC, which is very easy in that case because the coefficients of the transfer function figure 6 are: $\{B_1, B_2\}$ for the denominator, and $\{Ca_1, Ca_1 B_2\}$ for the numerator. There are 4 coefficients, and then their type is III because the number of parameters to identify is 3. The model is not structurally globally identifiable.

Second case : we put an effort sensor on the spring C_1. The model and the associated transfer function are represented on figure 8. In that case the gains of the causal loops and action chain are :

$$B_1 = \frac{-1}{I_1 C_1 s^2} \quad B_2 = \frac{-1}{R_1 C_1 s} \quad Ca_2 = \frac{1}{I_1 C_1 s^2} \qquad [9]$$

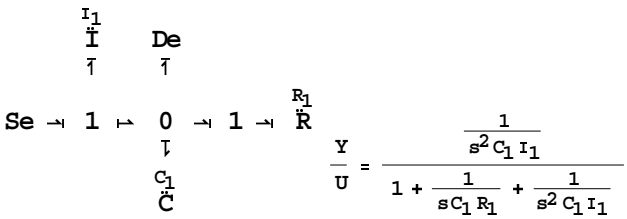

Figure 8: effort sensor on C element

The coefficients of the transfer function are: $\{B_1, B_2\}$ for the denominator, and $\{Ca_2\}$ for the numerator. They are already simple and 3, but the incidence matrix is not invertible, [10].

$$\mathrm{Det}\left[\begin{pmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{pmatrix}\right] = 0 \qquad [10]$$

Then according to the theorem the modified model of figure 8 is still not structurally globally identifiable

Third case : hence we try with a flow sensor on element R_1. This case is represented on figure 9

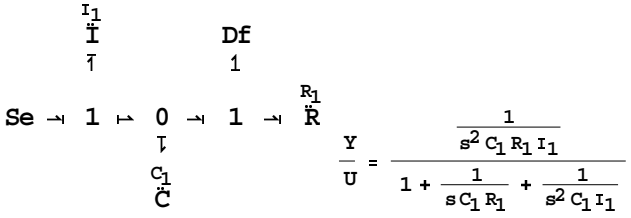

Figure 9: flow sensor on R element

The gains of the causal loops and action chain are :

$$B_1 = \frac{-1}{I_1 C_1 s^2} \quad B_2 = \frac{-1}{R_1 C_1 s} \quad Ca_3 = \frac{1}{R_1 I_1 C_1 s^2} \qquad [11]$$

The incidence matrix is given in equation [12].

$$\mathrm{Det}\left[\begin{pmatrix} 0 & -1 & -1 \\ -1 & -1 & 0 \\ -1 & -1 & -1 \end{pmatrix}\right] = 1 \qquad [12]$$

The coefficients of the transfer function, $\{B_1, B_2\}$ for the denominator, and $\{Ca_3\}$ for the numerator, are simple and their number is equal to 3, the incidence matrix is invertible, then the bond graph of figure 9 is structurally globally identifiable.

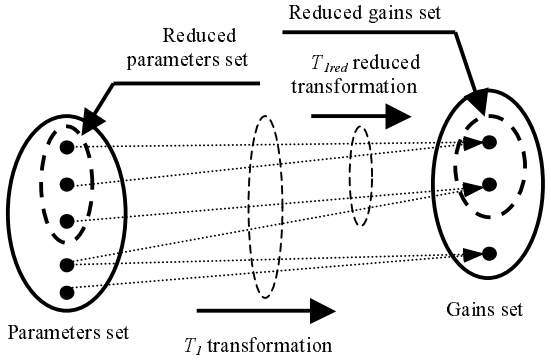

Figure 10: T_{1red} reduced transformation

In some cases the place of the sensor in the structure is fixed because of technological constraints. So in order to find some structure globally identifiable, another solution consists in the reduction of the number of parameters to identify. In order to test if a subset of parameters is structurally identifiable, we define some reduced transformations. The figure 10 represents the reduction of the transformation T_1, we also define a similar reduced T_2 transformation. From there it is possible to test the invertibility of a reduced incidence matrix and to test the type of some reduced PSC, and to conclude on the invertibility of a parameter subset.

CONCLUSION

After the study of the building mechanisms of the transfer function from a bond graph model, a test of structural global identifiability is given. This test involves a special incidence matrix and what we call "partially solved coefficients".

Bond graphs are very well suitable for the translation of the engineer needs into algorithmic language. Bond graphs explicit structural information of models, then engineers are able to graphically localize the places where they can place sensors. In the other way the non-identifiability can be localized in a graphical way with the use of the causal loops in the identifiability test. Addition of sensors or reduction of the number of parameters to identify can be guided by this test, in conformance with physical constraints. Finally this test allows some reliable identification computations. The whole process has been coded under *Mathematica* workspace, using and developing *Virtual Dynamics* bond graph toolbox.

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ANNEXES : some recalls on bond graph methodology

A bond graph consists of subsystems linked together by half arrows, representing power bonds. They exchange instantaneous power at places called ports. The variables that are forced to be identical when two ports are connected are the power variables, considered as functions of time. The various power variables are classified in a universal scheme, and called either effort $e(t)$ or flow $f(t)$. Their product $P(t) = e(t).f(t)$ is the instantaneous power flowing between the ports. Two other types of variables, called energy variables, turn out to be important in describing dynamic systems: the momentum $p(t) = \int e(t)dt$ and the displacement $q(t) = \int f(t)dt$ in generalized notation. Table 1 shows power variables for several physical domains.

	Power variables	
	Effort e	Flow f
Mechanical (Transl.)	force, F	velocity v
Mechanical (Rot.)	torque τ	angular velocity ω
Hydraulic	pressure P	volume flow rate Q
Electrical	voltage u	current i

Table 1. Generalized variables for several physical domains

A few basic types of elements are required in order to represent models in a variety of energy domains. Table 2 regroups basic 1-port elements, which respectively dissipate power (R), store energy (I, C) and supply power (sources). The detectors are used for sensors, supposed to be ideal (no power dissipated).

A causal stroke, placed perpendicularly to the bond, shows up the way the constitutive relations in an element have to be written, as shown Table 2. Table 3 regroups the 0 and 1 junction structure elements with their causality restrictions. They are power conservative.

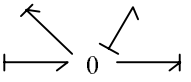
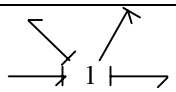
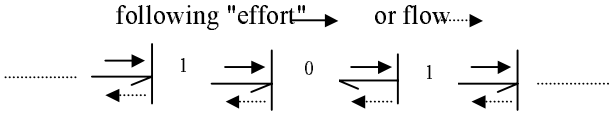
	BG symbol	General relation
0		$e_1 = \ldots = e_n = e_i$ $a_i f_i = -a_1 f_1 - \ldots - a_n f_n$
1		$f_1 = \ldots = f_n = f_i$ $a_i e_i = -a_1 e_1 - \ldots - a_n e_n$

Table 3 : 0 and 1 junctions

The bond graph is a graph, with causal paths and causal loops, followed by propagating effort or flow variable. A causal path is simple if it follows always the same variable (effort or flow)



The gains of the causal loops are equal to the product of the gains of the elements, table 4, encountered along the path of the propagating flow or effort variables.

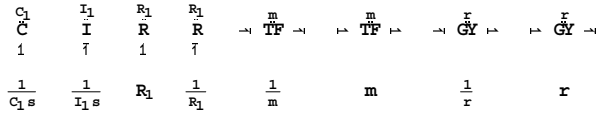


Table 4 : gains of the elements

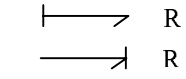
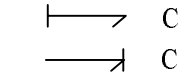
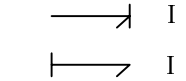
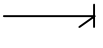
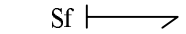
Elements	BG symbol	General relation	Linear relation	Physical elements
resistor R	 R R	$e = \phi_R(f)$ $f = \phi_R^{-1}(e)$	$e = R.f$ $f = G.e = e/R$	electrical resistor, mechanical damper
capacitor C	 C C	$e = \phi_C^{-1}(\int fdt)$ integral $f = d\phi_C(e)/dt$ derivat.	$e = 1/C \int fdt$ $f = C.de/dt$	electrical capacitor, mechanical spring
inertia I	 I I	$f = \phi_I^{-1}(\int edt)$ integral $e = d\phi_I(f)/dt$ derivat.	$f = 1/I \int edt$ $e = I.df/dt$	electrical inductance, mass, inertia
sources	Se  Sf 	e(t) given, f(t) arbitrary f(t) given, e(t) arbitrary		voltage supply, gravity current supply, pump

Table 2 : Passive and active elements