MULTI-OBJECTIVE OPTIMISATION OF FOOD REFLUENT TREATMENT PLANT DESIGN PARAMETERS THROUGH DESIGNED SIMULATION WITH SIGNIFICANT EXPERIMENTAL ERROR

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ABSTRACT

Some aspects of the research documented in [1] are further investigated by the authors to evaluate the possibility of improving plant engineering solutions for the case study and to highlight some limitations concerning the validity of the traditional optimal region identification techniques that arise under particular operating conditions.
With regard to the second point, the authors studied the opportunities and limit application conditions of the classic optimal region identification techniques and of the R.S.M. also through quantitative comparisons with empirical techniques based on the experimental method.
This study falls within that area of research already widely investigated by the authors and on which a more detailed analysis was carried out as evidenced by the recent work on designed simulation of complex systems (DOE techniques) and reported in [2].
On that occasion the problems that the classic techniques had in modelling the typical complex behaviours of industrial systems were solved using neural networks with even higher performance capabilities.
The study outlined in this paper is divided into 3 strictly interrelated points:

- a cost/benefit analysis relative to application of the Simplex Method;
- some further studies about the role, measurement and magnitude of the Pure Experimental Error;
- a multi-objective optimisation through fine mapping of the analysed dominion combined with the Montgomery-Bettencourt algorithm.

The results reported will prove, once again, that the traditional methods to solve some complex problems are not completely suitable and that alternative methods capable of expressing results that are sufficiently adequate for industrial applications must also be explored.

1. STATE-OF-THE-ART

The search for optimal conditions has become of fundamental strategic importance in every area of human activity, and especially in the industrial sector. This is the result of increasingly strong competition that imposes two fundamental objectives: cost reduction and best service level.
For problems solved using R.S.M., the literature proposes optimal region search techniques such as the well-known Simplex Method algorithm (in the basic version and the one modified by Nelder and Mead) and the Steepest Ascent/Descent algorithm (in the basic version and with a modified step).
Such techniques maintain their tested validity on both single-objective and multi-objective problems provided that they are inserted into systems with a low experimental error. To this regard, in the 1980s Mosca and Giribone already identified the Montgomery-Bettencourt algorithm as the most useful tool to find the absolute optimal conditions in the presence of homogeneous industrial objective functions such as the use coefficients of three groups of machine tools [3]. The situation becomes more complex, and thus requires greater analytical accuracy, if an attempt is made to identify absolute optimal conditions in the presence of non-homogeneous objective functions.
The same traditional techniques are not as efficient, due to their nature, when complex problems inserted within contexts with a high experimental error must be solved.
In fact, the problem of adequately evaluating and controlling the Pure Experimental Error is anything but solved and many experimenters often are unaware of how it can invalidate the results obtained also by using well-structured models with the subsequent serious consequences in the decision-making process.
In any case, the fact remains that from a technical viewpoint this problem can be solved even if it implies that the simulation times must become much longer [2].
Finally, the physical limitation of displaying functions and variables in multi-objective problems makes it almost impossible to identify the preferred directions along which to orient the analysis.

This is particularly true in industrial engineering where, without adequate mathematical-statistical skills, complex analytical techniques cannot be successfully applied.
The need to use a discrete and stochastic simulation model to handle a plant sizing problem leading to specific high-precision answers allowed the authors to highlight two important and often neglected methodological aspects of a technique that is being used more and more thanks to the widespread use of dedicated tools with low-cost calculating power. Specifically:

a) time evolution and quantitative control of the experimental error
b) the possibility, through adequate design of experiments to carry out using the model, of obtaining the absolute optimal conditions, in a certain experimental dominion, in the presence of a single-objective analysis and when searching for the best stationary point in the presence of a multi-objective analysis.

As indicated further in the paper, due to an unusual coincidence related to the intrinsic structure of the plant studied, the connection between the previous points a) and b) was such, without an accurate control about how the experimental error influences output data, that it nullified any possibility of using techniques to identify the optimal region.
Such a consideration obviously takes into account the fact that each time the experimental error is not controlled by the experimenter, the results, also using models that closely match the behavioural logic of the real system, might be very different from the "real" answers of the system. This is due to the entity of such a parameter, with all the consequences that can easily be imagined.
In fact, if we take just one moment to consider the situation, we can intuitively see how, in any problem considered within a stochastic regime, a specific result is never accepted without knowing at least one additional parameter that best fits the datum obtained within the general context (variance, confidence band, etc.).
Then, when the discussion shifts to the simulation of industrial systems, models in which dozens and often hundreds of frequency distributions interact, it might be clear how not knowing the experimental error makes the specific datum obtained in this manner from the experimentation reasonably unreliable.
What is most surprising is how this situation has persisted in many experimentations, both in the company and as reported in the literature, since consolidated methods for studying the evolution of the Mspe over time have been available for more than twenty years, like the one applied here. In fact, in addition to being used to constantly control the magnitude of the experimental error, thanks to Cochran's theorem, they also provide an efficient logical and methodological statistical validation and debugging tool of the simulation model.
With regard to optimisation, it should be noted how the low-cost and enhanced (and constantly increasing) calculating power available today allowed the authors to experiment with an optimum search method as an alternative to the classic methods, based on the approach to the optimal region and subsequent construction of that region in accordance with what was hypothesised by Mosca and Giribone in 1980.
The interesting results obtained confirm the validity of the proposed method.

2. SIMPLEX METHOD CRITICAL APPLICABILITY ANALYSIS

The Simplex Method algorithm, while maintaining its validity and performance superiority over the Steepest Ascent (n+1 simultaneous survey points in Simplex versus only 2 points in the Steepest), proved, as illustrated here below, to be extremely limited in the presence of a high experimental error.
In the research described in [1] it was necessary to identify a global optimum relative to three different objective functions - Biogas production [Nm^3/y]; Industrial water [m^3/y]; Accumulated volume in digester suspension [t].
The problem was handled, initially, by searching for the single optimum conditions. That search was complicated by the fact that for one of the three objective functions (Industrial water [m^3/y]) it was impossible to build a regressive model that could pass the statistical adaptation tests. The need to provide technically plausible answers and to limit experimentation times forced the authors to find a compromise. That compromise involved the construction of 3 regressive polynomials for the critical function obtained through the parameterisation of one of the 3 independent variables on three specific design values. The procedure used seemed to be a valid methodological compromise but, theoretically, was not beyond criticism: for this reason, the authors decided to carry out a new and rigorous analysis according to traditional methods by using the Simplex Method algorithm to identify the optimal region.
The region analysed (analysis dominion) is a cubic solid in space $\mathbb{R}^3$ bordered by the extreme values of the three independent variables analysed: Residues daily quantity [t/d]; Dry solid percentage [%]; Digester potentiality [%] – figure 1.
To ensure that the solutions would have adequate robustness, the authors decided to perform the analysis in two steps:

1) construction of a first simplex with initial tetrahedron in the centre of the dominion;
2) construction of a second simplex with initial tetrahedron in the optimal region identified by the first simplex.

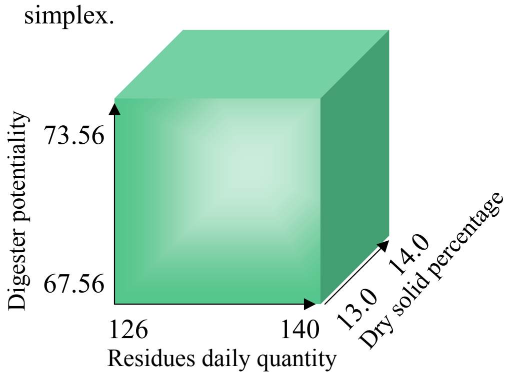


Fig. 1a –Simplex Method analysis dominion for the Industrial Water function

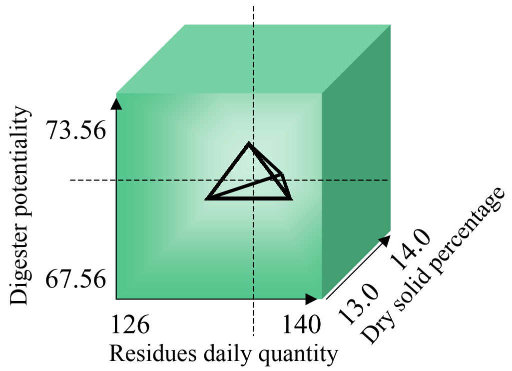


Fig. 1b – First simplex

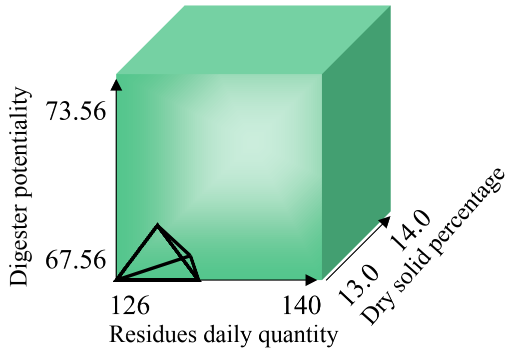


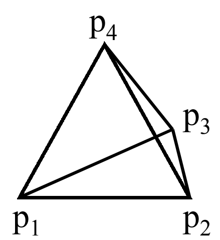
Fig. 1c – Second Simplex

Construction of the first simplex

As represented in figure 1b, implementation of the first simplex included the definition of an initial regular tetrahedron located in the central region of the analysed dominion.

The following is the code of the four points to define the initial tetrahedron:

$$p_1 = [0,0,0]$$
$$p_2 = [1,0,0]$$
$$p_3 = \left[\frac{1}{2}, \frac{\sqrt{3}}{3}, 0\right]$$
$$p_4 = \left[\frac{1}{2}, \frac{\sqrt{3}}{4}, \frac{1}{\sqrt{2}}\right]$$



which corresponds to the following decoded design points:

$$p_1 = [133,13.5,70.56]$$
$$p_2 = [135,13.5,70.56]$$
$$p_3 = [134,13.67,70.56]$$
$$p_4 = [134,13.59,71.27]$$

The traditional algorithm was applied using the plant simulation model developed in SIMUL8 and already described in detail in [1].

The tetrahedron was flipped 58 times and 34 of those around p_{24}, thus fully satisfying the convergence criterion, as suggested by the authors, according to which:

$$M = 1.65 \cdot n + 0.05 \cdot n^2$$

iterations are needed around a fixed vertex to identify the optimal region.

The optimal region identified is the one represented in figure 2a. It is identified by the following ranges of values of the independent variables:

Residues daily quantity [t/d]:	*126 – 128.7*
Dry solid percentage [%]:	*13.0 – 13.25*
Digester potentiality [%]:	*68.0 – 68.85*

However, the results reported do not match what emerged from the previous analysis [1] based on which the optimum, for the analysed function, should be located in the extreme lower corner of the cube delimiting the experimental region, i.e. for values of the independent variables equal to [126, 13.0, 67.56], with the third value of co-ordinates outside the range with respect to the optimal region identified by the Simplex.

To scrupulously analyse the reasons why problems arose with the algorithm, the authors, having never encountered anything similar in the numerous applications carried out in the past, decided to perform a new implementation of the Simplex Method starting from an initial tetrahedron located in the region including the point [126, 13.0, 67.56].

Construction of the second simplex

Implementation of the second simplex involved the definition of a regular tetrahedron located in the extreme region of the analysed dominion. The code of the points and the corresponding real decoded values are shown below:

$$p'_1 = [0,0,0]$$
$$p'_2 = [1,0,0]$$
$$p'_3 = \left[\frac{1}{2}, \frac{\sqrt{3}}{3}, 0\right]$$
$$p'_4 = \left[\frac{1}{2}, \frac{\sqrt{3}}{4}, \frac{1}{\sqrt{2}}\right]$$

$$p'_1 = [126,13.0,67.56]$$
$$p'_2 = [127,13.0,67.56]$$
$$p'_3 = [126.5,13.086,67.56]$$
$$p'_4 = [126.5,13.043,67.931]$$

Once again, using the simulator created, the tetrahedron was flipped 44 times, and 12 of those around p_{32}. As in the other case, this made it possible to satisfy the convergence.

The optimal region identified is the one represented in figure 2b. It is identified by the following ranges of values of the independent variables:

Residues daily quantity [t/d]:	*126 – 128.5*
Dry solid percentage [%]:	*13.0 – 13.5*
Digester potentiality [%]:	*67.56 – 68.5*

As easily noted in figure 2b, the optimal region does not coincide with the one identified by the first simplex. A single global optimal region is built in figures 2c and 2d.

It is more extended than the two identified and includes them: thus, this region was considered as the new dominion to be analysed and in which a model was built to search for the stationary point.

Single optimal regions | Global optimal region

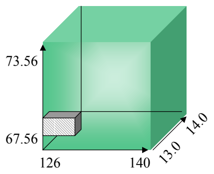


Fig. 2a – optimal region 1st simplex

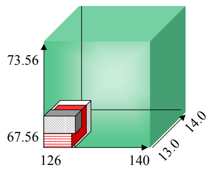


Fig. 2c – union of optimal regions

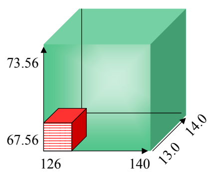


Fig. 2b – optimal region 2nd simplex

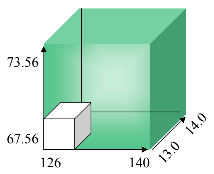


Fig. 2d – global optimal region

Construction of the regressive polynomial for a regular analysis

Traditionally, once the optimal region has been identified, a second-order model is built to specifically determine the stationary point and the nature of that point. It is commonly known, in a sufficiently restricted portion of the analysed dominion, that a second order model is generally suitable to represent the curvature of the real response surface.

In the case study, since the fit was so good for a model of that order, a suspicion arose that the curvature of the real response surface was so slight that it could even be described with a first-order regressive model.

This suspicion was confirmed using the usual Fisher double-tail statistical test through which it was possible to validate the following first-order polynomial:

$$Y_2 = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

$Y_2 =$ *industrial water [m^3/y]*

$x_1 =$ *residues daily quantity* [t/d]

$x_2 =$ *dry solid percentage* [%]

$x_3 =$ *digester potentiality* [%]

$$Y_2 = -24350.4 + 166.2914 x_1 + 1613.503 x_2 + 46.5705 x_3$$

The polynomial obtained confirmed the results of the study described in [1] with an optimum of the function in the point [126,13.0;67.56]. It follows then that the Simplex Method apparently has a problem in clearly identifying the optimal region also in what is seemingly a simple case since it does not have local optimums.

After a thorough analysis of all the experimental campaigns carried out and of the behaviour of the Simplex, the authors felt it was necessary to further investigate the Pure Experimental Error despite the fact that the curve of the MSpe had easily reached the stability area and that the magnitude of the error appeared to be mathematically acceptable.

3. MORE ABOUT THE PURE EXPERIMENTAL ERROR MEASURE

As a consequence, a new experimental campaign was carried out to specifically study the experimental error in the analysed dominion.

Based on the analysis of the Mspe carried out in the first part of the research [1] the optimum duration of the simulation is 210 days. Within this time span the error settled around a value of 10^{-2} for all the analysed objective functions. In particular, for the function y_2 (Industrial water [m^3/y]), the experimental error was 1.029%.

Despite the fact that the magnitude of the error would seem to be more than acceptable in terms of the technological result, since modifications of the second decimal digit are not significant, the values of the function y_2 in the points obtained by applying the simplex method are so similar to each other that such a magnitude of the experimental error easily masks the real response surface.

Therefore, it was decided to define a tetrahedron within the global optimal region and to perform various sets of simulations on its vertices, replicating each set on different time horizons to try to reduce the experimental error even further.

In the first experimental campaign, the analysis was carried out over a time span of 420 days, with three replications for each vertex of the tetrahedron. The results of y_2 are reported in table 1, while the normalised values needed to correctly measure the pure error are shown in table 2.

The magnitude of the pure experimental error decreased by almost one order of magnitude, settling around 0.6439%. This value, if we consider a confidence interval within the range of $\pm 3\sigma$, would make it possible to be at the limit of the confusion between the values reported in tables 1 and 2. Therefore, to increase the robustness of the results, it was decided to carry out an addition experimental campaign over a time span of 700 days. The results are reported in tables 3 and 4.

The magnitude of the experimental error was reduced even further to 0.2612%.

In this case, the confidence interval included within the range $\pm 3\sigma$ guarantees no confusion between the test vertices.

	replication 1	replication 2	replication 3
P1	42931.26456	42888.67402	42748.12524
P2	43267.95997	43225.0354	43083.38434
P3	43381.28783	43338.25084	43196.22876
P4	43274.03921	43231.10862	43089.43765

Tab. 1 – Values of y_2 on the vertices of the tetrahedron for t=420 days

	replication 1	replication 2	replication 3
P1	102.2172966	102.1158905	101.7812506
P2	103.0189523	102.916751	102.5794865
P3	103.2887805	103.1863115	102.8481637
P4	103.0334267	102.931211	102.5938992

Tab. 2 – Values of y_2 on the vertices of the tetrahedron normalised for t=420 days

	replication 1	replication 2	replication 3
P1	71665.36594	71618.51748	71541.85636
P2	72227.42681	72180.21093	72102.94857
P3	72416.61014	72369.27059	72291.80586
P4	72237.57517	72190.35265	72113.07944

Tab. 3 – Values of y_2 on the vertices of the tetrahedron for t=700 days

	replication 1	replication 2	replication 3
P1	102.3790942	102.3121678	102.2026519
P2	103.1820383	103.114587	103.0042122
P3	103.4523002	103.3846723	103.2740084
P4	103.196536	103.1290752	103.0186849

Tab. 4 – Values of y_2 on the vertices of the tetrahedron normalised for t=700 days

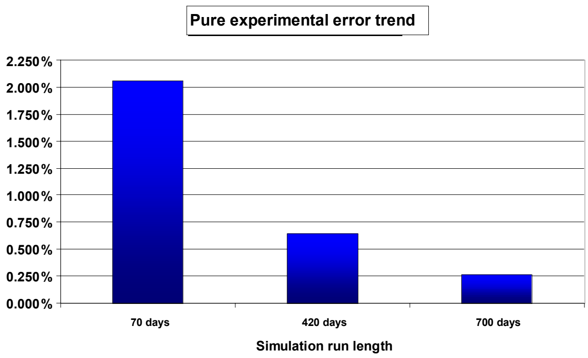


Fig. 3 –Reduction in pure experimental error

This result again confirms what the authors have indicated over the last few years, i.e. the fact that the magnitude of the experimental error tends to decrease, and even significantly, with respect to the first settling values, as run time increases.
To obtain reliable results from the application of traditional R.S.M. techniques, such as the Simplex Method, it may be necessary, as for the case being analysed, to perform runs over very long time frames: however, this may not be feasible due to the lack of adequate calculating power at sustainable costs (high-performance PCs are still very expensive and thus justified for research purposes but very difficult to justify in the industrial field).

4. MULTI–OBJECTIVE OPTIMISATION

The third phase of the study focused on solving multi-objective stochastic optimisation problems.
As already mentioned in paragraph 2, the objective of the research described in [1] was to identify a global optimum point relative to the three objective functions:

- Y_1 Biogas production [Nm^3/y];
- Y_2 Industrial water [m^3/y];
- Y_3 Accumulated volume in digester suspension [t];

which are not homogeneous and have different sizes.
To identify the global optimal region, the authors decided to use a different method than what was used in [1] to compare the potential of that method and to validate or repudiate their conviction about the possibility of improving the first solution.

The methodology utilised, as hypothesised by Mosca and Giribone already in 1980, involves a direct search of the optimum point by measuring the objective functions on points deriving from fine mapping of the analysis dominion.
Figure 4 shows the mapping of the analysed dominion (cubic solid in space $\mathbb{R}^3$ – figure 1a). The mapping generated 240 test points.
The co-ordinates of the new test points were defined by moving along the axes of the space $\mathbb{R}^3$ with the following steps Δ:

- Residues daily quantity [t/d]:
  $x_1 \in [126, 140], \Delta = 2$;
- Dry solid percentage [%]:
  $x_2 \in [13.0, 14.0], \Delta = 0.2$;
- Digester potentiality [%]:
  $x_3 \in [67.56, 73.56], \Delta = 1.5$.

Based on what emerged concerning the magnitude of the pure experimental error and following the problems encountered in the Simplex Method implementation, it was decided to carry out all the tests over a time frame of 700 days.
Finally, in this phase, a test was carried out on each of the 240 points defined and the value assumed by the three objective functions (for a total of 240 * 3 = 720 values) was measured with each simulation. Finally, the values found were graphically represented to better analyse the behaviour of the real system.

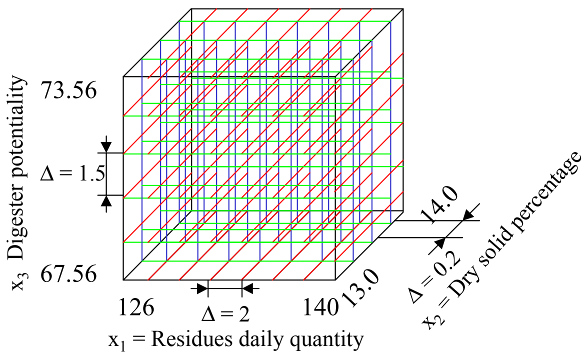


Fig. 4 – Mapping of the analysis dominion

Step 1
In this first step, attention was focused on the first two objective functions Y_1 and Y_2 since they are more interesting from a strictly plant engineering viewpoint.
Figure 5 illustrates the first graphic representation: the values assumed by the two objective functions in the 240 points of the mapped dominion together with the code used to define those points (i+j relative to the variables x_1 e x_2; k relative to the variable x_3) are represented on a Cartesian plane (Y_1, Y_2).
At this point, it should be recalled that the plant engineering technological objective to be achieved is to maximise Y_1, minimise Y_2 and maintain a value as close as possible to the average for Y_3.
Figure 5 illustrates the mapping of the values obtained for the three objective functions analysed: the functions Y_1 and Y_2 are represented on the Cartesian axes while the

values of function Y_3 are projected on the Cartesian plane and identified by the varying shades of grey. The parameters i, j and k are used to define the mapping steps.

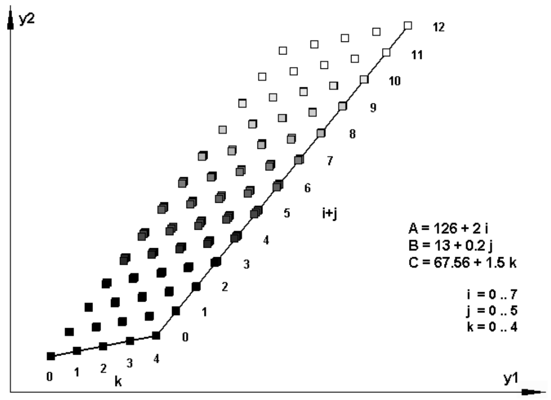


Fig. 5 – Mapping of the objective functions Y_1 and Y_2 on the dominion $\mathbb{R}^3$

For what concerns the functions analysed to this point, Y_1 and Y_2, the location of the Pareto–Optimal point is represented by the line that joins the extreme points of the two sides of the quadrilateral. In fact, with ordinate (Y_2) being equal, the line represents the points whereY_1 is a maximum. Instead, with abscissa being equal, the line represents the points whereY_2 is a minimum.
The analysis of the values of Y_1 and Y_2 on the 240 test points showed that there is a linear correlation between the values of the two objective functions: an example is provided in figure 6 which shows the correlation that links the two objective functions corresponding to the high value of the independent variable x_3. The values of the correlation coefficient for all other values of x_3 can be obtained from the graph in figure 7.

Step 2
After identifying the Pareto–Optimal points, we must identify the global optimal point.
In accordance with Montgomery and Bettencourt the location of the Pareto – Optimal points was obtained by joining the optimal points of the objective functions Y_1 and Y_2 considered individually (i.e. one at a time).

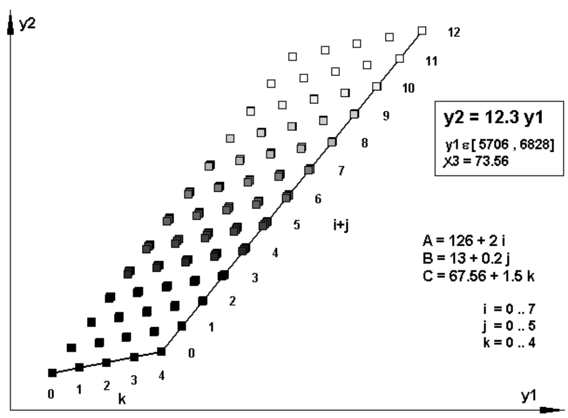


Fig. 6 – Correlation between the objective functions Y_1 and Y_2 in the dominion $\mathbb{R}^3$

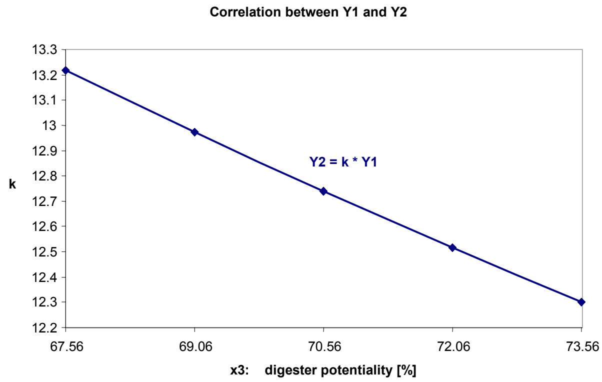


Fig. 7 – Correlation coefficient for different x_3 values

At this point we need a selection criterion that can be used to identify a specific optimal point that can then be converted into real values of the independent variables (plant engineering variables).
Thus, the authors decided to use the third objective function Y_3 as a constraint to reduce the optimal region that was previously identified.
This consideration was strongly justified by the fact that Y_3 also fell within the multi-objective optimisation problem, even though, from what is strictly a plant engineering point of view, it was not as interesting as the other two objective functions.
The decision to maintain the values of such a function within a range centered on the average value led to the graphic representation shown in figure 8: the grey segment that represents the region of interesting values for Y_3 intersects the Pareto – Optimal line, significantly reducing the global optimal region.
Figure 9 illustrates the graphic representation of the global optimal point identified in the analysis carried out in [1] and the lines along which it is possible to move to obtain even better points.
To make a quantitative comparison between the two applied methodologies, it is useful to recall the co-ordinates of the optimal point identified in [1] and the corresponding values of the three objective functions:

Independent variables
x_1 - Residues daily quantity [t/d] = 137 t/d x_2 - Dry solid percentage [%] = 13.0 % x_3 - Digester potentiality [%] = 70.56 %
Objective functions
Y_1 - Biogas production [Nm^3/y] = 5951.631 Nm^3/y Y_2 - Industrial water [m^3/y] = 75820.63 m^3/y Y_3 - Accumulated volume [t] = 2703.304 t

Comparing figure 9 with figure 6 we find that the location of the improved points is limited by the following ranges of steps along the three independent variables:

$$2 \leq i + j \leq 5 \qquad \text{and} \qquad k = 4$$

and thus by the area represented in figure 10 (A=x_1; B=x_2; C= x_3).

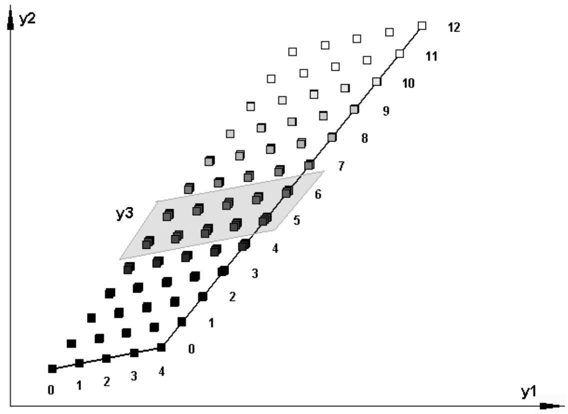


Fig. 8 – Global optimal region

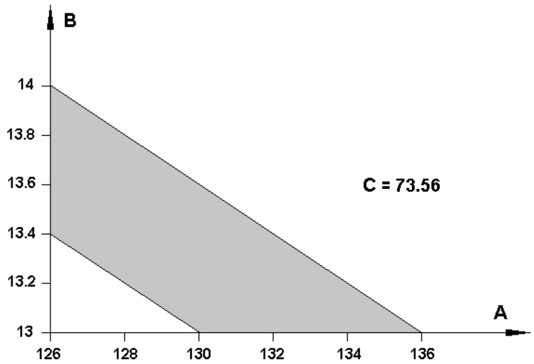


Fig. 10 – Narrow global optimal region

At this point, with the complete mapping of the dominion, we can read the values assumed by the three objective functions corresponding to the improved points and compare them with those calculated in the research described in [1].
As an example, we present the following two possible global optimums:

Global optimal 1	Global optimal 2
Independent variables	*Independent variables*
x_1 = 134 t/d x_2 = 13.0 % x_3 = 73.56 %	x_1 = 132 t/d x_2 = 13.2 % x_3 = 73.56 %
Objective functions	*Objective functions*
Y_1 = 6068.8 Nm^3/y Y_2 = 74647.09 m^3/y Y_3 = 2644.107 t	Y_1 = 6070.201 Nm^3/y Y_2 = 74664.23 m^3/y Y_3 = 2644.71 t

These are the points in which the three objective functions have better plant engineering values than those that were previously determined.
Having identified such a narrow optimal region it also becomes possible to apply additional decision-making rules, such as the weighing criterion and the utility function already widely described in [1] to satisfy industrial needs.

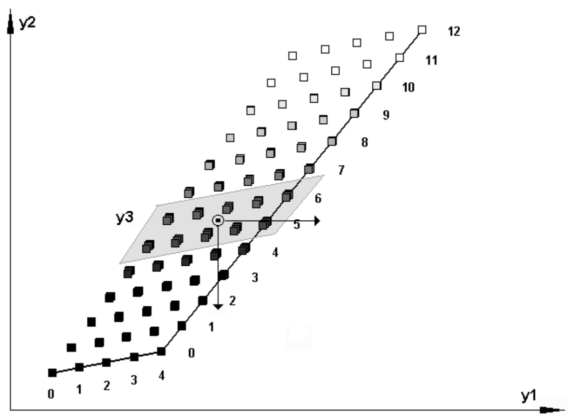


Fig. 9 – Lines of points that are better than the first global optimal region

5. FINAL CONSIDERATION

The research of the optimum region of the response surface obtained by a simulation model represents, at the beginning, an optimisation problem of a function with k variables. This function and the variability ranges of the independent variables are not known to the user.
Consequently, because of the past experience to study various complex model of industrial systems, it happens that also in apparently simple cases, with only two independent variables involved $f(x_1;x_2)$, the “intuitive” identification of variability ranges, where is contained the optimum of the response surface, can require an high number of simulation runs. This can be explained because, situation like these, could be underlined only at the end of an integral optimisation study.
This task requires:

- a Central Composite Design of 2nd order (that mean $N=2^2+2*2+5=13$ runs);
- the research of the better surface using the regression;
- the application of the classic analysis on the founded surface.

So, only after these steps, the user will know the project success or failure, with only the eventual advantage to identify the X_0 direction on which the searched optimum is situated.
This allows the user to reset the variability ranges and to reapply the complete project hoping to be much lucky.
So when is required to pass from 2 to k variables (k>2) the problem will assume “dramatic” sperimental dimension.
In this context the D.O.E. philosophy utilises gradient techniques (Steepest – Ascent) to obtain a guide methodology in the research of optimum region.
This technique, in the case of high stocasticity models, like industrial plant simulators are, appears too expensive in the convergence phase to the optimum or not able to study particular surface.
For this reason, from ’80, Mosca and Giribone, considering that a complete exploration of a surface requires a multidirectional analysis, preferred to utilise sequential projects.
In these projects the runs are execute one at the time and the choice of the next point is based on the information of the previous points; in particular, they decided to adopt projects of Simplex’s family.

The Simplex Method is not a real algorithm but a driven choice criterion for the next point. This allow to evaluate the function in a n dimension space (n=number of independent variables) in n+1 equidistant points.
During the optimisation of the methodology oriented to the simulation model, Mosca and Giribone identifies, like best method, the Nedler & Mead version of the Simplex Method. It was opportunely modified, respect to the base Simplex, in order to reduce the approaching phases to the optimum region.
The comparison between the classic D.O.E. methodology and the technique used by AA shows how the Steepest–Ascent cannot be compared with the Simplex already in the approaching phase to the optimum region.
Consequently a complete comparison between them, based on the total number of runs required, in most cases, this included, appear not sustainable.
It is possible, in fact, to observe that only in the case of classical mono hilly surfaces, however not known at the beginning of a simulation project, the Steepest gives good results.
The Simplex method reaches, with relatively few tests (less then 50% respect the Steepest), the optimum region, with the advantage to identify it with absolute certainty. This also happens in case of complex surface or in case of surface with particular crests.

CONCLUSIONS

The research described allowed the authors to improve the already very encouraging results obtained in the previous work and to test the applicability of methodologies that differ from those traditionally found in the literature.
In the specific case, the experimental methodology used proved the validity and efficiency but involved longer times and higher analysis costs. However, it is felt that in particularly complex cases and with a high experimental error, like the one presented, the approach used can help generate very advantageous operating conditions.

REFERENCES

[1] A. Frigato Bonello, P. Giribone, R. Mosca, S. Pozzi Cotto, "*Stochastic Optimal Design of an anaerobic digester for citrus residuals"*, Proceedings of ESS 2001 – Simulation in Industry, Marseille, October 2001

[2] A. Frigato Bonello, R. Mosca, F. Tonelli, "*Designed Simulation (Doe) of Complex Systems: New Frontiers in Using Neural Networks for R.S. Research part one: Theoretical Assumptions"*, Proceedings of WMC 2001, Phoenix, Arizona, January 2001

[3] R. Mosca, P. Giribone, "*The Nedlear-Mead method: a better sequential technique to get optimum region in discrete simulation of complex plants*", Proceedings of IASTED Int. Symposium "Modelling and Simulation", Lugano, Swiss, 1985

[4] R. Mosca, P. Giribone, R. Seiberl, "*An interactive generalized code for a multi-object analysis of discrete simulations for complex industrial plants*", Proceedings of IASTED Int. Symposium "Modelling and Simulation", Lugano, Swiss, 1985

[5] R. Mosca, P. Giribone*, "Metodi Gradientali per la Ricerca Dell'Ottimo negli Esperimenti Progettati di Sistemi Fisici",* Istituto di Tecnologie e Impianti Meccanici, University of Genoa, Italy, 1984

[6] D. C. Montgomery, V. M. Bettencourt, "*Multiple response surface methods in computer simulation*", Simulation, 1977

[7] L. Geoffryon, A. M. Dyer, J.S. Feinberg, "*An interactive approach for multi-criterion optimization with an application to the operation of an academic department*", Vol. 19 "Management Science", 1972

[8] M. J. Box, D. Davies, W. H. Swann, "*Non linear optimization technique*", Oliver & Boyd edition, 1969

[9] Y. A. Nedler, R. Mead, "*A simplex method for function minimisation*", Vol. 7th Computer Journal, 1965

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