A NEW METHOD FOR CHARACTERIZING LACE BASED ON A FRACTAL INDEX

A. Bigand
Université du Littoral, BP 649
62228 CALAIS Cedex
Tél. : 03 21 46 36 91
Fax : 03 21 46 06 86
e-mail : Andre.Bigand@lasl-gw.univ-littoral.fr

1. Introduction

The **fractal geometry**, issued of a synthesis of works done in Mathematics and in Physics since more of one century, proposed in the 70's new concepts to understand some complex phenomenon ([Man77]). The notions of fractional dimension and scale invariance have been recognized applicable quickly for the description of many natural objects, of the mountainous to the maritime facades while passing by the meteorological phenomenon, the porous surroundings ([Adl92]) or the chemical catalysis ([Avn89])... These tools make it possible to understand the phenomenon of growth far from the balance that appear in a spontaneous manner in many domains, as the dielectric straining or the dendritic growth. The physicists, the chemists, the meteorologists or the astronomers could have new quantitative measures thus to characterize the objects that they study.

The applications in signal processing appeared later, toward the beginning of the 80's. A characteristic of the first attempts is the essentially descriptive vision that there was to the work: some signals were analyzed and some behaviors fractals was or non raised, the most often under the shape of a scale invariance in a certain range of resolutions. Then a " fractal dimension " was deducted, and the developments stopped there.

Two important evolutions of different nature made it possible to enter in an "operational" phase in the beginning of the 90's. The first, that appears naturally in the developments of a this young disciplines, is the enrichment of the tools of basis of the theory in view of the applications to the variety of the natural phenomenon: from the characterization of a signal by its (only) " fractal dimension ", came to be added finer measures, as the lacunarity or the multifractal analysis; the models of fractal process, first perfectly auto-similar, diversified to take into account invariances in generalized senses; finally, the statistical methods of fractal signal processing improved to provide some estimators more robust, and applicable in more general situations.

The second evolution is more conceptual, and well adapted to signal processing. Instead of continuing to research fractal phenomenon (that means scale invariant) and to describe this invariance with the help of various measurements, one realized the profit that it could have there to apply some fractal tools to ordinary signals. In other words, instead of analyzing a signal to know if it is a fractal object, one makes it undergo some **fractal treatments** regardless of its possible scale invariance. Image processing provides a striking example of this change of point of view: fractal compression of images has been developed ([Fis98]), and not compression of fractal images! It is the same way for segmentation ([Veh96]), filtering ([Veh02]), or watermarking: any pictures are treated with fractal methods. This important evolution can be compared with the application of classic methods. The fractal measures or multifractal spectrum associated to a signal will be calculated while making some hypotheses on this one (adherence to a class of models, and "regularization" or "extension" of the signal (in the scales rather than in space)).

We applied this concept for the fractal characterization of pictures of laces. These pictures (figure 1) are indeed very rich, and so very complex to analyze. This complexity can be a handicap for the constitution of a data base adapted to this particular industry. Indeed, an important effort of standardization of the objective criterion permitting the classification of the motives of laces is set currently in motion (creation of a" thesaurus " for lace industry), and the fractal treatment of the pictures of laces makes it possible to get a reliable attribute. We first present the method used for the fractal treatment fractal (boxes method), the results we obtain and the possible extensions then for this work.

2. Basic theory and algorithm used

The notions of fractal object and fractal dimension are now well known ([Vos86], [Bar88], [Man82]). The mathematical analysis that is associated to a fractal object consists in covering this one by balls

Proceedings 14th European Simulation Symposium
A. Verbraeck, W. Krug, eds. (c) SCS Europe BVBA, 2002

of identical dimension to the one of the support, so the measure of a curve in the plan (2D) will require its recovery by "tablets" of diameter h, and of surface η^2 ([Tri93]).

2.1. Hausdorff dimension

Let's recall the manner of which Lesbesgue defines the "surface" of a set Γ in the plan. For a positive number given η, he considers the recovery of Γ by balls Bj of diameter rj lower than η. The set " surface of Γ " is bigger than the set than we try to define, in particular if the balls encroach one on the other. The most economic recoveries, in other words in plane geometry (2D), are those given by the lower value of the sum of the surfaces of tablets μ_η (Γ) necessary:

$$\mu_\eta(\Gamma) = \inf\left\{\sum_j \pi r_j^2/4\right\} \qquad (1)$$

The outside mass of Lebesgue consists then in putting:

$$\mu_\eta(\Gamma) = \lim \mu_\eta(\Gamma) \quad \text{with} \quad \eta \to 0 \qquad (2)$$

In the space of d dimension, it is also possible to define a measure of Lebesgue μ^d while using balls of d dimension. But the value d = 2 chosen is in some case too big; a mean exists then to reduce the total measure. The method is due to Hausdorff. It is adapted to the fractal curves , and is obtained in modifying the analysis of Lebesgue, while replacing r_j^2 by r_j^α and therefore as adding a parameter α on behalf of the value of the power 2 of the diameter of the ball. Either therefore, the measure of Haussdorff ([Bro92]) is about:

$$h_{\alpha,\eta}(\Gamma) = \inf\left\{\sum_j r_j^\alpha\right\} \qquad (3)$$

2.2. Minkowski – Bouligand dimension

To raise the dimension of Haussdorff, let's call Nη (Γ) the minimum number of balls presenting all same diameter η susceptible to cover the set Γ completely (all balls are then strictly identical). It results from the definition an overcharge of $H_{\alpha,\eta}$ (Γ) as:

$H_{\alpha,\eta}(\Gamma') \leq \eta^\alpha N_\eta(\Gamma)$

One deducts that for all values of α, as $H_\alpha = \infty$, one has:

$\lim (\eta^\alpha N_\eta(\Gamma)) = \infty$ for $\eta \to 0$

either again:

$\lim (\alpha \ln(\eta) + \ln N\eta(\Gamma)) = \infty$ for $\eta \to 0$

While putting $\Delta_{MB} = \lim\left(\dfrac{\ln N_\eta(\Gamma)}{\ln(1/\eta)}\right)$, the analysis drives to:

$\Delta_{MB}(\Gamma) \geq \alpha$

in the limit where $\eta \to 0$, either:

$\dim \Gamma \leq \Delta_{MB}(\Gamma)$

This new index Δ_{MB} is called Minkowski – Bouligand dimension or, for obvious reasons, logarithmic density of Γ. Let us consider the case of a curve or a set Γ in the space of dimension 2, the plan of the page, and let's define" the set of Minkowski" as the union of the balls centered on Γ of diameter η, that is to say $\cup B_\eta(\Gamma)$. Let us define A_2 (η) the area of the set $\cup B_\eta(\Gamma)$. It is easy to understand that this area A_2 (η) can be written as following:

$$A_2(\eta) = N(\eta) \cdot \eta^2$$

if η is the measure jauge and N (η) the minimum number of balls necessary to the recovery of Γ. Otherwise, the formula of definition of Δ_{MB} permits to write, when $\eta \to 0$:

$$N(\eta) \approx \eta^{-\Delta_{MB}}$$

The Minkowski dimension Δ_{MB} is given therefore by the relation:

$$A_2(\eta) \approx \eta^{2-\Delta_{MB}}$$

in the limit $\eta \to 0$, either again the relation:

$$\text{for } \Delta_{MB}(\Gamma) = \lim\left(2 - \frac{\ln A^2(\eta)}{\ln(\eta)}\right) \quad \text{with} \quad \eta \to 0 \qquad (4)$$

This formula is to compare to the expression of the length of a fractal curve, , $\lambda_\eta \approx \eta^{1-\Delta}$ ([Man82]). While affecting in the Γ curve a thickness η, it comes $\eta \cdot \lambda_\eta = A_2(\eta) \approx \eta^{2-\Delta}$, from where it results that $\Delta = \Delta_{MB}$.

The formula becomes widespread without difficulty in a space of dimension d under the form:

$$\Delta_{MB} = \lim\left(d - \frac{\ln A_d(\eta)}{\ln(\eta)}\right) \quad \text{with } \eta \to 0 \qquad (5)$$

where one designates by A_d (η) the d-dimensional volume of the set $\cup B_\eta(\Gamma)$.

If the method of assessment of Δ_{MB} is suitable enough easily to the calculation, for example when the fractal set is seen by means of a CCD sensor , it is unfortunately little precise, because it is rare that the alignment in the plane "ln/ln" is good and reliable, contrary to what is often affirmed. In a certain manner, the set of Minkowski is too thick to permit good adjustments. The most known method to determine the value of

Δ_{MB} then is the boxes method, proposed by Voss ([Vos86]).

2.3. Boxes method in R^2

It is about constructing a decreasing sequence η_n having for object to cover Γ by a network of squared stitches of side η_n. The value of the fractal dimension is gotten by deduction of the number N_n of squares meeting a point (white points in figure 4) of Γ. The dimension is then:

$$\Delta_{MB} = \lim \left(\frac{\ln N_n}{\ln(l/\eta_n)} \right) \quad \text{for } n \to \infty$$

$$(\eta \to 0) \qquad (6)$$

The slope in the diagram {ln (Nn) vs ln (l / hn)} corresponds therefore to the Δ_{MB} dimension. Very simple to use, this method presents however some inconveniences. In particular if l / η_n is not whole, the squares of side η_n are going to overflow generally on the left and on the right of the graph of Γ, what distorts the results and introduces some irregularities in the diagram, especially when η_n is big. Let's suppose the defined graph on [0, 1]. So that the projection on Ox of the squares meeting the graph is always included in [0, 1], it agrees to impose that the sequence η_n is dyadic, that means under the form $\eta_n \approx 2^{-n}$.

If we consider this sequence, that stretches enough quickly toward 0, the precision of the data are quickly overtaken and the process stops there: the diagram is only formed of a small number of points, insufficient to assure the validity of the result. Some methods exist to correct these problems, but it is however impossible to eliminate them all, because they are inherent to the simplest methods of treatment (for example, imprecision due to the fact that η_n only takes whole values and vary therefore always suddenly when one passes from η_n à η_{n-1}). However, in image processing, the important format of these images, associated to a least squares method evaluation of the slope of the cloud of points gotten with boxes of increasing size (algorithm of Keller, [Kel89], [Sar92]), give a good evaluation of the fractal dimension. We used this method with success in the case of lace patterns, as we are going to show it now.

3. Application

From the specific point of view of shape characterization of patterns, the main aim of the fractal approach is to find a measure to distinguish between curves with complicated contours (it is often the case with lace patterns). The main idea is to describe the complexity of the curve through a new parameter that makes it possible to identify a lot of complex patterns using a precise number, and thus to use this number in data bases indexation for example.

We now present figure 1 an example of lace pattern. This image is classically treated:

- Histogram treatment
- Filtering
- Contours detection

The result is presented figure 2. Then the Keller's algorithm (presented in section 2) is applied on this binary image, and the result is presented figure 3.

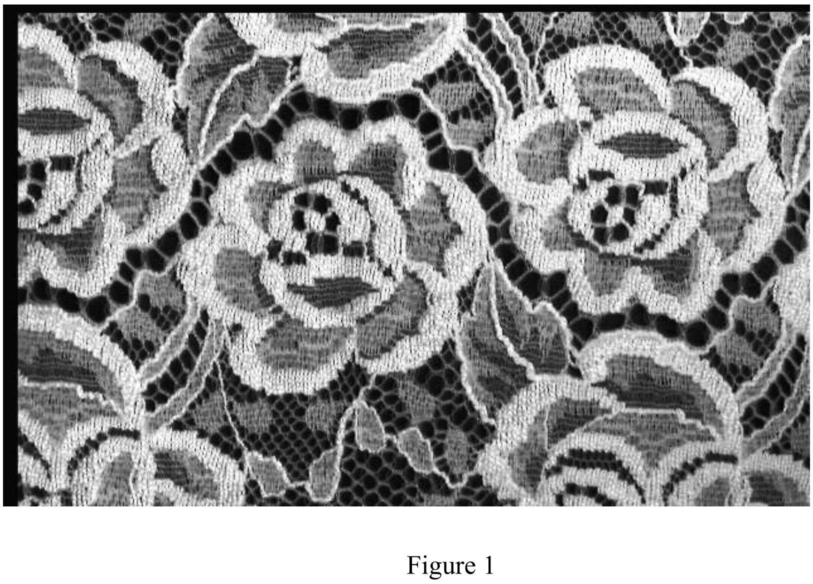

Figure 1

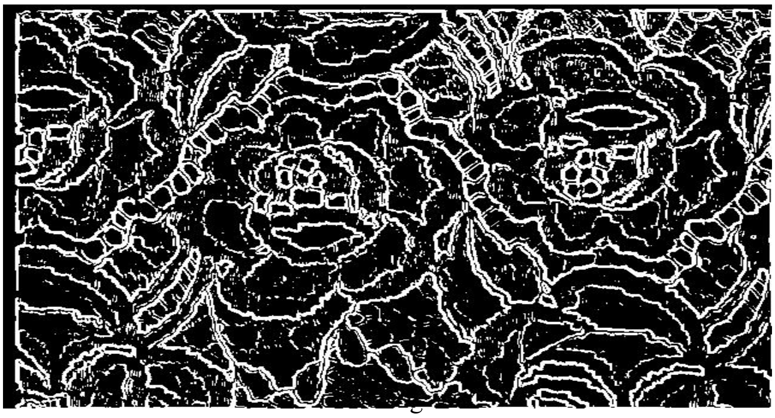

Figure 2

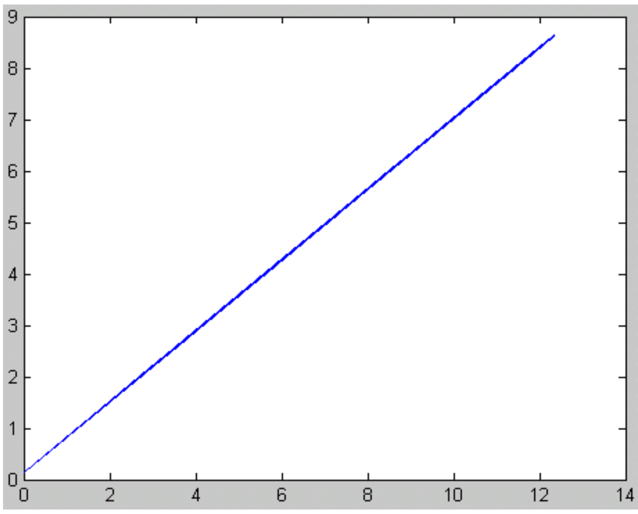

Figure 3

Pattern	Fractal index	« White » points
N°1	**0.6884434668**	**48244**
N°2	**1.2116501521**	**101367**
N°3	**1.2143531957**	**108005**
N°4	**1.2011347349**	**80536**
N°5	**1.2042286064**	**87065**
N°6	**1.2357822113**	**127380**

Figure 4: Results

We now present figure 4 the results (fractal index) from some different lace patterns. For the moment this treatment has been applied on some hundreds of patterns, and the results are very interesting: each pattern is perfectly characterised, and the fractal index is so very interesting to build a data base relative to lace patterns. Indeed lace patterns that have close designs are set with close indexes.

4. Conclusion

Pattern recognition requires the extraction of features from the images, and the processing of these features with a pattern recognition algorithm. In this paper, we presented some results which aimed at showing that fractal feature, based on the estimating fractal dimension, is relevant in pattern recognition tasks. The motivation behind using fractal transformation is to develop a high-speed feature extraction (for complex patterns), and then a high speed pattern index for graphical data bases. The problem considered here was the possibility to dispose a reliable feature to set numerous patterns (some hundred of thousands patterns!). Experiment results show that this approach allows us to obtain new and interesting descriptions of complex patterns. It would be interesting now to use a multi-fractal approach (or an other multiresolution method) to compute information conserving micro-features, and to obtain a finest description of lace patterns. We think that there will be a lot of applications possible for the future in the lace industry.

References

[Adl92] P. Adler, "*Porous media: geometry and transport",* Butterworth/ Heinemann, Stoneham, MA, 1992.

[Avn89] D. Avnir, "*The fractal approach to heterogeneous chemistry",* Wiley, New York, 1989.

[Bar88] M. F. Barnsley, R. L. Devaney, B. B. Mandelbrot, H-O. Peitgen, D. Saupe, R. Voss, " *The science of fractal images",* Springer-Verlag, Heidelberg, 1988.

[Bro92] G. Brown, G. Michon, J. Peyrière, "On the multifractal analysis of measures", *Journ. Of Stat. Phys.,* T. 66, pp. 775-790, 1992.

[Fis98] Y. Fisher, "*Fractal image encoding and analysis",* Chapman and Hall, New York, 1994.

[Kel89] J. M. Keller, S. Chen, R. M. Crownover "Texture description and segmentation through fractal geometry", *CVGIP,* 45, pp. 150-166, 1989.

[Man77] B.B. Mandelbrot "*Fractals: form, chance and dimension" ,* Freeman, San Francisco, CA, 1977.

[Man82] B.B. Mandelbrot "*The fractal geometry of nature" ,* W. H. Freeman and Co, New York, 1982.

[Sar92] N. Sarkar, B. B. Chaudhuri, "An efficient approach to estimate fractal dimension of textural image", *Pattern Recognition,* Vol. 25, n°9, pp. 1035-1041, 1992.

[Tri93] C. Tricot , "Courbes et dimensions fractales", *Springer Verlag,* 1993.

[Veh96] J. Lévy Véhel, "*Introduction to the multifractal analysis of images"* , in Fractal image encoding and analysis, Yuval fisher editor, Springer Verlag, 1996.

[Veh02] J. Lévy Véhel, "*Signal enhancement based on holder regularity analysis"* , IMA volumes in mathematics and its applications, 2002.

[Vos86] Voss R.F., "Random fractals: characterisation and measurement" in *Scaling phenomena in disordered systems,* R. Pynn and A. Skeltorp, *Plenum Press*, pp. 1- 11, 1986.