NON-LINEAR MODEL REFERENCE CONTROL OF pH PROCESS: AN EXPERIMENTAL STUDY

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ABSTRACT

The control of pH is important in many processes including wastewater treatment, chemical processes and biological processes. This paper considers a model reference non-linear controller developed by Jayadeva et al. (1990a). The method is tested using a 7-litre continuously stirred tank reactor to neutralise a strong acid using a strong alkaline solution. The method is first realised using a simulation of the process. Subsequently it is demonstrated on an experimental rig using real-time control. Experimental results confirm that a robust control of the process is achievable.

INTRODUCTION

In many processes, pH neutralisation is a very fast and simple reaction. In terms of practical control, it is recognised as a difficult control problem (Shinskey 1973; Pishvaie et al. 2000; Wright et al. 1991). The difficulties arise from high process nonlinearity (the process gain can change tens or hundreds of times over a small pH range) and from changes in the pH characteristics due to changes in influent concentration. Various techniques have been developed to control process pH. Young and Rao (1986) presented a variable structure controller ("sliding mode control for a neutralisation process") involving strong acids and bases. Parrish and Brosilow (1988) used non-linear inferential control in a simple simulated neutralisation process, using static estimation of the concentration of a single monoprotic weak acid. Kulkarni et al. (1991) presented non-linear internal model control for a simulated system of sodium hydroxide (NaOH) and hydrochloric acid (HCl). Li et al. (1990a) and Li and Biegler (1990b) presented non-linear feedback methods for a simulated neutralisation process. In the present work, a non-linear controller design is implemented. It uses a design procedure presented by Jayadeva et al. (1990b). The controller is implemented practically on a 7-litre reactor.

THEORY

In the present work, the design of a robust non-linear controller is introduced. It considers a model reference controller developed by Jayadeva et al. (1990a). The method is taken originally from a paper by Yuocef -Toumi and Ito (1987). The control scheme is illustrated in Figure.1.

Controller Design

Consider a single input and single output (SISO) state variable system of the form

$$x_1 = f_1(x_1, x_2, \ldots x_n) + g_1(x_1, x_2, \ldots, x_n)u + d_1(x, t)$$
$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad .$$
$$x_n = f_n(x_1, x_2, \ldots, x_n) + g_n(x_1, x_2, \ldots, x_n)u + d_n(x, t) \qquad (1)$$

$$y = c_1x_1 + c_2x_2 \ldots + c_nx_n \qquad (2)$$

where, u is a scalar manipulative input, $x_1, x_2, \ldots x_n$ are the states and y is a scalar output. f_i and g_i are nonlinear functions of state variables. $d_1, d_2, \ldots d_n$ represent general disturbances. The output variable y is a linear function of the state variable. $c_1, c_2, \ldots, c_n$ are constant scalars. Yuocef-Toumi and Ito presented a robust nonlinear feedback controller design for a general nonlinear multi-input state variable system, from which a least square solution for the manipulative variable was obtained. The method is applied to the specific form of Equation (1) and (2) to obtain an exact solution for the manipulative variable. Equation (1) and (2) can be written in vector form as

$$\dot{x} = f + gu + d \qquad (3)$$

$$y = cx \qquad (4)$$

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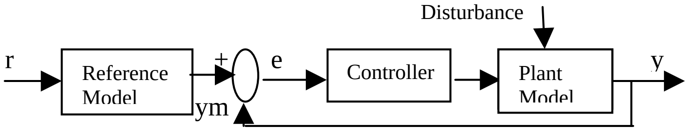


Figure 1: Model Reference Control Scheme

where the vectors f and g are functions of x, and c is a constant row vector. Let us assume the reference model in the scalar output y_m is given by

$$\dot{y}_m = \lambda_m y_m + b_m r \tag{5}$$

where, λ_m is the eigen value of reference model. The scalar e is defined as the difference between reference value and the process output. Therefore,

$$e = y_m - y \tag{6}$$

The control objective is to force the error to vanish with a desired dynamics:

$$\dot{e} = \lambda e \tag{7}$$

Where, λ is the eigen value for the error system. By combining Equations (3) – (7) we obtain the equation that governs the error dynamics. Therefore,

$$\dot{e} = \dot{y}_m - \dot{y} \tag{8}$$

$$= \lambda_m y_m + b_m r - cf - cgu - cd$$

$$as \qquad \lambda_m e = \lambda_m (y_m - y) \ \ therefore,$$

$$= \lambda_m e + (\lambda_m y + b_m r - cf - cgu - cd) \tag{9}$$

It is possible to determine the manipulative variable u in Equation (9) such that

$$(\lambda_m y + b_m r - cf - cgu - cd) = ke \tag{10}$$

From which we have the manipulative variable

$$u = (cg)^{-1} [b_m r - cf - cd - ke + \lambda_m y] \tag{11}$$

Therefore Equation (9) becomes

$$\dot{e} = (\lambda_m + k)\, e$$

$$\dot{e} = \lambda\, e \tag{12}$$

Where, k is a scalar error feedback gain. The error system eigen value λ can be assigned arbitrarily through proper choice of the error feedback gain k. The control law in Equation (11) is used to calculate u in order to get the desired error dynamics (Jayadeva et al. 1990a).

Now consider the application of the above control design to the model for the pH process described by McAvoy et al. (1972). The process consists of a strong acid flowing into a constant volume tank which is thoroughly mixed with a strong base. The feed flow rate of the base is to be controlled in such a way as to produce a neutral outlet from the tank. The equation describing this process is given by

$$\dot{x} = a_1 x - a_3 u(x + a_2) + a_1 D \tag{13}$$

Where, x is the deviation from neutrality. Note that, x and the pH value y are related by the non-linear equation:

$$x(t) = 10^{-y(t)} - 10^{y(t)} K_w \tag{14}$$

Where K_w = water equilibrium constant = 10^{-14}, $a_1 = F_1 / V$, F_1 is the acid flow in litres and V the volume of the mixing tank; $a_2 = C_{base}$ = concentration of base; $a_3 = 1/V$ are constant parameters; $u = F_2$, is the manipulative variable, base flow control in litres; $D = C_{acid}$ = concentration of acid = the disturbance variable. It is to be noted that Equation (14) is valid for the strong acid / strong base case only. For the general case, there are two model equations (Wright et al. 1991; Shinskey 1973).

Now, comparing Equation (13) with (3), we have,

$$f(x) = -a_1 x; \quad g(x) = -a_3 (x + a_2); \quad d(t) = a_1 D$$

And the output equation,

$$h(x, y) = x + 10^{y-14} - 10^{-y} = 0 \tag{15}$$

The control objective is to keep $y(t) = 7$ = constant in the presence of disturbances occurring in the process in general, making $y(t)$ follow a given reference trajectory. In the control design, the output equation is a linear function of the state variables. But, Equation (15) is a non-linear implicit output equation. Hence, for this nonlinear process, the controller design procedure requires to be suitably modified. Therefore we apply the following partial differentiation identity to Equation (15):

$$\frac{\partial h}{\partial y}\dot{y} + \frac{\partial h}{\partial x}\dot{x} = 0 \tag{16}$$

$$hence$$

$$\dot{y} = -\left\{ \frac{\partial h}{\partial x} \Big/ \frac{\partial h}{\partial y} \right\} \dot{x} \tag{17}$$

Using Equations (3), (6), (8) and (17) we get:

$$\dot{e} = \lambda_m y_m + b_m r + \left[(\partial h/\partial x)/(\partial h/\partial y)\right]\left[f(x) + g(x)u + d(t)\right] \quad (18)$$

$$= \lambda_m y_m + b_m r + J\left[f(x) + g(x)u + d(t)\right] \quad (19)$$

$$\text{where } J = (\partial h/\partial x)/(\partial h/\partial y) \quad (20)$$

If we make,

$$\lambda_m y_m + b_m r + J\left[f(x) + g(x)u + d(t)\right] = ke \quad (21)$$

then the control law is calculated as:

$$u = -(Jg)^{-1}(\lambda_m y + b_m r - ke + Jf + Jd) \quad (22)$$

Equation (19) becomes:

$$\dot{e} = (\lambda_m + k)e \quad (23)$$

$$= \lambda e \quad (24)$$

It is to be noted that since the disturbance term d(t) appears in the control law, it is essentially a combined feedback-feedforward control action (Jayadeva et al. 1990a). The expression for the control law of Equation (22) in terms of the plant variable y only is given by

$$u(t) = \left[(10^{y-14} + 10^{-y})(2.303)(\lambda_m r + b_m r - ke) - a_1(-10^{y-14} + 10^{-y}) + a_1 D\right] \Big/ \left[a_3(-10^{y-14} + 10^{-y}) + a_3 a_2\right] \quad (25)$$

EXPERIMENTAL SET UP

Figure 1 shows the experimental set up for the pH neutralisation system. The process stream (influent) consists of a diluted strong acid (HCl) and the titrating stream is a more concentrated strong base (NaOH). Table 1 consists of typical operating conditions. The process stream is fed through two feed tanks, and a 3-way valve is placed in the feed line, which allows switching between two different feed concentrations. A remote control peristaltic pump (RM pump) is used to control the flow rate of the titrating stream. The volume of the reactor vessel is kept constant at 5-litres with an over flow system.

An agitator is used to ensure proper mixing. The pH of the influent, the pH of the mixture in Continuously Stirred Tank Reactor (CSTR) and the influent flow are measured by a data acquisition system (National Instruments E series I/O card and a PC with LabVIEW Instrumentation package). The control objective is to maintain the pH value at the set point = 7. The control output is calculated according to the non-linear model reference control law

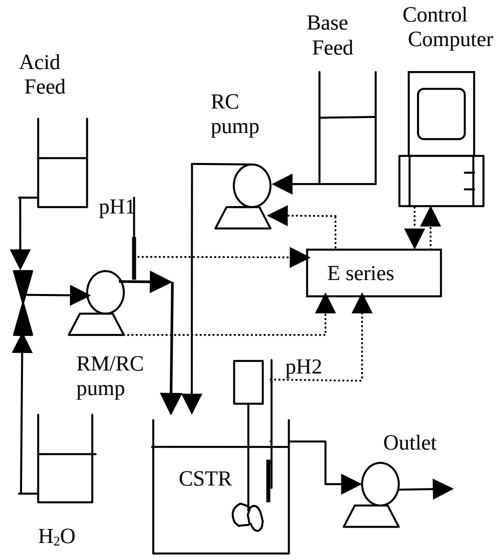


Figure 2: Laboratory set-up of the pH Neutralisation Process

(Equation (25)). The digital output is converted to an analogue output, and the signal is transmitted to a remote control peristaltic pump (RC pump) that controls the base flow rate. The sampling time for the measurements is 0.1 of a second and the control law is executed at approximately the same time (considering the time taken for control computation by the package and the operating system).

SIMULATIONS

A continuous time simulation of the controller was undertaken to confirm the results obtained by Jayadeva et al. (1990a). Figure 3a shows the open loop response of the plant for a 100% disturbance in the concentration of the influent at 1.4 seconds. Figures 3b and 3c are continuous controlled responses of the plant and the controller

Table 1- Typical operating conditions for the pH neutralisation process

Parameters	**Values**
Acid Flow (F_1)	Variable
Base Flow (u)	Manipulative variable
Conc. Of acid (D)	0.01M – 0.005M
Conc. Of base (a_2)	0.2M
Volume (V)	5 litres

respectively for the same disturbance at the same time. Figure 3d is the simulation response of the plant for a change in the operating point from pH 7 to pH 3 and the disturbance in concentration at 1.4 seconds. The controller responds robustly to both the disturbances. The robustness of the controller was also confirmed with a disturbance in the flow of the influent along with concentration.

Finally, the continuous controller is studied with sampled input and output signals before practical implementation. Hence zero order holds are applied to model this effect on the continuous process (Figure 4). The Simulink model incorporates the change in disturbance with respect to time as shown in Figure 4. The effect due to the change in influent flow was also studied with slight modification in the model. Analysis was done for the allowable sampling time for real-time implementation with Zero–order hold at both input source and output sampling.

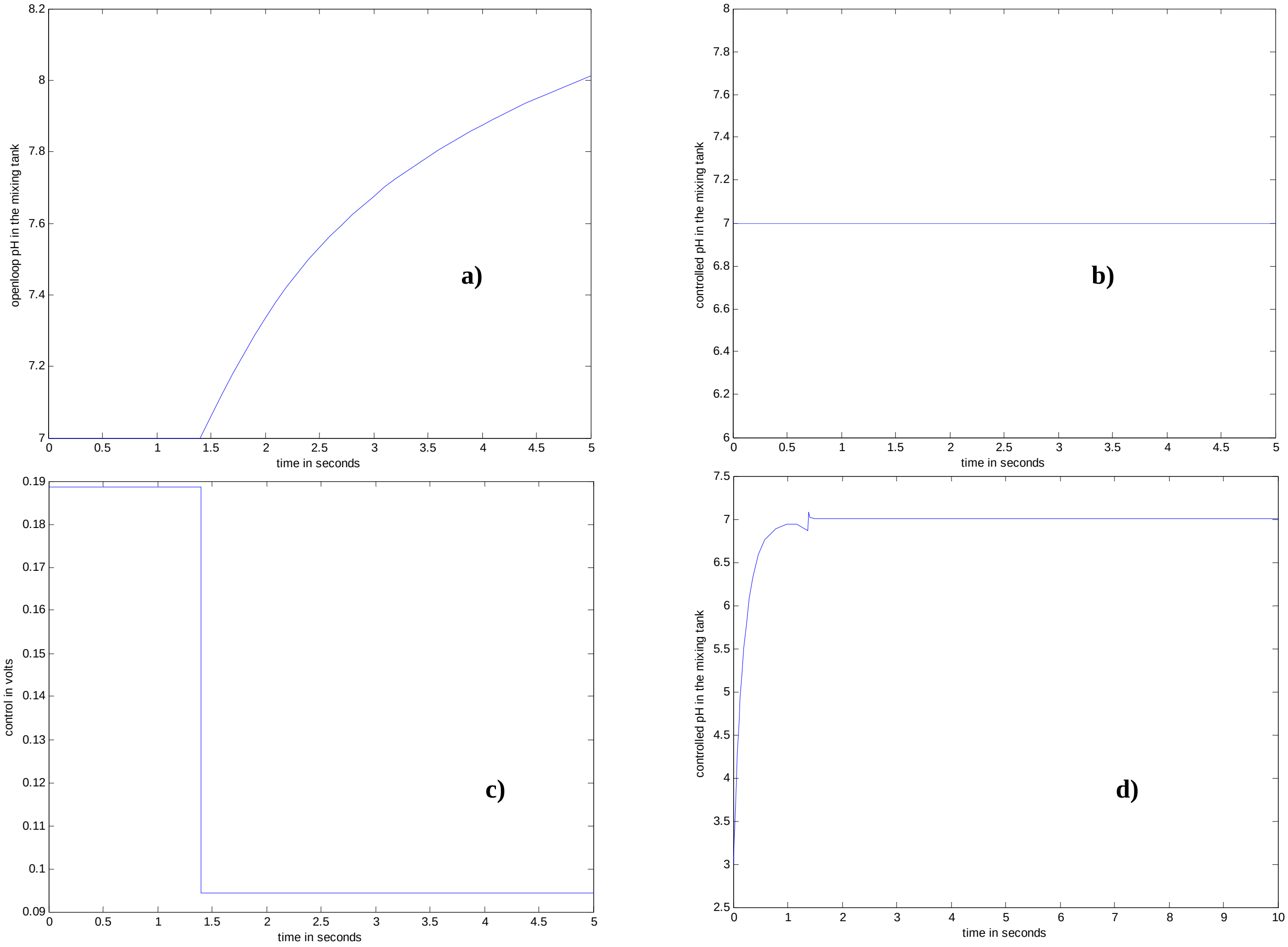
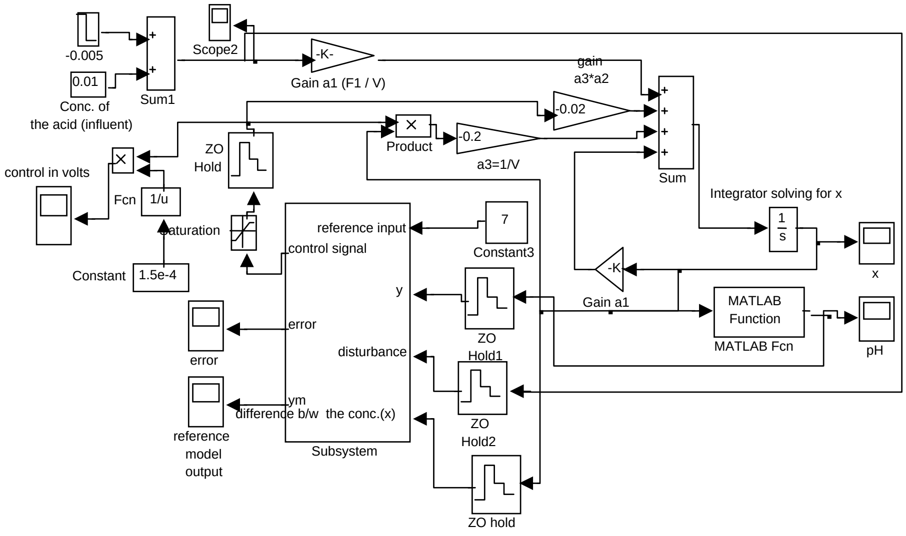


Figure 3: a) Step response simulation of the plant for a disturbance of 100% reduction in influent concentration.
b) Controlled pH value for the same disturbance at 1.4 seconds as in a).
c) Controller in action for the disturbance of 100% reduction in the influent concentration.
d) Controlled pH for a simulation of the controller at different operating point along with the disturbance at 1.4 seconds as in a)

REAL TIME IMPLEMENTATION OF THE CONTROLLER

The pH sensor is assumed to be linear and the temperature is assumed to be constant (Shinskey 1973). pH sensors have very high source impedance and it is therefore necessary to use a high input impedance buffer amplifier. A low pass filter is used to reject AC mains 50Hz.



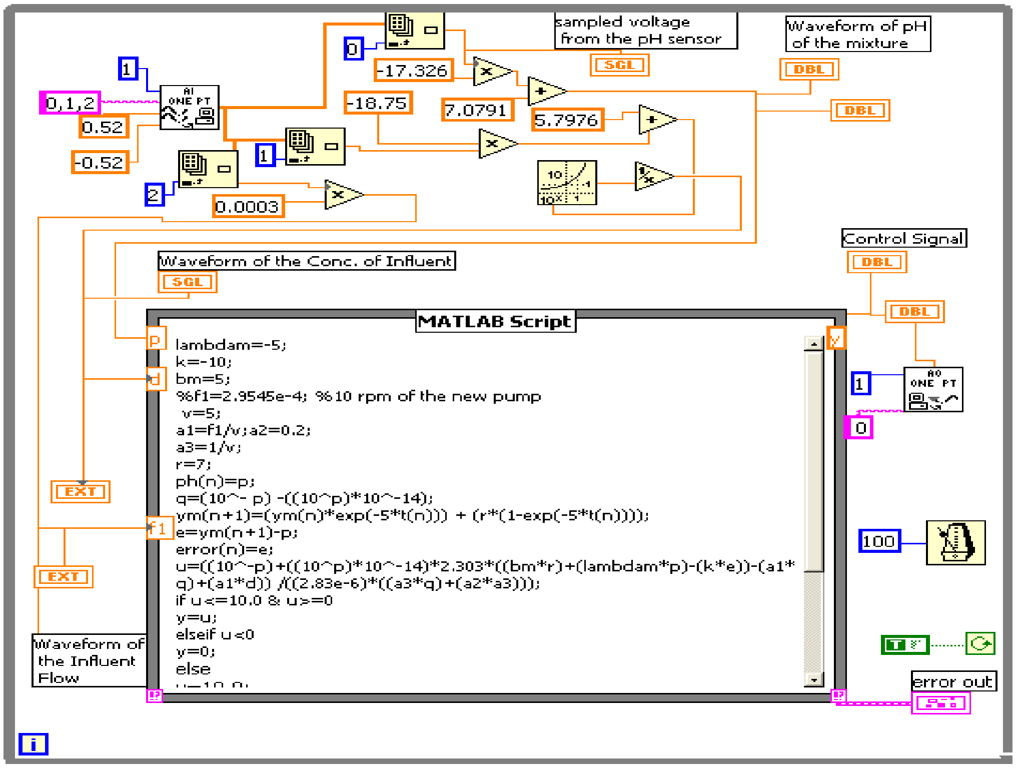
Closed Loop Simulation of the 7-Litre Experiment Rig with Reference Initial pH value at 7 and the plant initial value of x =0

Figure 4: Simulink Model of the Process.

Differential analogue input mode is preferred to single channel analogue input of the I/O card for sensor signal feed, as Common Mode Rejection Ratio (CMRR) is very high in this mode. The control signal range for the pump so that it responds linearly is 0-10 volts. The concentration of the solutions is accordingly chosen considering the constraint.

Software Platform

LabVIEW, a real-time virtual instrument package, is used to implement the control strategy. The easy accessibility of Matlab code within the LabVIEW environment is utilised for complete implementation and data retrieval. Figure 5 partly shows the VI diagram of the program for 0.1of a second sampling delay.



```
lambdam=-5;
k=-10;
bm=5;
%f1=2.9545e-4; %10 rpm of the new pump
 v=5;
a1=f1/v;a2=0.2;
a3=1/v;
r=7;
ph(n)=p;
q=(10^- p) -((10^p)*10^-14);
ym(n+1)=(ym(n)*exp(-5*t(n))) + (r*(1-exp(-5*t(n))));
e=ym(n+1)-p;
error(n)=e;
u=((10^-p)+((10^p)*10^-14)*2.303*((bm*r)+(lambdam*p)-(k*e))-(a1*
q)+(a1*d)) /((2.83e-6)*((a3*q)+(a2*a3)));
if u<=10.0 & u>=0
y=u;
elseif u<0
y=0;
else
```

Figure 5: Pictorial VI Block Diagram of the pH Process

ANALYSIS

The controller is tested for the most common disturbances, which are the change in the flow of the influent and the concentration. The experiment is conducted approximately for 3 minutes with the change in concentration after 1 minute (Figure 6) and then the change in flow after 2 minutes. The controller robustly responds for the disturbances with no apparent change in the pH of the mixture. It was inferred during simulation that, the maximum sampling time can be 0.2 second. But the response of the plant was not as quite the continuous time response (Figure 3). The reason can be studied further. The instantaneous control values were noted to be the same as the continuous time values.

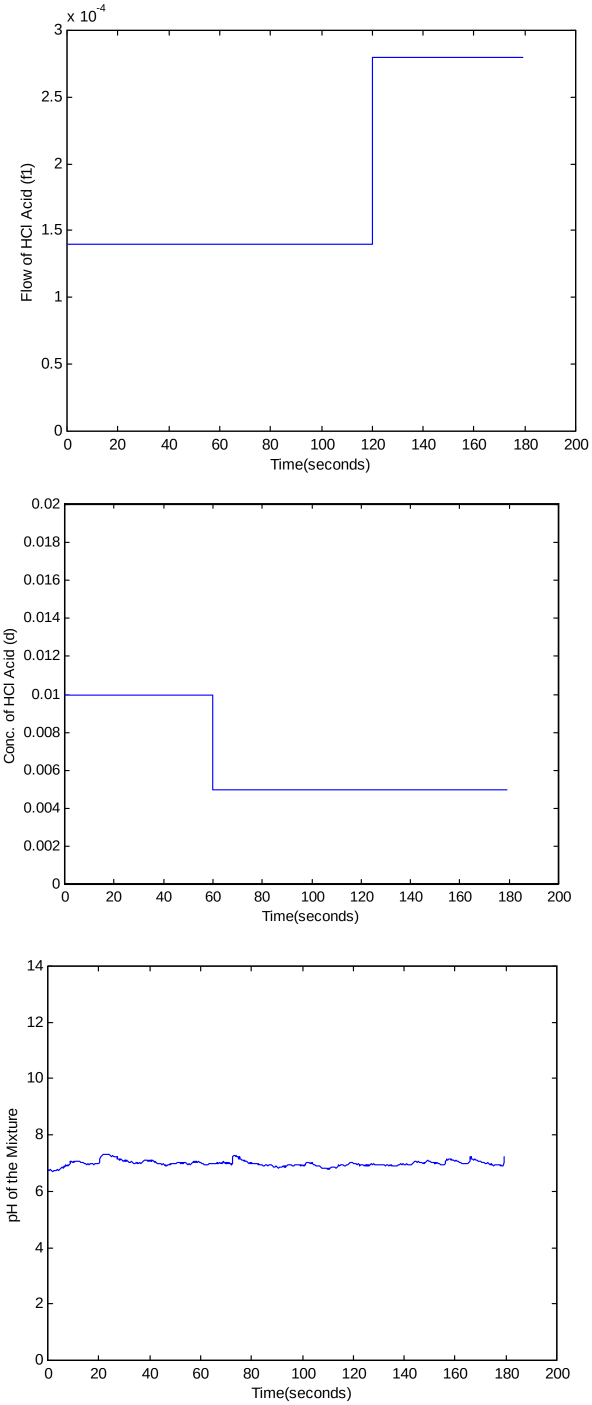


Figure 6: pH response of the mixture in the CSTR for the disturbances in the influent flow (after 2mins and concentrations (after1min)

CONCLUSION

This study is a part of the research to propose a non-linear adaptive control scheme for pH control of wastewater and implement it on an industrial scale for a water company in the United Kingdom. A lot of research is only simulation based understandably due to many factors such as cost etc. Therefore, the importance of real-time implementation has also been emphasised in this study. This study has opened doors for further investigation into simulation and real-time implementation. As this study aimed at exploring requirements to liaise software with hardware, the experimentation has been successful in doing so.

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