

# TOOLS OF MONTE CARLO SIMULATION IN INVENTORY MANAGEMENT PROBLEMS

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## KEYWORDS

Inventory Management, Supply Chain design, Monte Carlo Method, Spreadsheet Simulation, Simulation Software, Business Game, Cost Analysis.

## ABSTRACT

This paper describes the simulation model of supply chain and its implementation using general purpose tool and the simulation package. The output of Monte Carlo experiments taken both from spreadsheet formulas in Microsoft Excel and from graphical environment of Extend software was confronted and revised and then model was used to find the minimal inventory cost.

## A BRIEF DESCRIPTION OF THE INVENTORY MANAGEMENT'S BASIC CONCEPTS

According to (Heizer and Render 2001) there are four types of firm's inventory:

- Raw material inventory, which are at command of the firm. The main purpose of maintain of these items is to eliminate supplier variability in quality, quantity or delivery time.
- Work-in-process inventory, which are processed inside the firm. It means that changes has been made but not ended yet.
- Inventories which are devoted to maintenance, repair and operating. It refers to assure current running of plants and devices etc.
- Finished goods inventory: the completed products waiting for shipping. They arise because values of future demand are unknown.

There are two types of models of inventory control in relation to demand for other goods: dependent (for instance portable FM receivers and batteries) and independent (for instance TV receivers and FM receivers). In our paper, we will study the issues related only to independent demand.

The quality of implementation of inventory control policy is evaluated according to total inventory cost over certain time. Total inventory cost is composed of holding costs (also including insurance cost, handling etc.), ordering cost (also transportation cost, packing and setup cost, administration cost) and stockout cost (storage cost). The main purpose of model analysis is generally to minimize the total costs. One can assume sometimes that the demand is known or constant but

most often this assumption have to be rejected and it is necessary to specify the probability distribution of demand. This leads to a probabilistic model. That sort of models are sometimes evaluated by service level, which usually can be measured at a number of points in the supply chain e.g. X service levels means that the products is available X percent of the time or X out of a hundred customers will buy the product. To reduce stockouts one can increase inventory in comparison with results from non probabilistic model. Size of safety stock depend on stockout and maintain costs, including ordering and storage cost. There are two patterns (approaches) of inventory controlling and scheduling – push (for instance MRP, based on production schedules, developed for production stages according to demand forecasts) and pull system (for instance *just in time* and *kanban*). We will present the simulation model of a pull system. It is based upon the direct and immediate ordering products or components by the customer from the supplier who delivers it at required time and amount.

## IMPLEMENTATION OF THE INVENTORY MANAGEMENT BUSINESS GAME

Authors investigated the process of implementation of business game, based upon kanban approach which applies the probabilistic model of goods demand and time of fulfilling the order by the supplier.

Our considerations were based on inventory management model from (Heizer and Render 2001) but very similar approach is presented in (Lawrence and Pasternack 2001; Jensen and Bard 2003).

## Guidelines of the Inventory Management Business Game

There are two decision variables: order quantity and reorder point and two probabilistic components: daily demand and reorder lead time. The main purpose of simulation runs is to try out various schemes of order quantities and reorder point and to find (to minimize) the smallest total inventory cost. That means that inventory cost includes ordering cost, holding cost and stockout costs.

The elements of pull system model are as follows:

- Daily demands for items, collected from observation of past days. That historical frequency will be converted

into a probability distribution and cumulative distribution for the variable daily demand

- Lead time (in days), collected from observations of past orders. That historical frequency will be converted into a probability distribution and cumulative distribution for the variable lead time
- Beginning inventory (in units)
- Reorder point (in units)
- Order quantity (in units)
- Cost of placing order
- Holding cost per unit
- Cost of each lost sale (stockout).

Apart from the above mentioned, the additional assumptions were formulated:

- Orders (if necessary) are placed at the end of the day: the day's ending inventory is checked and if inventory is less or equal reorder point then order is placed.
- Lead time equal 1 day stands for that the order will arrive next morning after next day.
- After arriving at the morning, order quantity is added to inventory, before receiving of the new demand.

### Implementation of Monte Carlo Simulation in a Spreadsheet

Stages of spreadsheet's simulation building were formulated for instance at (Seila 2002). According to definition of Monte Carlo simulation (Lawrence and Pasternack 2001), simulated events take place randomly and match the description of the theoretical probabilities derived from acquired experiences. The process of fundamental importance in Monte Carlo simulation is called random number mapping and consists in matching the random number with simulated events (when they occur and how long they last). Many examples of applications of Monte Carlo method with Microsoft Excel spreadsheet as a base for making simulation experiments can be found. See (Heizer and Render 2001), (Jensen and Bard 2003) and (Lawrence and Pasternack 2001). We did similar task, but using more flexible formulas and data tables (see screenshots at Figures (1, 2 and 3) for collecting simulation outcomes. Figure (1) presents the frequencies of demand and lead time, probabilities and cumulative probabilities and the way of assigning random-number intervals to get the proper outcome in a computer simulation. Figure (2) shows one of 20-day simulation runs. Average and total number of units received, simulated demand, ending inventory, stockouts and placed orders are calculated. Figure (3) presents data table with one variable, used to collect 60 replications (runs) of simulated total stockouts during 20 days. A short procedure in Visual Basic is necessary to collect output values and to ensure correct calculation after sheet refreshing. Other (missing) values of problem description parameter, like reorder point, quantity order and beginning inventory are visible in Figures (1 and 2). We will use the data also to validate Monte Carlo approach model in Extend, Imagine That Inc. environment. It's worth mentioning, that outcomes of

our spreadsheet and the results discussed in (Heizer and Render 2001) are statistically similar. For example, during applying ANOVA test we obtained critical value of the F distribution (significance level =0.05, 1 and 58 degrees of freedom) equal 4 and source has F-value equal 1.55 and that means that both populations are not varied.

	A	B	C	D	E	F	G	H	I	J
30										
31		5	reorder point	30						
32		10	quantity order	80						
33										
		Demand	Frequency (periods)	Probability	Cumulative Probability		Beginning of random-number interval	Ending of random-number interval		
34										
35		0	10	0,033	0,033		1	1		
36		1	40	0,133	0,167		2	5		
37		2	60	0,2	0,367		6	11		
38		3	100	0,333	0,7		12	21		
39		4	60	0,2	0,9		22	27		
40		5	20	0,067	0,967		28	29		
41		6	10	0,033	1		30	30		
42		Total	300	1						
43										
44										
45										
		Lead time (periods)	Frequency	Probability	Cumulative Probability		Beginning of random-number interval	Ending of random-number interval		
46										
47		1	10	0,125	0,125		1	10		
48		2	25	0,313	0,438		11	35		
49		3	15	0,188	0,625		36	50		
50		4	30	0,375	1		51	80		
51		Total	80	0,625						
52										

Figures 1: First Part of our MS Excel Spreadsheet

The key formulas of our spreadsheet are as follows (starting from the cells in 4th row, from column B):

Number of units received: " =IF(L3=A4;\$B\$32;0)".

Beginning inventory: " =F3+B4".

Simulated demand: " =INDEX(\$B\$35:\$B\$41 MATCH(D4;\$G\$35:\$G\$41;1))"

First random number: " = \$E\$31-INT(RAND()\*\$E\$31)"

Ending inventory: " =IF(C4-E4>0;C4-E4;0)"

Stockouts: " =IF(C4-E4>0;0;E4-C4)"

Inventory not below reorder point?  
" =IF(F4<=\$B\$31;1;0)"

Placing order: " =IF(AND(H4=1;A4>=L3);1;0)"

Second random number:

“=IF(AND(H4=1;A4>=L3); \$E\$32-INT(RAND()\*\$E\$32);0)”

	A	B	C	D	E	F	G	H	I	J	K	L
1	Period (day)	Number of units received	Beginning inventory	First random number (01-30)	Demand (in units)	Ending inventory units	Lost sales in current period	Order needed?	To place order really (no order is outstanding)?	Second random number (01-80)	Lead time	Planned arriving period
2												
3					10							
4	1		10	19	3	7						
5	2		7	15	3	4		1	1	50	3	6
6	3		4	16	3	1		1				6
7	4		1	30	6	0	4	1				6
8	5		0	27	4	0	4	1				6
9	6	10	10	14	3	7						6
10	7		7	28	5	2		1	1	61	4	12
11	8		2	15	3	0	1	1				12
12	9		0	11	2	0	2	1				12
13	10		0	5	1	0	1	1				12
14	11		0	26	4	0	4	1				12
15	12	10	10	3	1	9						12
16	13		9	9	2	7						
17	14		7	15	3	4		1	1	21	2	17
18	15		4	12	3	1		1				17
19	16		1	21	3	0	2	1				17
20	17	10	10	18	3	7						17
21	18		7	6	2	5		1	1	62	4	23
22	19		5	3	1	4		1				23
23	20		4	2	1	3		1				23
24												
25	Average	1,5			2,75	3,05	0,9		0,2			
26	Total	30					18		4			

Figures 2: Second Part of Our MS Excel Spreadsheet

	N	O	P	Q	R	S
3	Total stockouts		Total stockouts		Total stockouts	
4	Replication		Replication		Replication	
5	1	24	21	14	41	10
6	2	12	22	24	42	19
7	3	22	23	15	43	13
8	4	12	24	8	44	18
9	5	21	25	13	45	25
10	6	7	26	18	46	16
11	7	32	27	15	47	17
12	8	13	28	16	48	15
13	9	12	29	18	49	21
14	10	7	30	23	50	17
15	11	11	31	20	51	20
16	12	17	32	15	52	14
17	13	17	33	10	53	12
18	14	8	34	21	54	20
19	15	13	35	1	55	25
20	16	16	36	17	56	19
21	17	22	37	13	57	16
22	18	22	38	20	58	14
23	19	14	39	4	59	14
24	20	18	40	15	60	19
25					Average	16,07
26					Std. Dev.	5,49

Figures 3: Tabularization of Simulation Outcomes of 60 Runs.

Simulated lead time:

“=IF(L3<=A4+1;IF(I4=1;INDEX(B47:B50;MATCH(J4;sG47:G50;1));0);0)”

Planned arriving day: =IF(IF(\$K4>0;\$A4+\$K4+1;\$L3)>=\$A4;IF(\$K4>0;\$A4+\$K4+1;\$L3);0)

Fourth row of Figure (3) in columns O, Q, S contains formula “= B26” and other rows in column O, Q, S contain formula “=TABLE(;P2)”.

## RUNNING MONTE CARLO SIMULATIONS MODEL IN EXTEND

### Introduction to Modeling with Extend software

The equivalent model was built in graphical environment of Extend software. Then, two models were compared and validated by confronting the outcomes. Modelling in Extend (Krahl 2002) consists in using pre-built modelling components to build and analyse system. This approach is also applied in other software like Arena, Flexim and others. As we see on Figure (4), results from this model implementation is similar (average 15.467 and standard deviation 5.95) to MS Excel spreadsheet outcomes, Figure (3). It is worth mentioning that it was necessary to make two corrections into object’s way in model. First, we have to secure to the arriving „demand” object a very small amount of time after the arriving of „order” object (it is beginning of the day), second – we have to add 1 day for all lead times shown in Figure (1) before placing this information in Input Random Number (the block with histogram icon). After introductory validation of our Extend model, we think that discrete event model built in Extend is more flexible, because it is possible to easily prolong simulated time and the number of replications (runs), to see the animation of objects during the run and to optimise model variables (minimization of total cost). We will try to prove it in the next section.

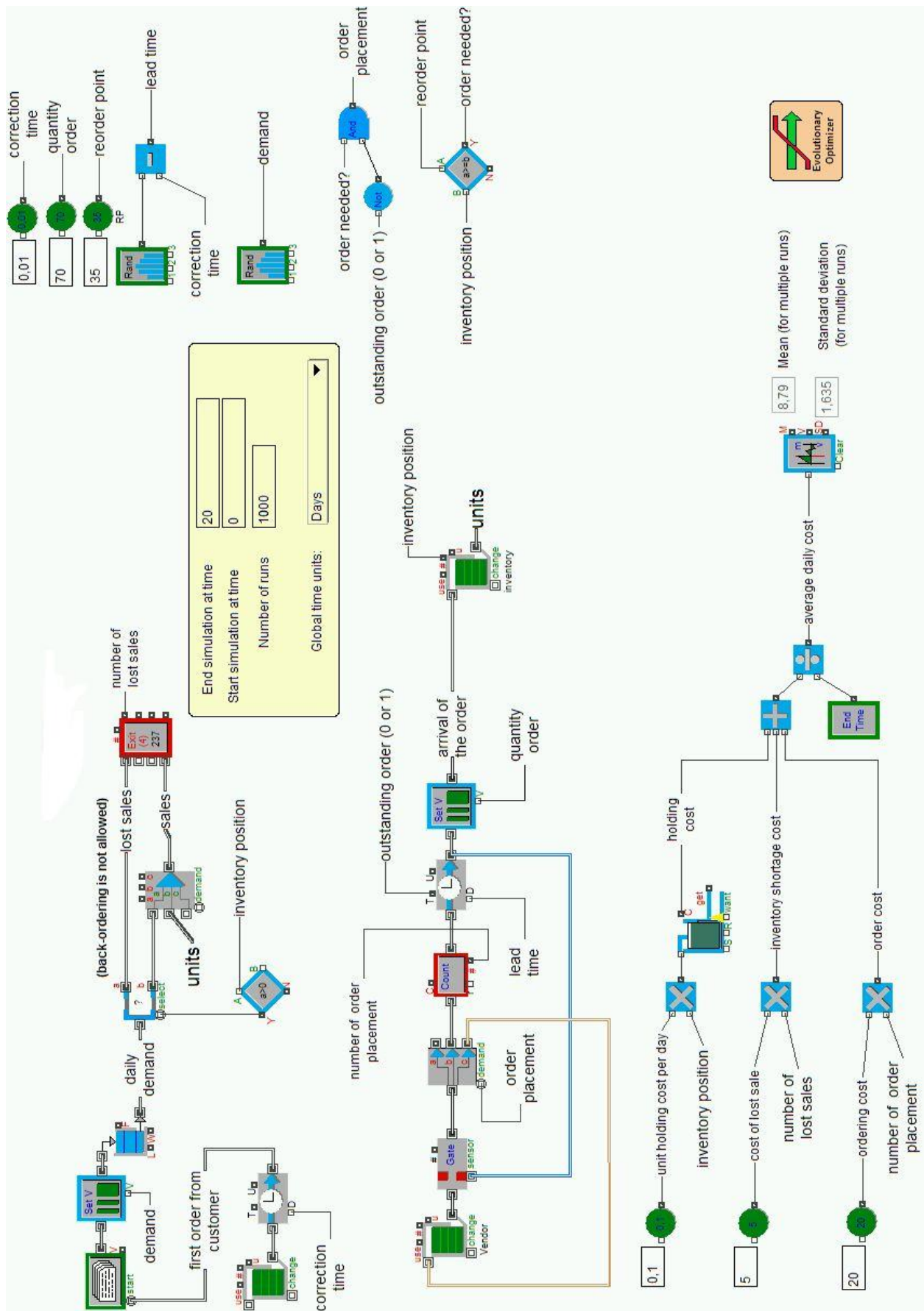
### Guidelines of the Inventory Management Business Game with Cost Accounting

We prepared the equivalent model to the previous one using the Extend software, but we extended it by cost coefficient taken from the example of inventory system simulation given in (Jensen and Bard 2003). Next, we validated it by comparing the optimal values found in our simulations with these presented in the book.

The set of assumptions are as follows: the daily demand oscillates between 6 and 16 units, lead time oscillates between 1 and 5 day, initial inventory is 50 units, cost per day of unit holding is 10 cents, cost of lost sale (stockout) is \$5, ordering cost is \$20, reorder point is 35 and order quantity is 70 units. Note: in this situation, lead time equal to 1 day means that the order will arrive next morning.







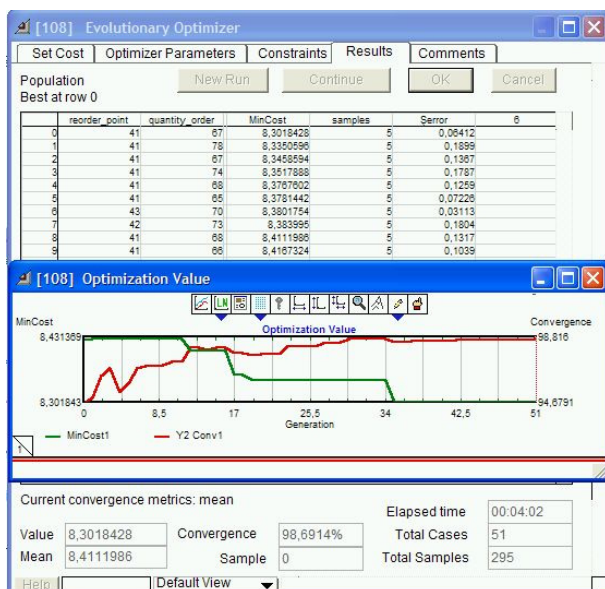
Figures 5: Inventory Management Model with Cost Calculations, Ready for Optimisation.

## Validation of Simulation Results and Parameter Optimisation

Results of 20-day simulation of inventory system (one of the replications) described in (Jensen and Bard 2003) book are as follows: average holding cost \$3.92, average order cost \$3, average shortage (lost of sale) cost \$0.25 and average total cost \$7.17.

The results of our experiment are presented on Figure (5): after 1000 replications of 20-day length period the average total cost is \$8.79 with standard deviation equal to 1.635. The results of 1000 replications given in (Jensen and Bard 2003) are very near to our outcome: the average simulated total cost for the mentioned values of reorder point and order quantity was \$8.77.

The next experiment was more sophisticated and it concerned the optimisation of average total cost. We changed values of reorder point and order quantity. We took an opportunity to use Extend ability to evolutionary optimisation that is approachable in „Evolutionary optimizer” block.



Figures 6: Results of Optimisation Process

We had to divide model parameters into two parts: independent variables (reorder point and order quantity) and dependent variable (in that case, goal function: average total cost of inventory) which will be minimized. We expanded length of simulation run to 1000 days because the beginning value of inventory could affect average cost and we would like to omit the bootstrap effect. Next step was to set minimum and maximum limit for variables. We assumed the reorder point (R) will be changed between 25 and 60 units and quantity ordered (Q) would be changed between 40 and 120 units, like in (Jensen and Bard 2003). They have located optimal solution at R=41 and Q=67 and it was \$8.17. Our solution (maximum samples per case = 5, maximum cases = 1000, member population size = 10 and termination if best and worst within 0.95) was

R=41 and Q=67 and estimated cost was \$8.30. The calculations took about 4 minutes. See Figure (6).

This paper is an excerpt of research in the field of application of simulation tools in supply chain management, beginning from 2002.

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