HIERARCHICAL EVOLUTIONARY SEARCH METHOD OF SOLVING NON-COOPERATIVE GAMES MODELS OF SELECTED DECISION-MAKING PROBLEMS FOR WATER RESOURCE MANAGEMENT

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ABSTRACT

In this paper we applied the parallel evolutionary strategy HGSNash to solving the decision making problem for water resource systems with external disagreement of interests. This problem is usually modelled as a non-cooperative strategic game. We define two different sets of water purifying cost functions for the water users and perform a simple experimental analysis. The efficiencies of HGSNash and others selected optimization methods were compared at the end of the paper.

INTRODUCTION

A water resource system can be defined as a system of methods of the water management in a given area. Some important decision problems connected with the system can be very difficult to solve, especially in the case of disagreement of the interests of the water users.

The disagreement can be internal or external. The recent methodology of decision making in the case of internal disagreement of interests exploits decision support systems which enable to solve multi-criteria problems. The external disagreement of interests of the decision makers is usually modelled as a non-cooperative game with the Nash equilibrium as the solution (Woźniak, 1995). This model can be used in practice because it does not oblige the decision makers to reveal their interests and it does not impose the centralized manner of the decision making. The Nash equilibria can be interpreted as steady states of a strategic game, in which each player holds correct expectations concerning the other players behavior (Straffin, 2004).

The problem of finding such equilibria points of the strategic games remains a challenging global optimization task. Algorithms for solving games have been studied from the beginning of game theory but usually they are very time consuming. HGSNash strategy applied in this paper is an evolutionary optimization method based on the Hierarchical Genetic Strategy (HGS) introduced by Kołodziej et al (Kołodziej et al, 2001).

The experimental analysis presented in this paper is a continuation of the research initialized in (Kołodziej et al, 2006). We defined a new set of water purifying cost functions for the water users. These functions define the case of better cost amortization in consequence of partial effluents recycling.

The remainder of the paper is organized as follows. Sections 2 and 3 contain the problem definition and the main idea of HGSNash. Results of performed numerical experiments are reported in section 4. The paper ends with some final remarks.

DECISION-MAKING PROBLEM FOR WATER RESOURCE SYSTEM

Let us consider that many water users pour their effluents in the common sewage treatment plant. A tax for the emission of effluents can be the reason of the disagreement of interests of the users.

The strategy decision variable of the user can be defined as the current pollution level of his effluents poured in the sewage treatment plant. The goal of the each user is to minimize the costs connected with the responsibility for the pollution of the natural environment. The cost function of the *i*-th user can be defined in the following way:

$$Q_i(x_1,...x_n) = k_i(y_i) + \alpha_i \cdot \sum_{j=1}^{n} x_j z_j + p_i(x_1,...,x_n) \quad (1)$$

where:

- $k_i\ (y_i)$ - the increasing and continuous function of the water purification cost paid by the *i*-th user,
- $y_i = b_i - x_i$ - the difference between the maximum (b_i) and the current (x_i) pollution level of water poured in the sewage treatment plant by the *i*-th user ($0 \le x_i \le b_i$),
- $t_i(x_1,...,x_n) = \alpha_i \cdot \sum_{j=1}^{n} x_j z_j$ - the *i*-th user tax for the water pollution (α_i - the tax coefficient, z_j - the amount of effluents discharged by

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the j-th user, $\sum_{j=1}^{n} x_j z_j$ - the level of water pollution in the whole sewage treatment plant)

- p_i $(x_1, \ldots, x_n)$ - the i-th user fine for exceeding the maximum pollution level q.

The p_i function is defined as follows:

$$p_i(x_i, \ldots x_n) = \begin{cases} 0, & if \sum_{j=1}^{n} x_j z_j \leq q \\ p_i^0, & oherwise \end{cases} \quad (2)$$

The values of parameters α_i, p_i^0 and q are estimated by the water resource management. These parameters are optimal for the users if the following condition is satisfied:

$$\sum_{j=1}^{n} x_j z_j \leq q \quad (3)$$

It was shown in (Petrosian and Zacharow, 1986) that parameters α_i, p_i^0 and q should be estimated according to the following rules:

$$\forall i \in N; \forall 0 \leq x_i^i \leq x_i^{''} \leq b_i :$$
$$k_i(b_i - x_i^{''}) - k_i(b_i - x_i^{'}) < \alpha_i z_i (x_i^{''} - x_i^{'}) \quad (4)$$

$$p_i^0 = k_i \left(b_i - \frac{b_i q}{\sum_{j=1}^{n} z_j b_j} \right) - k_i(0) - \alpha_i z_i \left(b_i - \frac{b_i q}{\sum_{j=1}^{n} z_j b_j} \right) \quad (5)$$

There exists an unique optimal solution satisfying those conditions. To find this solution we use the theory of non-cooperative games and the concept of the Nash equilibrium formulated in terms of the global minimization of the multi-cost function

Q: $S_1 \times \ldots \times S_n \rightarrow \mathbf{R}$ defined in the following way:

$$Q(s_1, \ldots s_n) = \sum_{i=1}^{n} \left[Q_i(s_1, \ldots s_n) - \min_{s_i \in S_i} Q_i(s_1, \ldots, s_n) \right] \quad (6)$$

where:

- $N = \{1,\ldots,n\}$ is the set of players,
- $S_1, \ldots, S_n$; ($card(S_i) \leq 2$; $i \in \mathbf{N}$) are sets of strategies for the players,
- $Q_1, \ldots Q_n$; Q_i: $S_1 \times \ldots \times S_n \rightarrow \mathbf{R}$ $\forall i \in \mathbf{N}$ are players cost functions,
- $s_1,\ldots,s_n$ are players decision variables, $s_i \in S_i$, $i=1,\ldots,n$.

The problem of the optimal estimation of these parameters is the main task for the management of the water resource system. The optimal solution for this problem should imply the optimal decisions of users, which decrease the values of their cost functions.

The game cost function has non-negative values and its global minimum is zero. The minimizing procedure for that function is composed of two cooperated units:

- **Main unit** - which solves the problem of the global minimization of the Q function,
- **Subordinate unit** - which solves the problems of the minimization of the users cost functions Q_i.

We need to minimize the cost functions of the users in the subordinate unit to compute the values of the cost function Q. We usually define a non-gradient optimization algorithm such as Powell algorithm for that unit.

The gradient computation for the function Q is impossible in many cases. Thus the non-gradient global optimization algorithms such as Powell, modified Controlled Random Search or evolutionary algorithms could be recommended as the main a unit algorithms.

We applied Hierarchical Genetic Strategy (HGS) (Kołodziej et al, 2001; Kołodziej, 2001) as the main mechanism of the optimization process of the game cost function Q.

THE MAIN IDEA OF HGSNash

HGSNash, introduced in (Kołodziej et al, 2006), is a parallel evolutionary strategy designed for the detection of the Nash equilibria as the solutions of the non-cooperative games. It is based on the Hierarchical Genetic Strategy (HGS).

HGS is a very effective tool in solving ill-possed global optimization problems with multimodal and weakly convex objective functions (Kołodziej, 2001). High efficiency of the strategy comes from the concurrent search in the optimization landscape by many small populations. The sequences of these populations are defined as the evolutionary dependent processes. HGS was successfully applied as a method of solving some practical engineering problems, for example for the estimation of the geometric errors of the Coordinate Measure Machine (Kołodziej et al, 2004).

HGSNash can be defined as a simple adaptation of the Hierarchical Genetic Strategy (HGS) to the global optimization of the multi-cost game function Q defined by the formula (3). This adaptation requires a modification of the main evolutionary mechanism in HGS due to the parallel structure of the multi-cost function optimization procedure (see Fig. 1). As the subordinate unit in HGSNash we applied the Powell non-gradient optimization algorithm.

The main unit algorithm in HGSNash is depended on the HGS implementation type. We apply the binary HGS with Simple Genetic Algorithm (SGA) as the evolutionary mechanism in the case of the binary HGSNash implementation and the floating point HGS with Simple Evolutionary Algorithm (SEA) in the case of HGSNash real implementation. We define SEA algorithm as (μ,μ)-ES with Gaussian mutation and proportionate selection. SEA is also called Evolutionary Search with Soft Selection (ESSS) in some papers (Galar, 1885).

A fixed number of iterations of HGSNash defines a metaepoch of the given period. The rough scheme of the *k*-periodic metaepoch procedure is presented on Figure 1.

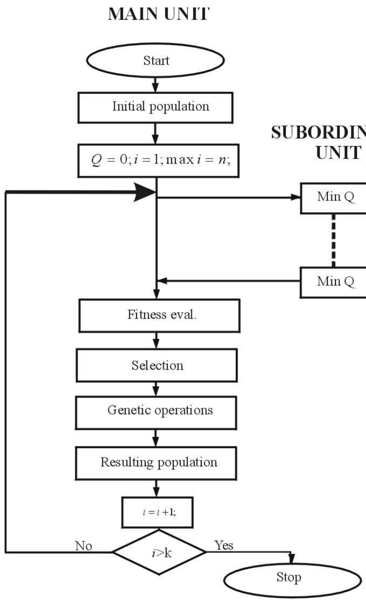

Figure 1: A metaepoch of fixed period *k* in

All genetic operators defined originally for HGS, i.e. sprouting operator and prefix comparison operator (see (Kołodziej et al, 2001; Schaefer and Kołodziej, 2002) and (Wierzba te al, 2003) for details) can be applied also in HGSNash. Figure 2 shows the example structure of binary implementation HGS after running of 3 metaepochs.

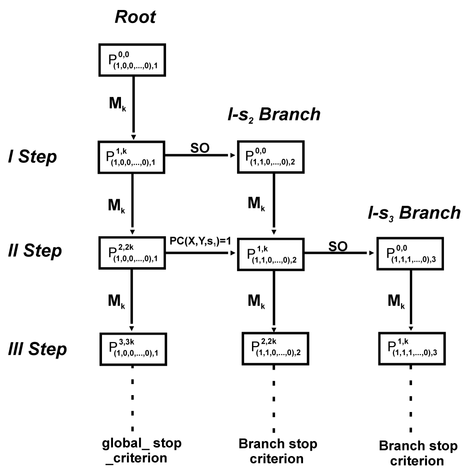

Figure 2: A binary HGS structure after running 3 metaepochs

In this paper HGSNash with SEA as the evolutionary mechanism was applied to solve the decision-making tasks for the systems with external disagreement of interests. We performed some numerical experiments and compared their results with the results of some similar tests obtained for the single population SEA and ACRS PW algorithm specially designed for Nash equilibria detection (Ślepowrońska, 1996).

NUMERICAL EXPERIMENTS AND RESULTS

In this section we reported the results of some simple numerical experiments performed for the verification of the efficiency of HGSNash in solving the decision-making problem of water resource system. We solved that problem for two different variants of the users water purifying functions and three groups of input parameters values.

The HGSNash performance was compared with other selected optimization methods like a single population evolutionary strategy and modified Advanced Controlled Random Search as the main unit algorithms in the 2-steps Nash detection procedure (Ślepowrońska, 1996).

Test suite

Let us assume that there are only two water users. We considered two variants of the users water purifying cost functions.

Case 1

In the first case we assumed that the users costs of the water purifying increase proportionally to the current pollution levels of water poured in the sewage treatment plant. For the first user those costs increase two times faster than for the second one. The water purifying functions in this case are defined by the following formulas:

$$k_1 = b_1 - x_1 \tag{7}$$

$$k_2 = 0.5 \cdot (b_2 - x_2) \tag{8}$$

Case 2

In the second case we changed the cost water purifying function for the second user from linear to the quadratic one. Now the values of water purifying cost for that user, small in the beginning, can grow very fast and exceed the first user's costs, which was impossible in the previous case. The water purifying functions in this case are defined by the following formulas:

$$k_1 = b_1 - x_1 \tag{9}$$

$$k_2 = \frac{4}{3} \cdot (b_2 - x_2)^2 \tag{10}$$

The plots of those functions are presented on the Figure 3.

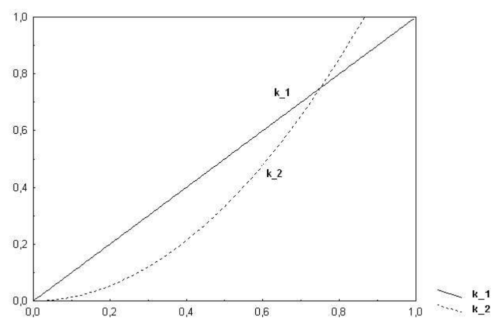

Figure 3: Plot of the water purifying functions k_1 and k_2 in Case 2.

All experiments were performed for three different sets of the input parameters values.

Set 1

$\alpha_1 = 0.2 \cdot \frac{x_1}{k_1}$ $\alpha_2 = 0.8 \cdot \frac{x_2}{k_2}$ $q=1$

$z_1=0.3$ $z_2=0.3$

$b_1=1$ $b_2=1$

Note that in this case the cost of water purifying is low for both users.

Set 2

$\alpha_1 = 0.8 \cdot \frac{x_1}{k_1}$ $\alpha_2 = 0.2 \cdot \frac{x_2}{k_2}$ $q=1$

$z_1=0.8$ $z_2=1.6$

$b_1=1$ $b_2=1$

In this case we have exchanged the coefficients in the tax rates and we changed the capacity of effluence poured by the users. These changes should show the significant differences in the users behavior.

Set 3

$\alpha_1 = 0.2 \cdot \frac{x_1}{k_1}$ $\alpha_2 = 0.8 \cdot \frac{x_2}{k_2}$ $q=1$

$z_1=1.2$ $z_2=1.2$

$b_1=1$ $b_2=1$

In the last case we again exchanged the coefficients in the tax rates. The capacity of effluence is greater than the maximal pollution level set for both users.

Parameter setting

We applied 3-levels HGSNash strategy for solving the test problems defined in the previous section. The 3-levels floating point HGS (Wierzba et al, 2003) was defined as the main unit algorithm in HGSNash structure. The genetic operations in SEA were reduced to the Gaussian mutation with the standard deviation defined as *a mutation parameter*. We considered two cases of the evaluation of small and big populations on every HGSwithSEA level. The values of parameters for the 3-levels HGSwithSEA in both cases are presented in Tables 1 and 2.

Table 1: Values of parameters for the 3- levels floating point HGS in the case of small level populations.

Parameter	Level 1	Level 2	Level 3
Population size	10	10	10
Mutation parameter	1	0.5	0.25
Metaepoch period	100	100	100

Table 2: Values of parameters for the 3- levels floating point HGS in the case of big level populations.

Parameter	Level 1	Level 2	Level 3
Population size	100	50	50
Mutation parameter	1	0.5	0.25
Metaepoch period	100	100	100

The Powell optimization algorithm was defined as the subordinate unit algorithm in HGSNash. We accepted the maximal number of metaepochs executed in the single run (which was 500 in every experiment) as the stop criterion.

The HGSNash performance was compared with the following algorithms:

- single population SEA with Gaussian mutation and no crossover,
- Advanced Controlled Random Search Algorithm with Powell method - ACRS PW

ACRS PW algorithm was introduced in (Ślepowrońska, 1996) as a hybrid method of Nash equilibria detection for non-cooperative games. The main unit algorithm in ACRS PW is based on two-steps optimization method with Controlled Random Search (CRS) algorithm in the first step and Powell method in the second step. The main goal of the CRS algorithm is to indicate some neighborhoods of the potential solutions, in which the Powell method could be started (see Ślepowrońska, 1996) for details). The subordinate unit algorithm in ACRS PW is also Powell algorithm.

The values of initial parameters for SEA algorithm in cases of small and big populations are presented in Table 3 .

Table 3: Values of parameters for SEA in the cases of small (a) and big (b) populations

Parameter	(a)	(b)
Population size	30	200
Mutation parameter	0.25	0.25

We accepted the value of the mutation parameter defined for the most accurate process in HGSNash as the mutation parameter value for SEA. The population size values in SEA correspond to the sum of the values of those parameters on all levels of HGSNash. The stop criterion for that algorithm was defined as a maximal number of iterations, which was 50000 in all experiments.

The values of the initial parameters for ACRS PW are presented in Table 4.

Table 4: Values of parameters for ACRS PW

Parameter	Value
Number of start points	50
Search accuracy in step1 (ε)	0.005
Search accuracy in step2 (γ)	0.05

The optimal number of start points for CRS algorithm was set to $n\ldots 25$ (n is the number of objective function variables) according to the W.L. Price experimental studies (Price, 1983). The search accuracy parameter ε is the maximal possible value of the objective function at the points detected by CRS as the potential solutions. The maximal number of iterations of the CRS algorithm was 1000 in every experiment.

The search accuracy γ parameter is the radius of the neighborhood of the proper solution found by the Powell algorithm in the second step. The γ parameter defines the termination condition for the whole algorithm.

Results of the experiments

Each experiment was repeated 30 times for all algorithms. Tables 5-9 report experimental results obtained in two cases of users water purifying cost function and three sets of users total cost functions parameters ("*Case i -Set j*", *i=1,2; j=1,2,3*). In tables 5 and 6 we compare the efficiency in finding the global optimum in cases of small and big populations. In the case of ACRS PW we define as the "population" the number of initial points for CRS step. "Small population" for that algorithm contained 10 initial points while "big population" - 50. The parameter **(nr)** means the number of runs of the particular algorithms, in which the global optimum was found. We assumed the value of the objective function in optimal solutions should be not greater than 0,005.

Table 5: The values of **(nr)** in the case of small populations.

Case	HGSNash	SEA	ACRS PW
Case 1-Set1	30	25	19
Case 1-Set2	30	20	18
Case 1-Set3	30	21	18
Case 2-Set1	30	17	13
Case 2-Set2	30	13	9
Case 2-Set3	30	11	11

Table 6: The values of **(nr)** for HGSNash, SEA and ACRS PW in the case of big populations.

Case	HGSNash	SEA	ACRS PW
Case 1-Set1	30	19	25
Case 1-Set2	28	18	22
Case 1-Set3	28	18	22
Case 2-Set1	19	13	19
Case 2-Set2	13	7	17
Case 2-Set3	13	7	17

The most efficient was HGSNash with small populations on each its level. It always found an optimum according to the defined criterion. Note that the same method with big populations is not as good. It confirms the theoretical and experimental studies on the populations dynamics in evolutionary algorithms performed in (Karcz-Dulęba, 2004). ACRS PW algorithm was better in case of 50 initial points than in case of 10 initial points.

In what follows we will report the results only for evolutionary algorithms with small populations and ACRS PW with 50 initial points.

In tables 7 and 8 we present the best solutions found by three applied algorithms and the average time (in seconds) needed for such results.

Table 7: The best solutions found by HGSNash, SEA and ACRS PW

Case	HGSNash	SEA	ACRS PW
Case 1-Set1	(0,977; 0,002)	(0,995;0,002)	(0,999;0,000)
Case 1-Set2	(1,000;0,996)	(0,977;0,884)	(1,000;1,000)
Case 1-Set3	(0,663;0,021)	(0,553;0,011)	(0,833;0,000)
Case 2-Set1	(0,802;0,003)	(0,788;0,018)	(0,999;0,000)
Case 2-Set2	(0,889;0,933)	(0,762;0,899)	(0,999;0,828)
Case 2-Set3	(0,032;0,751)	(0,022;0,667)	(0,034;0,795)

Table 8: The comparison of the average time needed for finding the optimal solution.

Case	HGSNash	SEA	ACRS PW
Case 1-Set1	3	7	32
Case 1-Set2	5	12	44
Case 1-Set3	5	11	38
Case 2-Set1	8	15	65
Case 2-Set2	11	25	79
Case 2-Set3	10	33	88

The obtained results show that HGSNash is the fastest method and can find the Nash equilibrium in each case. SEA is about 2 times slower than HGSNash. It could not find the global optimum in 2 cases. ACRS PW is also very slow and in some cases could find the Nash point.

In the end we compared the average fitness evaluations for the evolutionary algorithms. The results are reported in table 9.

Table 9 : The comparison of the fitness evaluations for HGSNash and SEA

Case	HGSNash	SEA
Case 1-Set1	2765	7248
Case 1-Set2	3678	10956
Case 1-Set3	5766	17564
Case 2-Set1	37382	100528
Case 2-Set2	25994	102856
Case 2-Set3	119821	625027

The computational cost for HGSNash measured in fitness evaluation is about 4 times lower in the most cases than for SEA.

CONCLUSIONS

A water resource system can be defined as a system of methods of the water management in the given area. Some important decision problems connected with the system can be very difficult to solve, especially in the case of disagreement of the interests of the water users.

The external disagreement of interests of the decision makers is usually modelled as a non-cooperative game with the Nash equilibrium as the solution. This model can be used in practice because it does not oblige the decision makers to reveal their interests and it does not impose the centralized manner of the decision making.

The experimental results show that HGSNash could be effective in finding the optimal solutions for the decision-making problems of water resource system in the case of coexistence of two users with external conflict of interests. This method was compared with the single population evolutionary algorithm and specialized hybrid algorithm based on Controlled Random Search and the Powell optimization algorithm.

Presented method should be examined in some more difficult cases and compared with others evolutionary methods. The application of HGSNash for solving other engineering problems modeled as the cooperative and non-cooperative games is the main goal of our future work.

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