AN ENVIRONMENT FOR STABILITY ASSESSMENT OF SINGLE-SOURCE ACTIVE NOISE CONTROL SYSTEMS IN 3D FREE-FIELD PROPAGATION

N. Jafferi and M. O. Tokhi
Department of Automatic Control and Systems Engineering,
The University of Sheffield, UK
E-mail: cop03nj@sheffield.ac.uk

KEYWORDS
Active noise control, geometrical design, single-input single-output control structure

ABSTRACT

This paper describes the development of an environment for the assessment of stability of active noise control (ANC) systems in 3D propagation. A single-input single-output (SISO) feedforward control structure (FFCS) is considered. The environment allows geometrical design of the system complying with system stability. Results demonstrating such phenomena are presented in the paper. The environment thus designed and developed may be used for assessment and evaluation of ANC system designs.

INTRODUCTION

Active noise cancellation is based upon the intentional superposition of acoustic waves to result in destructive interference. This is realized by artificially generating sound waves that interfere with the noise, thereby using the destructive interference of the component waves to reduce the level of noise (Nelson and Elliot 2000; Tokhi and Leitch 1992; Tokhi and Veres 2002). This paper presents an investigation into the development of an environment for the assessment of stability of an ANC mechanism within a fixed controller framework. An ANC system is designed utilising a SISO control structure to yield optimum cancellation of noise, emanating from a primary source, at an observation point in a free-field propagation medium.

Simulations are often used to discover and analyze new ideas. Utilizing simulation technologies in engineering education has many benefits. There is great potential to develop simulation as a tool for education and research. The environment presented in this paper allows the geometrical design of the system to comply with system stability. The performance results are presented in a graphical user interface to allow interaction between the environment and the user.

In this work, the concept of parametric analysis of the cancellation process (Tokhi and Leitch 1992; Tokhi 1999; Tokhi and Veres 2002) is used. This is a frequency-domain analysis approach based on power spectral density functions. Derivation of the algorithm has previously been reported (Tokhi and Leitch 1992; Tokhi 1999; Tokhi and Veres 2002). This paper considers system stability related to geometrical arrangement of system components in an ANC system.

THE FEEDFORWARD ACTIVE NOISE CONTROL STRUCTURE

Figure 1 shows a schematic diagram of a single-source ANC system in the FFCS configuration and Figure 2 shows the corresponding block diagram of the system. The wave emitted by the primary source is detected by a detector. The detected signal is fed to a controller for phase and amplitude adjustment. Then, the processed signal is transferred to a secondary source. The result of superimposed secondary and primary signals is observed at an observation point in the medium. E(s), F(s), G(s) and H(s) represent transfer functions of path through r_e, r_f, r_g and r_h respectively. M(s), C(s) and L(s) represent transfer functions of the detector, controller and secondary source respectively. D(s), P(s), Po(s), S(s), So(s), O(s) represent the detected signal, primary signal at the source and observation point, secondary signal at the source and observation point and the combined primary and secondary (observed) signal at the observation point respectively.

The objective with the FFCS is to reduce the level of noise to zero at the observation point. Thus,

$$Po(s) = -So(s) \qquad (1)$$

Manipulating the above using the block diagram in Figure 2 yields the controller transfer function as:

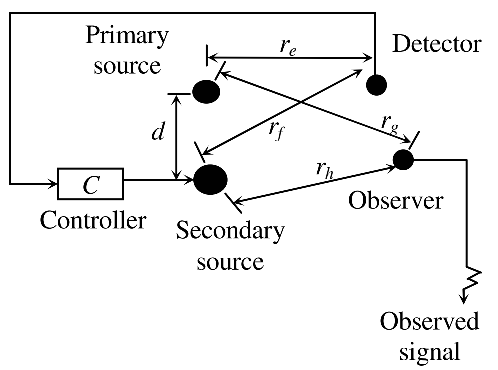

Figure 1: Schematic diagram of FFCS.

Proceedings 21st European Conference on Modelling and Simulation
Ivan Zelinka, Zuzana Oplatková, Alessandra Orsoni ©ECMS 2007
ISBN 978-0-9553018-2-7 / ISBN 978-0-9553018-3-4 (CD)

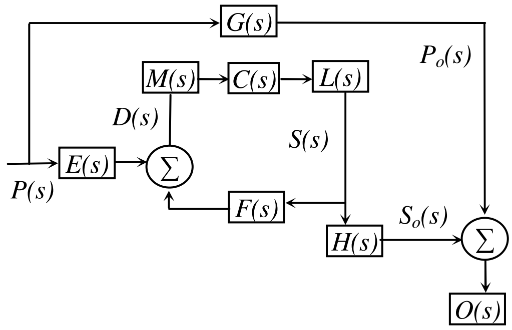

Figure 2: Block diagram of FFCS

$$C(s) = M(s)^{-1}\Delta(s)^{-1}G(s)\, H(s)^{-1}L(s)^{-1} \qquad (2)$$

where $\Delta(s)$ is an $n \times n$ matrix given by

$$\Delta(s) = [G(s)\, H(s)^{-1}F(s) - E(s)] \qquad (3)$$

The detector gives a combined measure of the primary and secondary waves that reach the detection point. The secondary source radiation reaching the detector thus gives rise to acoustic feedback. Therefore, the closed loop formed by the detector, controller, secondary source and the acoustic path between the secondary source and the detector can cause the system to become unstable. A stability analysis of this loop is essential at the design stage.

SISO STABILITY ANALYSIS

The acoustic path between the secondary source and detector forms a feedback loop in the FFCS. Consider the controller transfer function C(s) described by equation (2) representing the required transfer function for optimum system performance.

From the block diagram in Figure 2, the secondary signal S(s) can be written as:

$$S(s) = M(s)\, C(s)\, L(s)\, [P(s)\, E(s) + F(s)\, S(s)] \qquad (4)$$

Simplifying this between P(s) E(s) and S(s) yields

$$\frac{S(s)}{P(s)E(s)} = \frac{M(s)C(s)L(s)}{1+X(s)} \qquad (5)$$

where $X(s) = -M(s)\, C(s)\, L(s)\, F(s)$ (6)

In order for the system to be stable, the denominator in equation (5) should have roots in the left-hand side of the s-plane. The stability analysis in this work is based on the Nyquist stability criterion, which uses a graphic polar plot of transfer function X(s). Representing X(s) for periodic waves ($s = j\omega$) with magnitude $B(\omega)$ and phase $\theta(\omega)$ yields

$$X(j\omega) = B(\omega)\, e^{j\theta(\omega)} \qquad (7)$$

Thus, in accordance with the Nyquist stability criterion, for the ANC system to be stable, the following situation should be true;

$$B(\omega)<1 \text{ when } \theta(\omega) = -(2n+1)\pi;\ n=0,1,2,\ldots \qquad (8)$$

This can be expressed graphically by evaluating $X(j\omega)$ from $\omega = 0$ to $\omega = \infty$ to form the polar plot. Thereby if the point -1 lies at the left-hand side then the system is considered to be stable, whereas if the point -1 lies at the right-hand side of the plot then the system is unstable. The design criterion in this work uses the concept of relative stability of the inherent feedback loop. This procedure results in a robust design under stationary conditions. The frequency domain stability conditions are derived and interpreted as spatial conditions on the geometry of the ANC system (Tokhi and Leitch 1992).

Gain and phase margins

The gain margin is defined as:

$$k_g = \frac{1}{B(\omega)} \text{ when } \theta(\omega) = -180^\circ \qquad (9)$$

Equations (8) and (9) show that if the gain margin k_g is greater than unity, the system will be stable; while if it is less than unity, it will mean that the system is unstable.

Phase margin can be expressed in terms of the phase $\theta(\omega)$ as:

$$k_\theta = \theta(\omega) + 180^\circ \text{ when } B(\omega) = 1 \qquad (10)$$

Equations (8) and (10) show that, the phase margin must be positive for a minimum phase system to be stable.

Spatial conditions

Substituting C(s) from equation (2) into equation (6) and using equation (3), for periodic waves ($s = j\omega$), and simplifying yields

$$X(j\omega) = \frac{1}{\frac{E(j\omega)H(j\omega)}{F(j\omega)G(j\omega)} - 1} = \frac{1}{Q(\omega)e^{j\varphi(\omega)} - 1} \qquad (11)$$

where $Q(\omega)$ and $\varphi(\omega)$ denote the magnitude and phase, respectively as:

$$\frac{E(j\omega)H(j\omega)}{F(j\omega)G(j\omega)} = Q(\omega)e^{j\varphi(\omega)} \qquad (12)$$

Simplifying equation (11) yields the magnitude $B(\omega)$ and phase $\theta(\omega)$ of $X(j\omega)$ as:

$$B(\omega) = [Q^2(\omega) + 1 - 2Q(\omega)\cos\varphi(\omega)]^{-0.5} \qquad (13)$$

$$\theta(\omega) = \tan^{-1}\left[\frac{Q(\omega)\sin\varphi(\omega)}{1 - Q(\omega)\cos\varphi(\omega)}\right] + 2m\pi \qquad (14)$$

where $m=0, \pm1, \pm2, \ldots$

In order to relate the stability of the system to the geometrical arrangement of system components in the medium, the interpretation of the gain and phase margins according to the locations of system components is important to the analysis and design of ANC system.

$E(j\omega)$, $F(j\omega)$, $G(j\omega)$ and $H(j\omega)$ are the transfer functions of the acoustic paths through the distances r_e, r_f, r_g and r_h respectively; thus

$$E(j\omega) = \frac{A}{r_e} e^{-j2\pi r_e/\lambda}, \quad F(j\omega) = \frac{A}{r_f} e^{-j2\pi r_f/\lambda}$$

$$G(j\omega) = \frac{A}{r_g} e^{-j2\pi r_g/\lambda}, \quad H(j\omega) = \frac{A}{r_h} e^{-j2\pi r_h/\lambda} \tag{15}$$

Representing these in the form of magnitude and phase based on pressure interpretation and substituting into equation (12) yields $Q(\omega)$ and $\varphi(\omega)$ as:

$$Q(\omega) = \frac{r_g r_f}{r_e r_h} = \left(\frac{r_g}{r_h}\right)\left(\frac{r_e}{r_f}\right)^{-1} \tag{16}$$

$$\varphi(\omega) = \frac{2\pi}{\lambda}[(r_h - r_g)-(r_f - r_e)] = \frac{2\pi}{\lambda}[(r_{gh} - r_{ef})] \tag{17}$$

where $r_{gh} = r_h - r_g$ and $r_{ef} = r_f - r_e$. Having only information on the distances r_e, r_f, r_g and r_h and the signal wavelength is enough for the purpose of explaining the transfer function $X(j\omega)$. It is thus noted that locations of the detector and observer relative to the primary and secondary sources in the medium influence directly the stability of the system.

For the gain margin $k_g>1$, two possible cases of $Q(\omega) <1$ and $Q(\omega) \geq 1$ are considered. For the case $Q(\omega) \geq 1$, equation (16) yields:

$$Q = \left[\frac{r_g}{r_h}\right]\left[\frac{r_e}{r_f}\right]^{-1} \geq 1 \text{ or } \frac{r_g}{r_h} \geq \frac{r_e}{r_f} \tag{18}$$

Let the distance ratio, $a=r_g/r_h$. This will define a family of spheres in the medium. For $1>a\geq 0$, the centre of the sphere is closer to the location of the primary source, and for $1<a\leq\infty$, it is closer to the location of the secondary source.

If $a=1$, a plane surface is defined bisecting perpendicularly the line joining the primary and secondary sources. Thus, for $a<1$ the detection point should remain on or inside the sphere r_e/r_f; for $a=1$, the detection point should be on or inside the region on the side of the plane containing the primary point; and for $a>1$, the detection point should be on or outside the sphere.

In conclusion, a system with $r_{gh} - r_{ef} = 0.5(2n+1)\lambda$ will be stable when

$k_g=1+Q(\omega) \geq 2$

for $\varphi(\omega)=(2n+1)\pi$ and $Q(\omega) \geq 1$; $n=0, ,\pm 1, \ldots$

For the case $Q(\omega) <1$, $r_{gh}-r_{ef}=0.5n\lambda$, the following conclusions can be drawn:

1) The system is stable when

   $k_g=1+Q(\omega)>1$ for $\varphi(\omega) = (2n+1)\pi$

2) The system is unstable when

   $k_g=1+Q(\omega) <1$ for $\varphi(\omega) = 2n\pi$

Substituting for $B(\omega)$ from equation (13) into equation (10) and simplifying yields:

$$0<Q(\omega) \leq 2 \tag{19}$$

and the range of $\varphi(\omega)$ as equivalent for the distance differences, $r_{gh} - r_{ef}$:

$$(n-0.25)\lambda < r_{gh}-r_{ef} < (n+0.25)\lambda \tag{20}$$

for $0<Q(\omega) \leq 2$; $n=0, \pm 1, \ldots$

The phase angle $\theta(\omega)$ is

$$\theta(\omega)=\begin{cases} \tan^{-1}\dfrac{Q(\omega)\sqrt{4-Q^2(\omega)}}{2-Q^2(\omega)}+2m\pi \\ \quad \text{for} \quad 0\leq\varphi(\omega)<\dfrac{(4n+1)\pi}{2} \\ -\tan^{-1}\dfrac{Q(\omega)\sqrt{4-Q^2(\omega)}}{2-Q^2(\omega)}+2m\pi \\ \quad \text{for} \quad \dfrac{(4n-1)\pi}{2}<\varphi(\omega)<0 \end{cases} \tag{21}$$

The phase margin k_θ is thus

$$k_\theta=\begin{cases} \tan^{-1}\dfrac{Q(\omega)\sqrt{4-Q^2(\omega)}}{2-Q^2(\omega)}+(2m+1)\pi \\ \quad \text{for} \quad 0\leq\varphi(\omega)<\dfrac{(4n+1)\pi}{2} \\ -\tan^{-1}\dfrac{Q(\omega)\sqrt{4-Q^2(\omega)}}{2-Q^2(\omega)}+(2m+1)\pi \\ \quad \text{for} \quad \dfrac{(4n-1)\pi}{2}<\varphi(\omega)<0 \end{cases} \tag{22}$$

where $0<Q(\omega) \leq 2$ and m and n are integers.

The range $0<Q(\omega)\leq 2$ yields $re/rf\geq 0.5a$ which defines the loci of detection points to be outside of a region of half the distance ratio.

For a minimum-phase condition, the system will be stable when the phase margin k_θ assumes positive values, while the system will be unstable when the phase margin assumes negative values.

SIMULATION RESULTS

A simulation environment was developed using MATLAB and C programming language (Hanselman and Littlefield 2001). It allows the user to identify regions of the propagation medium as loci of detection and observation points for which the system will be stable.

The panel shown in Figure 3 is for a SISO system with a single frequency signal. The input parameters contain coordinates of the primary source, secondary source, detector and observer, as well as the selected frequency involved. The panel shows plot of the locations of sources in 2D coordinates and the identified regions for the observation and detection points of the system with the magnitude, Q(ω) values. It will further show allowed region of the detection point based on specified coordinates of the observation point. Exploring the influence of a certain frequency on the system and numerous geometrical arrangements of system components will help the user to understand the characteristics of the system and assess system stability.

Figure 4 demonstrates the process of identifying stability and calculation of phase margin and gain margin for which the SISO system will be unstable, with the primary source at (500mm,0), secondary source at (–500mm,0), detector at (300mm, 200mm) and observer at (150mm, 0) with a range of frequency from 1 Hz to 1000 Hz.

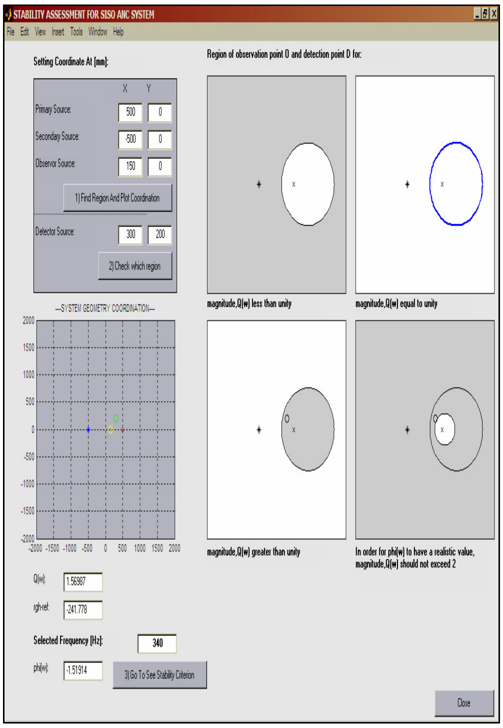

Figure 3: Interface panel of the entire interactive environment for a single-source ANC system.

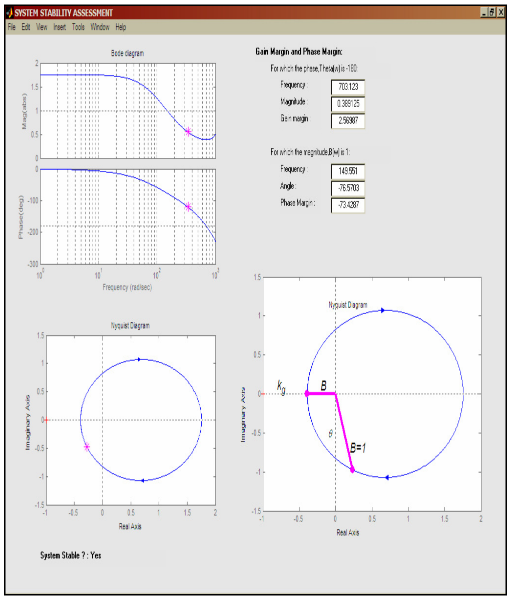

Figure 4: Stability assessment of the spatial arrangement for a SISO structure.

CONCLUSIONS

It has been demonstrated that suitable design and implementation of an ANC system can be achieved and verified through a suitable geometrical arrangement of system components. It has been shown that by using minimum information specific to an ANC system, including relative distances between sources and the signal frequency, system stability may be easily estimated. The simulation environment developed and presented in this paper proves to be very useful for the assessment of stability performance of the system. Moreover, the environment helps to enhance the user's learning and understanding of the system. Further investigation into this technique could be undertaken extend the environment for stability assessment of multiple-source ANC systems.

REREFENCES

Hanselman, D.C. and B. Littlefield. 2003. *"Mastering MATLAB 6, A comprehensive Tutorial and Reference"*, Prentice Hall

Nelson, P. A. and S. J. Elliot. 2000. *Active Control of Sound*, Academic Press, London

Tokhi, M. O. and R. R. Leitch. 1992. *Active Noise Control*, Clarendon Press, Oxford.

Tokhi, M.O. 1999. "The design of active noise control systems for compact and distributed sources", *Journal of Sound and Vibration, Vol. 225(3),* 401-424

Tokhi, M. O. and S. Veres. 2002. *Active Sound and Vibration Control, theory and applications*, The Institute of Electrical Engineers, Michael Faraday House, Herts.

AUTHOR BIOGRAPHIES

Noormaziah Jafferi was born in Johor, Malaysia and went to Takushoku University, where she studied mechanical system and obtained her degree in 2002. She is currently a research student at The University of Sheffield.

Dr. M. O. Tokhi is a Reader at the University of Sheffield. His research interests include active control of noise and vibration, adaptive/intelligent and soft computing approaches for modelling and control of dynamic systems, high-performance real-time computing and biomedical applications of robotics and control. Further details may be found at: http://www.shef.ac.uk/acse/staff/mot.