A BEHAVIORAL MODEL OF SIMULTANEOUS EXTREME RETURNS

Zsolt Tulassay
Department of Finance
Corvinus University of Budapest
8 Fővám tér, Budapest 1093, Hungary
E-mail: zsolt.tulassay@uni-corvinus.hu

KEYWORDS

simultaneous extreme returns, tail dependence, heterogeneous-agent models

ABSTRACT

A multivariate stylized fact of financial markets is the frequent occurrence of simultaneous extreme returns. A simple agent-based behavioral model is presented that accounts for this phenomenon. Joint extremes are generated by heterogeneous fundamentalist traders whose perceptions of the effect of a common news factor on asset values become aligned during market stress. Simulation results are compared to an empirical investigation of two major Hungarian stocks.

INTRODUCTION

A fundamental insight of finance is the idea that diversification is beneficial. It is based on the observation that asset prices move together imperfectly. For a long time, ever since Markowitz founded portfolio theory, these comovements were described by a linear correlation structure, where the benefits of diversification result from imperfect correlations between returns. During the same period, however, several financial market crashes occurred, in the course of which asset prices fell simultaneously, displaying comovements that were much stronger than usual. Market wisdom summarizes this as "correlations rise to one in times of market stress". While correlations certainly do not become one, joint occurrence of such extreme returns, whether negative or positive, might still seriously affect portfolio risk and diminish the benefits of diversification.

The occurrence of such simultaneous extreme returns is necessarily a multivariate phenomenon that requires multivariate analysis. The statistical properties of *univariate* return data have been extensively studied in the financial literature. These properties, commonly referenced to as the stylized facts of financial returns, include heavy tails, change and clustering of volatility, aggregational Gaussianity and a particular scaling behavior, among others (see e.g. Dacorogna et al., 2001; Cont, 2001). Much less attention has however been devoted to the systematic study of the corresponding *multivariate* properties of returns. Notable exceptions include (Breymann et al., 2003); a brief overview of multivariate stylized facts is given in (McNeil et al., 2005). Still, in recent years several statistical tools have been developed to handle such phenomena. There are now time series models like multivariate GARCH models that allow for a time-varying and volatility-dependent correlation structure. Stationary models of simultaneous extremes include various classes of copulas and multivariate EVT models.

There is a lack, however, in models that try to explain these multivariate phenomena in behavioral terms. Non-equilibrium, heterogeneous-agent models have been remarkably successful in replicating univariate stylized facts, while also providing insights into the price formation process (see e.g. Arthur et al., 1997; Brock and Hommes, 1998; Lux and Marchesi, 2000; Farmer and Joshi, 2002). However, very few papers have examined the interrelations and dependence of several asset markets in a heterogeneous-agent framework.

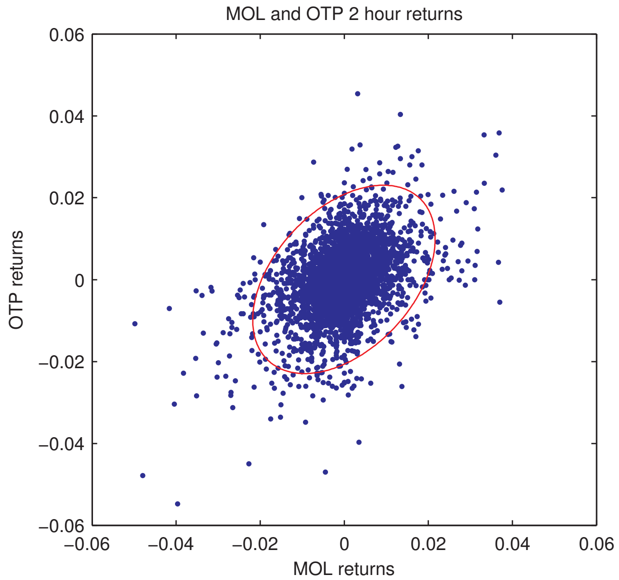
This paper tries to fill this gap by studying asset price dynamics and dependence in a multi-market, heterogeneous-agent framework. Based on the multi-market model of (Westerhoff, 2004), I present a simple behavioral model that can account for extremal dependence in financial returns. The mechanism behind the model is based on the idea that in times of market turmoil (i.e. after a large news shock) the reactions of fundamentalist agents, being heterogeneous themselves, become aligned and highly sensitive to later news.

EMPIRICAL INVESTIGATION

In order to be able to discuss, let alone measure (and test for) extremal dependence, it is necessary to give a sufficiently precise definition first. This is formulated via the *tail dependence coefficient* – a notion that usually arises in the copula literature. In this section I present a non-parametric estimation of tail dependence, based on the empirical cumulative distribution function (cdf) of the data.

The data investigated cover intradaily (2-hour) and daily bivariate log-returns of Hungarian stocks MOL and OTP, from January 2002 to April 2007. These two stocks were selected because they are among the most frequently traded in the Hungarian stock market. Besides that, the two companies are from different industries (energy and finance, respectively), therefore simultaneous extreme returns are not likely to be caused by industry-

Proceedings 22nd European Conference on Modelling and Simulation ©ECMS Loucas S. Louca, Yiorgos Chrysanthou, Zuzana Oplatková, Khalid Al-Begain (Editors)
ISBN: 978-0-9553018-5-8 / ISBN: 978-0-9553018-6-5 (CD)



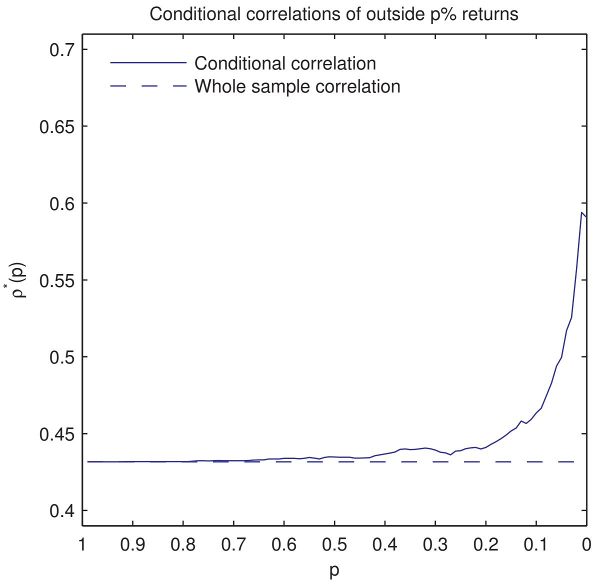


Figure 1: MOL and OTP 2 hour returns scatterplot with the ellipse containing 95% of the data (left); conditional correlations of return data outside ellipses of progressively larger size (right)

related shocks. Additionally, an extended sample of daily returns (ranging from January 1998 to October 2007) was also investigated. The purpose of this was to include data from the Russian financial crisis of August 1998, during which several simultaneous extreme returns occurred. The inclusion of this period alters results for the daily data considerably.

Tests of elliptical symmetry

Though not directly related to extremal dependence, it is quite instructive to test the data for elliptical symmetry – in other words, to test whether a single linear correlation coefficient (or, in higher dimensions, correlation matrix) is sufficient to describe dependence between returns. Elliptical distributions have the property that their densities' contours are ellipses. It is also true that conditional distributions of elliptically distributed random vectors lying outside such quantile ellipses retain the same correlation matrix as the original random vector. For a more detailed discussion of elliptical distributions, see (McNeil et al., 2005).

Thus, for an elliptically distributed random vector X, the conditional correlation $\rho^*(p)$, defined as

$$\rho^*(p) = \rho\left(X \,|\, h(X) \geq c\left(p\right)\right)$$

is expected to remain approximately stable with p approaching 0 (where $h(x) = (x - \widehat{\mu})' \, \widehat{\Sigma}^{-1} \, (x - \widehat{\mu})$, $c(p)$ is a multivariate analogue of the univariate quantile function and $p \in [0, 1]$). In contrast, estimates for the MOL and OTP 2 hour return data show that the conditional correlation is increasing in the tails (see Figure 1). The plot suggests that extreme events might be more strongly correlated than normal ones. However, this method is only exploratory and does not allow one to come to any formal conclusion. Elliptical symmetry can be tested instead by standardizing the data and testing for spherical symmetry. In the polar representation, angles should be uniformly distributed; for the data investigated here, this is rejected at 95% in both the intradaily and the extended daily sample.

The tail dependence coefficient

A shortcoming of the standard linear correlation coefficient discussed above is that it is intimately linked to the marginal distributions. A better dependence measure would be one that only depends on the dependence structure, that is, on the *copula* of a multivariate distribution. The *tail dependence coefficient*, discussed in this section, is such a measure. Moreover, using the tail dependence coefficient it is possible to define extremal dependence precisely.

The tail dependence coefficient λ is essentially the limiting conditional probability of quantile exceedances. One can distinguish between upper and lower tail dependence. Upper tail dependence is the (limiting) probability that X_2 exceeds its q-quantile, given that X_1 exceeded its q-quantile. Similarly, lower tail dependence is the (limiting) probability that X_2 falls below its q-quantile, given that X_1 fell below its q-quantile. In financial applications we are more concerned with downside risk, therefore here I am mainly interested in the lower tail dependence coefficient:

$$\begin{aligned}
\lambda\left(q\right) &= \Pr\left(X_2 \leq F_2^{\leftarrow}(q) \mid X_1 \leq F_1^{\leftarrow}(q)\right) = \\
&= \frac{\Pr\left(F_1(X_1) \leq q, F_2(X_2) \leq q\right)}{\Pr\left(F_1(X_1) \leq q\right)} = \frac{C(q,q)}{q} \\
\lambda &= \lim_{q \to 0^+} \lambda(q)
\end{aligned}$$

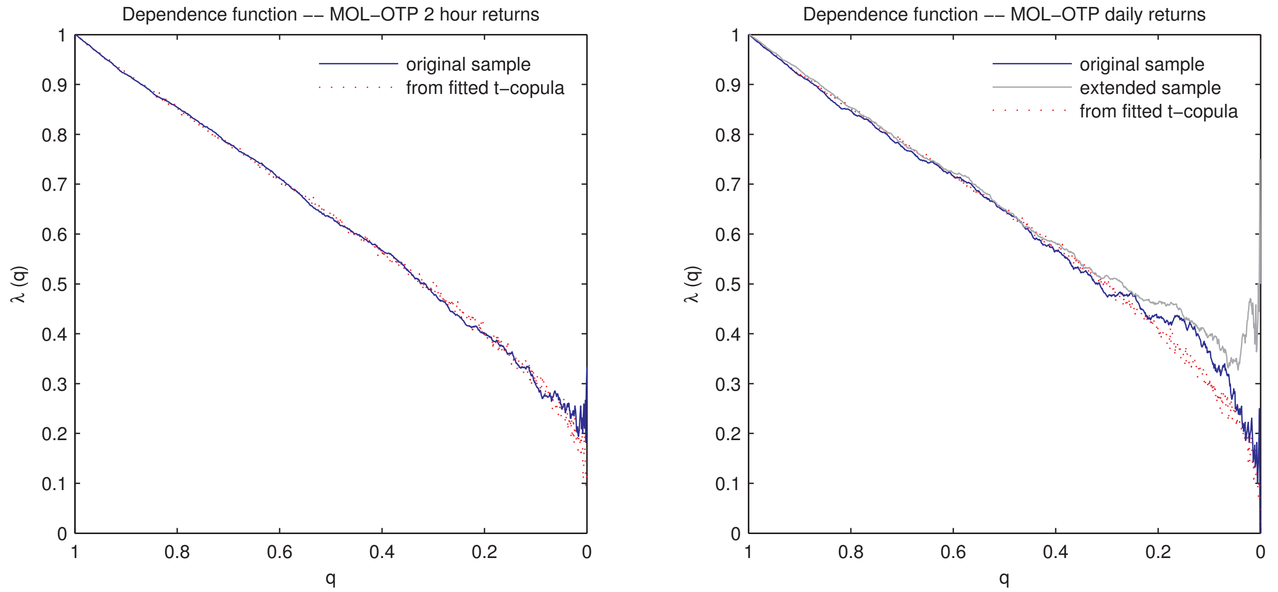


Figure 2: Dependence functions $\lambda(q)$ of MOL and OTP 2 hour (left) and daily (right) returns. Dotted lines are dependence functions from fitted t-copulas.

I will call $\lambda(q)$ the quantile-based dependence function; C is the (unique) copula of the multivariate distribution. If $\lambda = 0$, then X_1 and X_2 are *asymptotically independent* in the lower tail; if $\lambda \in (0, 1]$ then they are said to show *(asymptotic) lower tail dependence*. It can be shown that the Gaussian copula (and thus, in particular, the multivariate normal distribution) is asymptotically independent for $\rho < 1$, while the t copula (and therefore the multivariate t-distribution) is asymptotically dependent for $\rho > -1$ (see e.g. McNeil et al., 2005).

Estimating $\lambda(q)$ from the data is straightforward; it is even more appealing since this can be done in a non-parametric way, using the empirical cumulative distribution (in the copula parlance this step is usually referred to as constructing a pseudo-sample of observations). Figure 2 displays the estimated dependence functions for the 2 hour and the daily return datasets. Table 1 presents the estimates of the tail dependence coefficient (averages for the smallest 2% of q's). Intradaily returns seem to be asymptotically dependent. For the daily returns, the picture is less clear; including data on the Russian financial crisis however results in a much higher λ.

	$\widehat{\lambda}$
2 hour returns	0.2278
daily returns (extended sample)	0.4576
daily returns	0.1575

Table 1: Estimated tail dependence coefficients (average of the last 2% of $\lambda(q)$ as $q \to 1$).

Based on the estimated dependence functions, extremal dependence is clearly present in intradaily returns. It is also present in daily returns – although somewhat less pronouncedly if the crisis data is omitted. Thus, its effects should not be neglected and it is worthwhile to investigate possible kinds of behavior that could result in such joint extreme outcomes.

MODELING SIMULTANEOUS EXTREMES

In the remainder of this paper I present a model that provides a behavioral explanation for the occurrence of these simultaneous extreme returns. The model follows the heterogeneous-agent, non-equilibrium modeling strand of literature, which in recent years has been successful in replicating univariate stylized facts of financial markets. The current model is a modified, stochastic version of the model presented in (Westerhoff, 2004).

Assets

There are two risky assets ($k \in \{1, 2\}$), traded in two separate, symmetric asset markets of equal size. The fundamental values of assets are governed by two different kinds of shocks. One is a common news process N_t, which affects all assets simultaneously, but to a different, asset-specific extent (determined by a coefficient $\alpha^{(k)}$). The other kinds of shocks, denoted by $\varepsilon_t^{(k)}$, are idiosyncratic, asset-specific, and independent across assets (as well as across time). Their effect on the fundamental value is controlled by the coefficient $\beta^{(k)}$. The (log) fundamental values therefore evolve as follows:

$$F_{t+1}^{(k)} = F_t^{(k)} + \alpha^{(k)} N_t + \beta^{(k)} \varepsilon_t^{(k)},$$

where $N_t \sim$ iid $N(0, 1)$ and $\varepsilon_t^{(k)} \sim$ iid $N(0, 1)$, and the squares of α and β sum to one, thus resulting in a random walk for F. One can interpret N_t as incorporating common, macroeconomic or political news that affect the entire economy, and $\varepsilon_t^{(k)}$ as company-specific news (e.g.

quarterly reports) or shocks which affect only a particular company.

Price dynamics

Prices of the two assets are determined on order-driven markets, in the spirit of (Kyle, 1985) and (Farmer, 2002). There are three types of agents operating in the markets: market makers, fundamentalists and chartists. Orders are initiated by the latter two types against market makers; these fulfill excess demand or absorb excess supply, and adjust prices according to a loglinear market impact function. This results in the following price dynamics:

$$S_{t+1}^{(k)} = S_t^{(k)} + a^{(M,k)} \left(D_t^{(F,k)} + W_t^{(k)} D_t^{(C,k)} \right)$$

where $S^{(k)}$ is the log price of asset k, $D^{(F,k)}$ and $D^{(C,k)}$ are the orders of fundamentalists and chartists in market k, and $W^{(k)}$ is the fraction of chartists currently active in market k (see below for details). The (positive) coefficient $a^{(M,k)}$ determines the sensitivity of market makers' price adjustment reactions to orders; thus it is essentially a measure of liquidity, or depth, of market k.

Agents' trading strategies

Agents base their trading decisions on their price expectations: they buy if they expect an increase in the price and vice versa. Their demand (orders) is as follows:

$$\begin{aligned} D_t^{(F,k)} &= a^{(F,k)} \left(E_t^F \left[S_{t+1}^{(k)} \right] - S_t^{(k)} \right) \\ D_t^{(C,k)} &= a^{(C,k)} \left(E_t^C \left[S_{t+1}^{(k)} \right] - S_t^{(k)} \right) \end{aligned}$$

where $a^{(F,k)}$ and $a^{(C,k)}$ are positive reaction coefficients.

The basic difference between the two types of traders (fundamentalists and chartists) is in how they form their expectations. *Chartists* are simple trend-followers:

$$E_t^C \left[S_{t+1}^{(k)} \right] = S_t^{(k)} + b^{(C,k)} \left(S_t^{(k)} - S_{t-1}^{(k)} \right) \qquad (1)$$

Chartists' presence in a market is reflected by $W_t^{(k)}$, the fraction of them active in market k at time t. They enter a market if they find it attractive enough, where attractiveness is measured as follows:

$$A_t^{(k)} = \log \frac{1}{1 + f^{(k)} \left(S_t^{(k)} - S_{t-1}^{(k)} \right)^2}.$$

The relative percentage of chartists trading in market k at time t is given by the discrete choice model of (Manski and McFadden, 1981), a common vehicle of heterogeneous-agent models:

$$W_t^{(k)} = \frac{\exp\left(\gamma A_t^{(k)}\right)}{\sum_k \exp\left(\gamma A_t^{(k)}\right)},$$

where the parameter γ reflects the intensity of choice.

In contrast, *fundamentalists* are themselves heterogeneous; they differ in their perceptions of the fundamental value, but they all expect prices to revert back to this (perceived) fundamental value:

$$E_t^{F,i} \left[S_{t+1}^{(k)} \right] = S_t^{(k)} + b^{(F,k)} \left(\widehat{F}_t^{(k,i)} - S_t^{(k)} \right) \qquad (2)$$

where i refers to the specific fundamentalist trader.

Perceptions of fundamental values

The essential part of the model is how fundamentalists form and update their $\widehat{F}_t^{(k,i)}$ perceptions of fundamental values. It is assumed that the true values of both types of news (N_t and $\varepsilon_t^{(k)}$) are known to fundamentalists at time t. However, they cannot assess correctly the specific *extent* to which each of this news affects the true fundamental values – that is, they do not know the true $\alpha^{(k)}$ and $\beta^{(k)}$ coefficients. Instead, they have different, subjective, time-varying *perceptions* about the effects of news on the fundamental values. These will be denoted by the coefficients $\widehat{\alpha}_t^{(k,i)}$ and $\widehat{\beta}_t^{(k,i)}$.

Each fundamentalist trader thus has a subjective perception of the fundamental value of asset k:

$$\widehat{F}_{t+1}^{(k,i)} = \widehat{F}_t^{(k,i)} + \widehat{\alpha}_t^{(k,i)} N_t + \widehat{\beta}_t^{(k,i)} \varepsilon_t^{(k)}$$

These are then used to form subjective price expectations according to Equation (2).

The fundamental behavioral assumption concerns how fundamentalist traders' perceptions of the effect of news vary over time. The basic idea is as follows: fundamentalists differ in their sensitivities to news; however during a market turmoil traders start to ascribe much higher importance to news than usually, as they lose their anchors of value. In highly volatile times traders cannot distinguish between important and unimportant information, so they regard every piece of information as possibly highly important and react accordingly (that is, overreact).

In terms of the model there is a population of $\widehat{\alpha}$'s and $\widehat{\beta}$'s, the members of which all rise together and become much less dispersed during a market turmoil. This shows up as follows: with N_t large enough in absolute terms, a "panic term" ξ_t kicks in, and by raising all perceived α's simultaneously this makes all fundamentalists much more sensitive to news:

$$\widehat{\alpha}_t^{(k,i)} = \widehat{\alpha}_{t-1}^{(k,i)} + \xi_t + c^{(k)} \left(\alpha^{(k)} - \widehat{\alpha}_{t-1}^{(k,i)} \right) + d^{(k)} \eta_t^{(k,i)}$$

$$\xi_t = \begin{cases} 0.5 & \text{if } |N_t| > 2 \\ -0.01 & \text{otherwise} \end{cases}$$

$$\eta_t^{(k,i)} \sim \text{iid } U(-1,1)$$

An additional random term η ensures that fundamentalist traders remain heterogeneous.

Different sensitivities thus become aligned and high during market turmoils. This is the mechanism that "generates" simultaneous extreme returns. Over time these

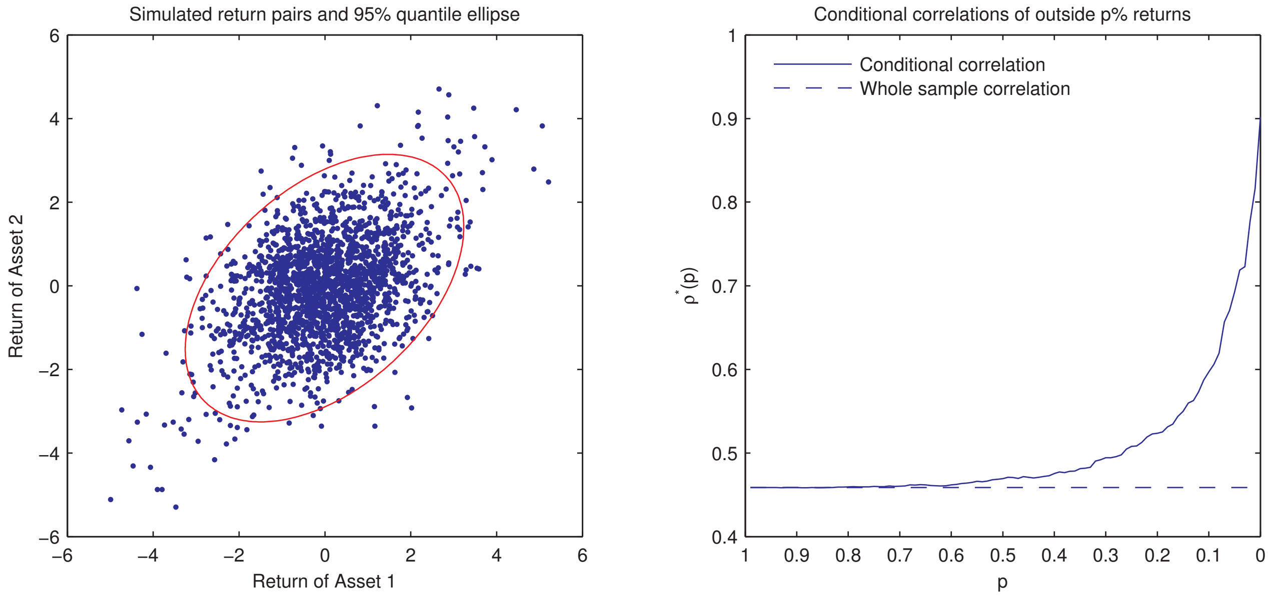


Figure 3: Scatterplot (left) and conditional correlation plot (right) of simulated returns

sensitivities eventually return to more moderate levels and will be dispersed around the true value, or even go below it. This allows the price of assets to break away from their fundamental values in the short run. However, the market on average "gets the price right" in the long run, even if individual perceptions about the fundamental values show considerable variation over time and across traders.

SIMULATIONS

I performed simulations for 2000 consecutive trading periods to see whether the model is able to produce joint extreme returns. Table 2 contains the parameter values used for simulations; N and M denote the number of fundamentalists and chartists, respectively. As there is little empirical guidance on how to pick the parameter values of such models, I based them mostly on parameter choices used by (Westerhoff, 2004).

T	2000	f	1
N	100	γ	2
M	100	α	$\sqrt{0.2}$
$a^{(M)}$	1	β	$\sqrt{0.8}$
$a^{(F)}b^{(F)}$	1	c	0.05
$a^{(C)}b^{(C)}$	0.5	d	0.1

Table 2: Parameter values of the simulations

In the simulations, the $\widehat{\alpha}$'s of fundamentalists were initially uniformly distributed around the true α's, while the true β's were known to the agents.

Figure 3 displays the scatterplot of returns of a typical simulation and the conditional correlation plot calculated from the same simulated return series. The plots display features that are roughly similar to those of real world data displayed in Figure 1: extreme returns cluster at the lower-left and upper-right corners of the scatterplot, signalizing a relatively high occurrence of joint extremes, and conditional correlation is rising towards the tails. Likewise, elliptical symmetry is rejected at 95% confidence level.

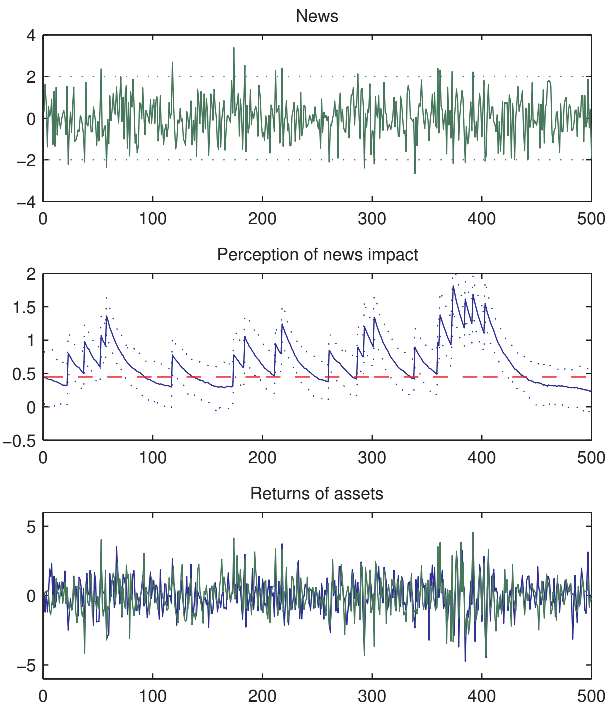


Figure 4: Common news N_t (top); variation of average α's over time (middle, solid line) and the true α (dashed line); returns of assets (bottom).

Figure 4 reveals the inner workings of the model. The

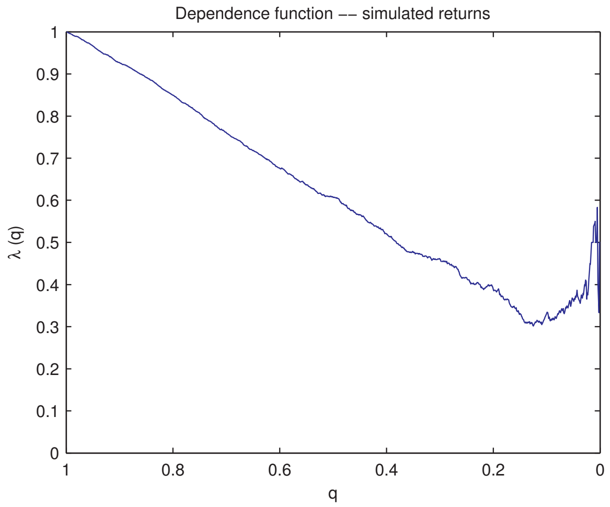


Figure 5: Dependence function of the simulated returns.

top panel displays a realization of the N_t common news over 500 trading periods; the middle panel plots the variation of fundamentalists' average α's over time (along with the 5% and 95% quantiles); the lower panel shows the asset returns. It is clear that periods with higher volatility correspond to periods with higher perceptions of news impact (triggered by large news).

Figure 5 displays the dependence function calculated from a randomly chosen simulated return series. It clearly shows lower tail dependence. While limiting behavior is different from the intradaily sample, it is roughly similar to its counterpart calculated from the extended daily sample.

Simulated returns also reproduce a number of univariate stylized facts. First, they display excess volatility: prices deviate from fundamental values in the short run, but revert back in the long run. In fact, as there is no market consensus among fundamentalists on the true fundamental value, deviations from fundamental values ("bubbles") can last for quite a long time. There is no significant autocorrelation in returns, which is in line with hypotheses of market efficiency. There is however autocorrelation in the second moments, which signals clustered volatility. In addition, returns are fat-tailed: tails follow a power law with a tail index slightly above 3.

CONCLUSION

The model presented above demonstrates that it is possible to replicate the multivariate stylized fact of simultaneous extreme returns using non-equilibrium, heterogeneous-agent models. Additionally, the model also reproduces a number of univariate stylized facts. These results are promising.

The main behavioral assumption in the model is the aligned reaction of heterogeneous fundamentalists. While this mechanism is plausible enough, it is nevertheless important that any behavioral assumption one entertains in an agent-based model be well-grounded. Therefore, stronger empirical behavioral underpinnings are probably needed to support the model's assumptions.

REFERENCES

Arthur, W. B., Holland, J. H., LeBaron, B., Palmer, R., and Tayler, P. (1997). Asset Pricing under Endogenous Expectations in an Artificial Stock Market. In Arthur, W. B., Durlauf, S. N., and Lane, D. A., editors, *The Economy as an Evolving Complex System*, volume II, pages 15–44. Addison-Wesley.

Breymann, W., Dias, A., and Embrechts, P. (2003). Dependence Structures for Multivariate High-Frequency Data in Finance. *Quantitative Finance*, 3(1):1–16.

Brock, W. A. and Hommes, C. H. (1998). Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model. *Journal of Economic Dynamics and Control*, 22(8-9):1235–1274.

Cont, R. (2001). Empirical Properties of Asset Returns: Stylized Facts and Statistical Issues. *Quantitative Finance*, 1:223–236.

Dacorogna, M., Gençay, R., Müller, U., Olsen, R. B., and Pictet, O. V. (2001). *An Introduction to High-Frequency Finance*. Academic Press.

Farmer, J. D. (2002). Market force, ecology and evolution. *Industrial and Corporate Change*, 11(5):895–953.

Farmer, J. D. and Joshi, S. (2002). The price dynamics of common trading strategies. *Journal of Economic Behavior & Organization*, 49:149–171.

Kyle, A. S. (1985). Continuous Auctions and Insider Trading. *Econometrica*, 53(6):1315–1335.

Lux, T. and Marchesi, M. (2000). Volatility clustering in financial markets: A micro-simulation of interacting agents. *International Journal of Theoretical and Applied Finance*, 3:675–702.

Manski, C. F. and McFadden, D. L., editors (1981). *Structural Analysis of Discrete Data with Econometric Applications*. MIT Press.

McNeil, A. J., Frey, R., and Embrechts, P. (2005). *Quantitative Risk Management*. Princeton University Press.

Westerhoff, F. H. (2004). Multiasset Market Dynamics. *Macroeconomic Dynamics*, 8:596–616.

AUTHOR BIOGRAPHY

ZSOLT TULASSAY is a Ph.D. student at the Department of Finance at Corvinus University of Budapest. His main topics of interest include quantitative risk management, the microstructure of financial markets and agent-based modeling. His e-mail address is `zsolt.tulassay@uni-corvinus.hu`.