DIFFERENT RISK-ADJUSTED FUND PERFORMANCE MEASURES: A COMPARISON

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ABSTRACT

Traditional risk-adjusted performance measures, such as the Sharpe ratio, the Treynor index or Jensen's alpha, based on the mean-variance framework, are widely used to rank mutual funds. However, performance measures that consider risk by taking into account only losses, such as Value-at-Risk (VaR), would be more appropriate. Standard VaR assumes that returns are normally distributed, though they usually present skewness and kurtosis. In this paper we compare these different measures of risk: traditional ones vs. ones that take into account fat tails and asymmetry, such as those based on the Cornish-Fisher expansion and on the extreme value theory. Moreover, we construct a performance index similar to the Sharpe ratio using these VaR-based risk measures. We then use these measures to compare the rating of a set of mutual funds, assessing the different measures' usefulness under the Basel II risk management framework.

INTRODUCTION

Evaluation of mutual fund performance is a key issue in an industry that has been rapidly evolving over the last few years; however, there is no general agreement about which measure is best for comparing funds' performance. Traditional fund performance measures are the ones developed by Sharpe (Sharpe 1966), Treynor (Treynor 1966) or Jensen (Jensen 1968) that use the mean-variance approach using the standard framework described in Markowitz (Markowitz 1952), namely using variance and covariance in the form of sigma or beta. However, another popular way to measure risk is the Value-at-Risk defined as *the maximum loss corresponding to a given probability over a given horizon.* Traditional VaR calculations assume that returns follow a normal distribution (Jorion, 2001), however, Favre and Galeano (Favre and Galeano 2002) introduced the modified VaR, which adjusts risk, taking into account skewness and kurtosis using the Cornish-Fisher expansion (Cornish-Fisher 1937). Another approach would be to model only the tail of the distribution, in order to precisely predict an extreme loss in the portfolio's value. Extreme value theory (EVT) provides a formal framework to study the tail behavior of the distributions. The use of EVT for risk management has been proposed in McNeil (McNeil 1998), Embrechts (Embrechts 2000) and Gupta and Liang (Gupta and Liang 2005), among others.

In this paper we evaluate traditional risk-adjusted measures that are based on the mean-variance approach like with others that use VaR to quantify risk exposure, empirically testing the appropriateness of each within a sample of UK mutual funds.

The novelty of this paper lies in the use of the VaR calculation of losses using EVT and applying it as a risk measure to construct a performance index similar to the Sharpe ratio. Using EVT allows for a better estimation of the distribution of extremes and, consequently, provides a better estimation of the risk associated with a portfolio. We will also compare the rating of mutual funds using the different risk-adjusted performance measures.

This paper is organized in five sections. Section 2 reviews the different classical performance measures used in the analysis and introduces the modifications to obtain more accurate estimations. In section 3 we present the data and the sample statistics. Empirical analysis is presented in section 4 as well as differences in funds' ranking. The final section provides a brief summary and some concluding remarks.

PERFORMANCE MEASURES

Performance measures are used to compare a fund's performance, providing investors with useful information about managers' ability. We divide risk-adjusted performance measures into two types: traditional performance measures, based on the mean-variance approach, and VaR-based measures.

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Traditional mutual fund performance measures are the Sharpe ratio, the Treynor index and the Jensen's alpha. The aim of the Sharpe ratio is to measure risk-adjusted performance of a portfolio. It quantifies the reward per unit of total risk:

$$S_i = \frac{R_i - R_f}{\sigma_i}, \quad (1)$$

where R_i represents the return on a fund, R_f is the risk-free rate and σ_i is the standard deviation of the fund. Treynor index measures the return earned in excess of a riskless investment per unit of market risk assumed.

$$T_i = \frac{R_i - R_f}{\beta_i}, \quad (2)$$

where R_i represents the return on a fund, R_f is the risk-free rate and β_i is the beta of the fund. Finally, Jensen's alpha measures the performance of a fund compared with the actual returns over the period. The required return of a fund at a given level of risk β_i can be calculated as:

$$R_i = R_f + \beta_i (R_m - R_f), \quad (3)$$

where R_m is the average market return during the given period. The alpha can be calculated by subtracting the required return from the fund's actual return.

Nonetheless, there are other ways to measure risk. One of the most popular is Value-at-Risk (VaR). Value at Risk is defined as the expected maximum loss over a chosen time horizon within a given confidence interval, that is:

$$P(loss > VaR) \leq 1 - \alpha, \quad (4)$$

where α is the confidence level, typically .95 and .99. Formally, Value-at-Risk is a quantile of the probability distribution F_X, or the x corresponding to a given value of $0 < \alpha = F_X(x) < 1$, which means

$$VaR_\alpha(X) = F^{-1}(x), \quad (5)$$

where F_X^{-1} denotes the inverse function of F_X.

We present four different approaches to VaR: normal VaR, historical VaR, modified VaR and extreme value VaR. Normal VaR assumes that the portfolio's rate of return is normally distributed. Historical VaR uses historical returns to calculate VaR using order statistics. Let $R^{(1)} \geq R^{(2)} \geq \cdots \geq R^{(T)}$ be the order statistics of the T returns, where losses are positive; then the $VaR_\alpha(R) = R^{(T\alpha)}$. The modified VaR takes into account not only first and second moments but also third and fourth ones. It uses the Cornish-Fisher expansion (Cornish-Fisher 1937) to compute Value-at-Risk analytically. And finally, extreme Value VaR uses Generalized Pareto Distribution (GPD) to obtain VaR. The upper tail of $F(x)$ may be estimated by:

$$\hat{F}(x) = 1 - \frac{N_u}{n}\left(1 + \hat{\xi}\frac{x-u}{\hat{\sigma}}\right)^{-1/\hat{\xi}} \quad \text{for all } x > u \quad (11)$$

To obtain the VaR_α we invert (11), which yields

$$VaR_\alpha = u + \frac{\hat{\sigma}}{\hat{\xi}}\left(\left(\frac{n}{N_u}(1-\alpha)\right)^{-\hat{\xi}} - 1\right), \quad (12)$$

where u is the threshold, $\hat{\xi}$, $\hat{\mu}$ and $\hat{\sigma}$ are the estimated shape, location and scale parameters, n is the total number of observations and N_u the number of observations over the threshold.

DATA

Our data set comprises monthly returns on 239 UK mutual funds over 11 years, from January 1995 to December 2005. The data were provided by Morningstar. Owing to the construction of the data set, we use only the data for funds that are active for the entire period. All mutual funds are measured gross of taxes, with dividends and capital gains, but net of fees. To illustrate the different methodologies and for the shake of simplicity, we have chosen the highest ten and lowest ten monthly averages return for the whole period. Table 1 summarizes detailed statistics of those funds.

Table 1: Descriptive statistics of the top ten and bottom ten average return funds.

Panel A: Bottom 10 funds					
Mean	Std.	Max.	Min.	Sk.	Ku.
-0.002	6.672	14.934	-21.498	-0.236	-0.136
0.050	6.632	17.934	-18.002	0.032	0.103
0.080	5.636	13.712	-11.713	-0.019	-0.567
0.127	5.633	13.778	-11.638	0.002	-0.548

0.146	6.651	19.202	-15.840	0.297	-0.198
0.147	6.378	18.895	-20.506	-0.132	0.335
0.171	7.713	26.480	-21.025	0.143	0.531
0.212	6.234	26.352	-20.446	0.579	4.066
0.216	6.754	16.394	-13.878	0.244	-0.498
0.244	6.931	19.498	-15.877	0.345	-0.244
Panel B: Top 10 funds					
Mean	Std.	Max.	Min.	Sk.	Ku.
1.319	5.058	12.182	-14.900	-0.390	0.231
1.331	6.670	29.765	-21.934	-0.049	3.103
1.414	4.959	15.328	-14.597	-0.252	0.500
1.470	4.671	13.823	-10.472	-0.170	0.285
1.526	5.033	15.327	-13.883	-0.231	0.270
1.605	4.611	10.282	-16.008	-0.966	1.623
1.632	4.898	16.023	-17.239	-0.570	1.922
1.697	8.399	24.776	-18.913	0.282	0.740
1.771	6.453	27.880	-12.853	0.665	1.454
3.499	17.921	60.549	-70.528	-0.130	1.929

Mean=sample mean; Std. = Standard deviation; Max.=maximum observed monthly return; Min.= minimum observed monthly return; Sk.= Skewness; Ku. = Kurtosis

An examination of Table 1 shows that several funds have an asymmetric distribution. Upper fund returns have mostly negative skewness, indicating that the asymmetric tail extends toward negative values more than toward positive ones; however bottom fund returns show more negative skewness. According to the sample kurtosis estimates, the top ten fund returns show a higher level of kurtosis than the bottom ones. Table 1 also shows the highest and lowest month return for each fund: the highest one-month return is 60.5%, while the highest loss corresponds to the same fund (79.5%). These results indicate that some funds do not follow a normal distribution.

EMPIRICAL RESULTS

In this section we report and discuss results of the different performance measures. Table 3 shows the results of the classical performance measures: Sharpe, Jensen and Treynor, as well as the ranking of the fund that would result from those indexes. As expected, the bottom ten has a smaller index than the top ten. Moreover, for the bottom ten we have negative Sharpe and Treynor indexes and Jensen's alpha, meaning that those funds are not able to beat the market.

Table 3: Classical mutual fund performance measures

Panel A: Bottom 10 funds						Panel B: Top 10 funds					
Sharpe	Rank	Jensen	Rank	Treynor	Rank	Sharpe	Rank	Jensen	Rank	Treynor	Rank
-0.025	10	-0.372	10	-0.234	10	0.228	7	1.014	9	2.346	7
-0.017	9	-0.311	9	-0.169	9	0.175	10	0.959	10	1.602	10
-0.015	8	-0.075	3	2.371	1	0.252	5	1.119	8	2.724	2
-0.007	7	0.008	1	0.237	2	0.279	3	1.165	7	2.656	3
-0.003	6	-0.166	6	-0.039	8	0.270	4	1.229	6	2.944	1
-0.003	5	-0.221	8	-0.027	7	0.312	1	1.278	4	2.530	4
0.001	4	-0.191	7	0.007	6	0.299	2	1.298	3	2.475	5
0.007	3	-0.078	4	0.105	4	0.182	9	1.267	5	1.648	9
0.008	2	-0.091	5	0.101	5	0.249	6	1.414	2	2.386	6
0.011	1	-0.075	2	0.145	3	0.186	8	2.910	1	2.236	8

The other performance indexes are based on VaR. We have estimated $VaR_{0.05}$ and $VaR_{0.01}$ using the four different approaches discussed in section 2. To offer EVT VaR results we proceed with the generalized Pareto distribution (GPD) estimation. A crucial step in estimating the parameters of the distribution is the determination of the threshold value. We have selected the proper threshold, fitting the GPD over a range of thresholds looking at what level ξ and σ remain constant. Because of space constraints and for the ease of exposition, we do not present a detailed parameter estimation.

$VaR_{0.05}$ and $VaR_{0.01}$ estimates are shown in Table 4. As expected, results for normal VaR and Cornish-Fisher VaR are similar for funds with little asymmetry and kurtosis. EVT VaR tends to be higher than the other VaR thresholds when the return distribution is not normal and presents asymmetries and kurtosis, which is more frequent in the top group. Additionally, the bottom 10 funds have, in general, higher VaR values than the top ones, which means they are more susceptible to extreme events.

Table 4: VaR results of the different approaches

Panel A: Bottom 10 funds							
$VaR_{0.05}$				$VaR_{0.01}$			
Norm-VaR	Hist-VaR	Mod-VaR	EVT-VaR.	Norm-VaR	Hist-VaR	Mod-VaR	EVT-VaR
10.977	10.289	11.449	13.697	15.524	15.259	16.608	18.641
10.858	11.237	10.784	13.193	15.378	14.085	15.384	16.626
9.190	9.531	9.286	10.481	13.031	11.412	12.365	11.572
9.138	9.442	9.197	10.295	12.977	11.317	12.246	11.542
10.794	9.956	10.270	12.231	15.326	11.796	13.787	14.650
10.344	9.967	10.542	12.896	14.690	14.894	15.849	17.793
12.516	11.522	12.122	14.638	17.772	19.225	17.976	20.182
10.043	10.870	8.544	12.685	14.292	13.872	18.350	19.652
10.893	10.107	10.501	11.636	15.496	13.081	13.650	13.437
11.158	10.321	10.527	12.599	15.881	11.837	14.037	14.932
Panel B: Top 10 funds							
$VaR_{0.05}$				$VaR_{0.01}$			
Norm-VaR	Hist-VaR	Mod-VaR	EVT-VaR	Norm-VaR	Hist-VaR	Mod-VaR	EVT-VaR
7.001	7.098	7.552	9.846	10.448	12.814	12.460	15.452
9.641	9.927	9.315	14.439	14.186	17.759	19.269	21.918
6.743	7.242	7.054	10.248	10.123	10.121	11.740	13.567
6.213	7.252	6.415	9.228	9.397	9.862	10.344	10.320
6.753	7.192	7.061	10.283	10.183	10.121	11.458	13.031
5.979	6.669	7.175	10.774	9.121	13.062	15.765	22.774
6.425	5.800	7.058	11.375	9.763	12.029	14.613	21.464
12.118	13.104	11.332	15.303	17.842	17.991	17.804	21.459
8.844	8.103	7.489	10.002	13.242	11.567	13.354	12.715
25.978	22.294	25.948	34.442	38.191	36.590	48.099	53.696

Norm-VaR: normal-VaR; Hist-VaR: Historical-VaR; Mod-VaR: modified VaR using Cornish-Fisher expansion; EVT-VaR: extreme-value-VaR calculated form the GPD estimation.

With regard to performance measures and rankings, Table 5 shows the results of the modified Sharpe ratio using the $VaR_{0.05}$ in Table 4. The rank from each ratio is also in the table. The bottom group exhibits a very small ratio, and they show similar ranking regardless of the method used for calculation. However, the results of the top group show more differences. To find out if they really produce similar results, we compared the rank order correlations of the top ten funds. Table 6 shows the Spearman and Kendall rank correlation.

Table 5: VaR-based performance measures

Panel A: Bottom 10 funds							
Norm-VaR	Rank	Hist-VaR	Rank	Mod-VaR	Rank	EVT-VaR	Rank
-0.015	10	-0.016	10	-0.015	10	-0.012	10
-0.011	9	-0.010	9	-0.011	9	-0.009	9
-0.009	8	-0.009	8	-0.009	8	-0.008	8
-0.004	7	-0.004	7	-0.004	7	-0.004	7
-0.002	6	-0.002	6	-0.002	6	-0.002	6
-0.002	5	-0.002	5	-0.002	5	-0.001	5
0.000	4	0.000	4	0.000	4	0.000	4

0.005	3	0.004	3	0.005	2	0.004	3
0.005	2	0.005	2	0.005	3	0.004	2
0.007	1	0.008	1	0.007	1	0.006	1

Panel B: Top 10 funds

Norm-VaR	Rank	Hist-VaR	Rank	Mod-VaR	Rank	EVT-VaR	Rank
0.165	7	0.163	7	0.153	7	0.117	7
0.121	10	0.117	9	0.125	10	0.081	10
0.185	5	0.172	6	0.177	6	0.122	6
0.210	3	0.180	5	0.203	3	0.141	2
0.201	4	0.189	4	0.193	5	0.132	4
0.241	1	0.216	2	0.201	4	0.134	3
0.228	2	0.253	1	0.208	2	0.129	5
0.126	9	0.117	10	0.135	8	0.100	8
0.181	6	0.198	3	0.214	1	0.160	1
0.128	8	0.150	8	0.128	9	0.097	9

Not surprisingly, the Sharpe and the various VaR ratios exhibit higher correlations than do the Jensen and Treynor ratios. This is not unexpected, since Jensen and Treynor calculations include only the systematic component of risk, while the other measures also include residual risk. The normal VaR measure and the Sharpe ratio rate the ten top funds in the same order.

Table 6: Spearman and Kendall correlation of the performance measures for the top 10 funds

	Sharpe	Jensen	Treynor	Norm-VaR	Hist-VaR	Mod-VaR	EVT-VaR
Sharpe	1	0.248	0.782	1.000	0.891	0.770	0.745
Jensen	0.2	1	-0.006	0.248	0.394	0.394	0.297
Treynor	0.644	-0.067	1	0.782	0.636	0.588	0.673
Norm-VaR	1.000	0.200	0.644	1	0.891	0.770	0.745
Hist-VaR	0.733	0.378	0.467	0.733	1	0.879	0.782
Mod-VaR	0.644	0.378	0.378	0.644	0.733	1	0.927
EVT-VaR	0.600	0.244	0.511	0.600	0.600	0.867	1

Statistics in the top half of the matrix represent Spearman rank correlation coefficients; numbers in the lowest half correspond to Kendall values.

The modified VaR and EVT VaR also show a high correlation. With respect to the Kendall correlation, which is easier to interpret, the value of 0.867 indicates that there is an 87.6 percent greater chance that any pair will be ranked similarly than differently. Consequently, the chance of disagreement in the ranking between modified VaR and EVT VaR is 6.6%; however, the chance between the Sharpe ratio and EVT VaR is 20%.

Taking into account that only modified VaR and EVT VaR ratios allow asymmetry and kurtosis, the results of those measures would be more accurate in the calculation of performance measures for funds with non-normal returns. As a preliminary conclusion, these findings indicate that the comparison between Gaussian funds and non-normal ones is better done using those measures, since they are better able to capture the risk behavior among them.

Our sample data do not show a high degree of asymmetry, and so the Sharpe and normal VaR ratios are highly correlated. In other words, risk measured through variance and the 0.95 quantile loss leads to the same ranking of performance measures.

CONCLUSIONS

We studied monthly returns of UK mutual funds, and we selected for the study the ten funds with the lowest and highest monthly average returns. For the distribution of the bottom ten, we reject that they follow normal distribution in all but one case. On the other hand, the upper ten show a higher degree of asymmetry and kurtosis, and we can reject normality in half of the cases. Also, we have calculated VaR using four different approaches, and EVT VaR is the one that gives higher results for probabilities of 0.05 and 0.01.

Regarding the ranking of performance measures, from the bottom sample we obtained the same ranking regardless of the measure used, except for the Jensen and Treynor measures, which also show a high rank correlation. However, for the top data set, the ranking is not the same. If we consider rankings from the modified Sharpe index calculated with the Cornish-Fisher VaR and EVT-VaR, more accurate measures in the presence of non-normal distribution, both are highly correlated and present a lower correlation with the other measures. So we recommend employing when trying to rank the performance of different funds, especially in the presence of non-normal data, such as returns from hedge funds or more frequently sampled returns.

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