MORTALITY RATES OF SPANISH DEPENDENTS: A JOINT CORRECTION APPROACH

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ABSTRACT

The quantification of mortality, especially the one corresponding to the dependents insured, is nowadays a core problem when pricing long term insurance. The usual fit in Spain shows a big gap, even more important when adjusting older ages. What we show and present in the following pages is a joint correction of the extramortality rates departing from the well-known Rickayzen and Walsh reciprocal function of one plus an exponential. When comparing it wit the usual additive and multiplicative corrections, the joint one yields better results, fitting better to the real Spanih sutuation and thus should be choosen and applied by the insurance sector when working on the subject.

INTRODUCTION

A key element in the pricing of long term care insurance is the quantification of mortality, especially the one corresponding to the dependents insured.

In the actuarial literature on mortality of the dependents insured, there is an unanimous opinion in regard to the mortality rates of dependents are different and higher than those of the overall mortality expressed in general mortality tables used by insurers for the valuation of normal hazards, and of course, appreciably higher than the mortality of insured autonomous. The following expression is therefore assumed:

$$q_x^d > q_x > q_x^a \qquad (1)$$

A different question is, and resolved through different procedures too, the treatment given to the mortality rates when it comes to measure and to express in a mathematical way the present asseveration. Then, there is no single approach but, in contrast, different corrections can be found of the overall mortality for heads in situation of dependence.

The first step in the delivery of adjusted mortality rates consists in having a mortality table of heads in a state of dependence, with gross values, whose definition corresponds to the contractually used in the policy as the one generating benefits in a long term care situation, and that employs equivalent criteria for obtaining the ranking of the long term care situation to those used in the derivation of the incidence rates. The starting raw data come from French survey HID 98-01 .

Differences in the mortality of dependents, according to its level, induce not just higher and different overmortality values but also the functional expressions in which these values are modelled, so it's usually associated additive adjustments to the great dependencies and multiplicative to the less severe ones, as can be seen in (Gatenby 1991).

Once raw mortality of dependents data are derived, different formulas are proposed for correcting these mortality rates; the chosen procedure will minimize the squared deviations function.

Base table for correction

The mortality of dependents is usually derived from overall mortality statistics or mortality of healthy heads. In this case annuity tables are chosen because in this business insurer must also face the survival risk, as (Pitacco 2002) evidences. In the present paper we will fit French statistics, HID 98-01, corrected for Spanish population, using as base statistics the PERMF-00P tables.

Adjustment of mortality rates

There are different ways to adjust the mortality rate of the insured in a long-term care situation, all of them using as a starting point the following expression due to (Ainslie 2000).

$$q_x^d = q_{x+\delta} \times \beta(x) + \alpha(x) \qquad (2)$$

where

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$\beta(x)$ Factor of the general mortality rate, that can be depending on the age;

$\alpha(x)$ Additive surcharge on the overall mortality, which may depend in turn on the age

δ Whole number of years to add to the overall mortality rates.

There are then different approaches, all deriving from the previous general form, depending on the different functional expressions that the general formula finally takes. The choice of a specific fit will finally turn on the level of dependence which in turn is closely related to the kind of pathologies generating the state of long-term care.

The usual correction by adding years to the standard mortality rate hazards is frequently replaced using multiplicative or additive factors, or a mixture of them, even if they don't yield the same values. In the present paper the fitting of the extramortality will not follow that way.

The best approach to the raw mortality data of the great dependents corrected for the Spanish population will come from the comparison of them to the chosen standard mortality tables, the PERMF-00P ones.

THE ADDITIVE CORRECTION

In case that, in the general calculation formula, equals to one, the correction over the overall mortality rates will follow the additive correction. That family shows different fitting possibilities, either through a fixed constant either through other sort of corrections directly depending on the age. We will follow the additive correction proposed by Rickayzen and Walsh known as "reciprocal of one plus an exponential ".

Fixed correction

In addition to its simplicity, this type of adjustment has the property that, as indicated in Ainslie , its weight decreases, in relative terms, with regard to the overall mortality, which is always increasing in the range of ages of elderly.

The materialization of the mortality of dependents under this assumption would remain as

$$q_x^d = q_x + \alpha \qquad (3)$$

This correction has been widely used in actuarial literature because it better fits the mortality of great dependents than other sort of approaches, and its implementation is quite easy. See, for instance, MacDonald and Pritchard ; they propose an additive correction for Alzheimer's institutionalized sufferers of 0.17291. Gatenby considers a value of 0.1 for the extramortality of dependents.

The derivation of the fitting parameters is done through ordinary least squares (OLS) (López shows a comprehensive explanation of the use of OLS for deriving the parameters) and yields the following results :

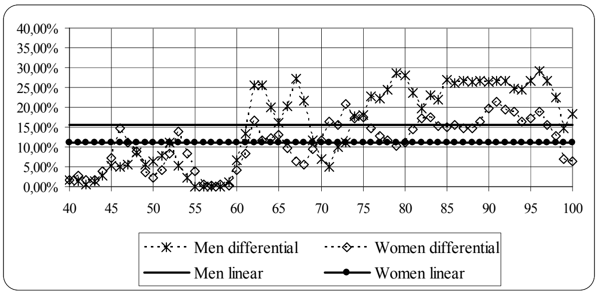

Figure 1: Raw differentials in the mortality rates of dependents and its fit. Additive adjustment from a fixed correction

The additive fit from a fixed correction overestimates the extramortality in younger ages and underestimates those from elderly. This same effect can be seen in overall mortality values already adjusted for dependents, showing that this fitting procedure is not the most appropriate one.

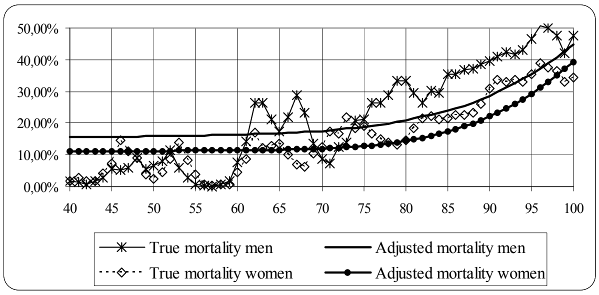

Figure 2: Mortality rates for great dependents. Raw vs. fitted data Additive adjustment from a fixed correction

Age-dependent correction

This additive fitting procedure over the overall mortality rates has the advantage of considering age as an independent variable in a functional way.

The approach to the overmortality proposed by Rickayzen and Walsh in the modelling of the UK data fall into this type of corrections. Because of the lack of British data, they use raw data from the USA, derived from the report of the Society of American Actuaries (SOA), which considers data of the log-term care surveys between 1982 and 1984. The statistics of transition states were obtained from people who were receiving benefits from the Medicare public system. As a result it was observed that:

•Mortality rates increase with the disability level.
•The mortality ratio among those failing more than 3 activities of daily living (ADLs) and those not failing any decreases with age.
•The differences in mortality among those who fail over 3 ADLs and those not failing any has, roughly speaking, an additive behaviour in mortality with a value of 0.15.

With these starting premises Rickayzen and Walsh propose an additive correction from a reciprocal function of one plus an exponential of the following type:

$$q_x^d = q_x + \frac{\delta}{1+\lambda^{x_i - x}} \qquad (4)$$

where

δ Maximum value (of asymptotically convergence) to incorporate depending on the age.

λ Slope factor

x_i Inflection age where the curve changes its shape, from convexity to concavity.

This sort of additive modification of mortality is based on the following considerations, that characterize the proposed function and that is used in both Rickayzen and Walsh as in Leung :

•There is a weak relationship between the mortality of healthy heads and the one of those who are dependents;
•The overmortality is significantly lower in younger ages;
•For less severe dependencies no overmortality is applied.

In our paper the derivation of the , and values come from an OLS procedure with regard to the Spanish gross-up estimated great dependency data, according to the following expression :

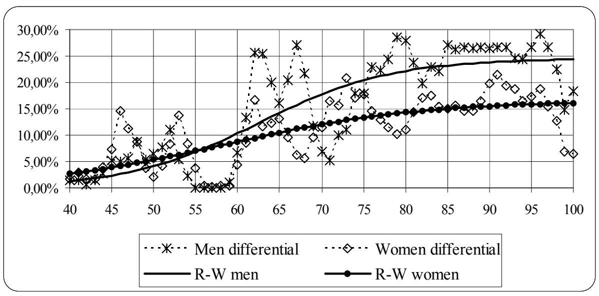

Figure 3: Raw differentials in the mortality rates of dependents and its fit Additive adjustment. Rickayzen and Walsh proposed correction

This additive age-dependent adjustment fits much better the overmortality than the fixed correction additive one. The already mortality-adjusted overall values for dependents under this procedure are shown in the following figure.

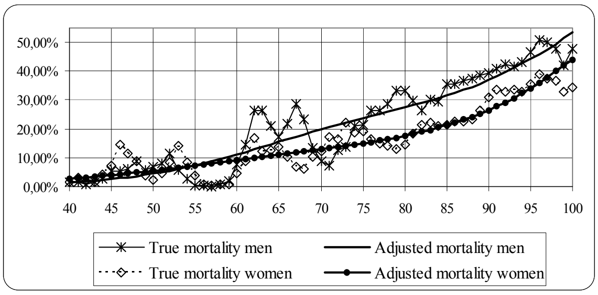

Figure 4: Mortality rates for great dependents. Raw vs. fitted dataAdditive adjustment. Rickayzen and Walsh proposed correction

THE MULTIPLICATIVE CORRECTION

Taking the general expression for a null $\alpha(x)$ we will find a multiplicative correction of the overall mortality rates according to the following expression:

$$q_x^d = q_x \times \beta(x) \qquad (5)$$

Fixed correction

As the prior case of additive fixed correction, this easy to implement procedure and its very intuitive interpretation justifies its widely use , especially for those less severe dependence levels. The disadvantage this method presents comes from its poor adjustment for great dependencies because these statistically weakly depend on overall mortality rates. In the figure below we can appreciate this assertion, after fitting through OLS the differential of mortality among tables PERMF-00P and mortality for great dependents, considering the Spanish modified HID 98-01.

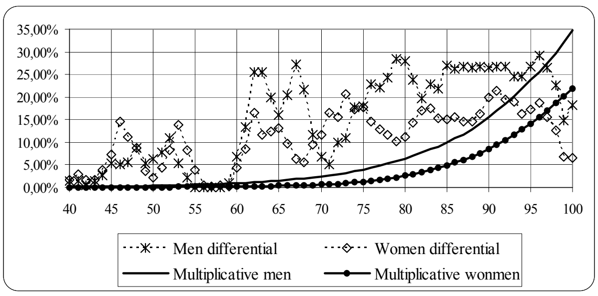

Figure 5: Raw differentials in the mortality rates of dependents and its fit Multiplicative adjustment with fixed correction

The overall values of mortality for dependents under this procedure, shown in the following figure, reflect the bad mortality fit for them:

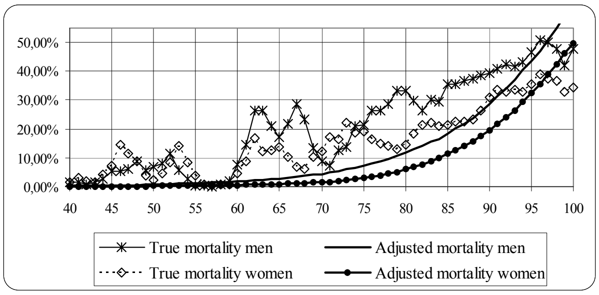

Figure 6: Mortality rates for great dependents. Raw vs. fitted data Multiplicative adjustment with fixed correction

Age-dependent correction

In order to improve the goodness of the previous fit, from a multiplicative modification of mortality rates, and taking into account, as observed in SOA, that the ratio of the mortality of dependents with regard to the overall mortality decreases with age, some authors propose a correction of the multiplicative fit. Boladeras proposes a linear overmortality age-decreasing correction, according to the next expression :

$$q_x^d = q_x \times \max\left[\left(\omega - x\cdot\varphi\right);1\right] \qquad (6)$$

where

ω Maximum multiplicative correction to take into consideration on the standard mortality rates to reflect the mortality of dependents.

φ Reduction on the mortality rates of dependents applied to each age

The figure below reflects the OLS fit of the differential of mortality among PERMF-00P tables and mortality for great dependents, considering the Spanish modified HID 98-01 using the multiplicative age-dependent correction.

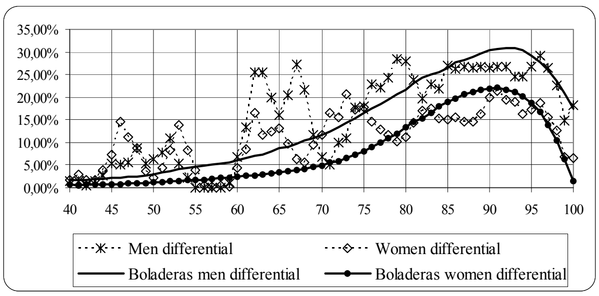

Figure 7: Raw differentials in the mortality rates of dependents and its fit Multiplicative adjustment with age-dependent correction

Our results for the present fitting considerably differ from those from Boladeras, with notably different values for men and women. In case of men, equals to 18.90 and takes a value of 0.1731, as the values for women are 20.59 and 0.1954 respectively.

The overall values of mortality for dependents under this procedure, shown in the following figure, reflect a better fit in the mortality of dependents than in the case of the fixed multiplicative correction:

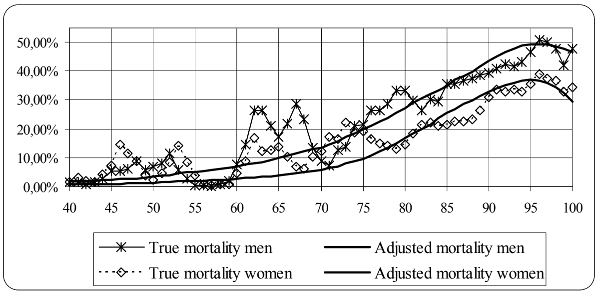

Figure 8: Mortality rates for great dependents. Raw vs. fitted data Multiplicative adjustment with age-dependent correction

The achieved results in the latest modelled ages, with declining mortality rates, do not recommend the use of this fitting procedure.

JOINT CORRECTIONS

Corrections with additive and multiplicative amendments are employed together to fit the extramortality; they try to by-pass the deficiencies of the prior studied procedures, incorporating on one hand a mortality not linked to the overall mortality rates, and on the other a mortality depending on the behavior of the overall mortality.

Linear correction

The linear correction, which considers the overall mortality as the independent variable, is the more widespread case. So we have

The OLS fit of the mortality differential among PERMF-00P tables and great dependents mortality considering the Spanish modified HID 98-01, using the joint linear correction, yields the following results:

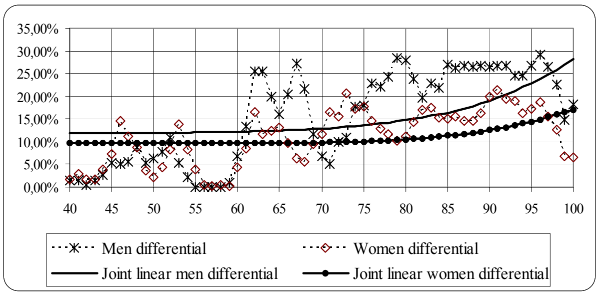

Figure 9: Raw differentials in the mortality rates of dependents and its fit Joint linear adjustment

As in the case of fixed multiplicative correction, this approach overestimates the additional mortality of dependents in ages under 60 and underestimates the one for those over 70 years.

The aggregated mortality values for dependents according to this procedure, as shown in the following figure, evidence the lack of the fit in the mortality of dependents:

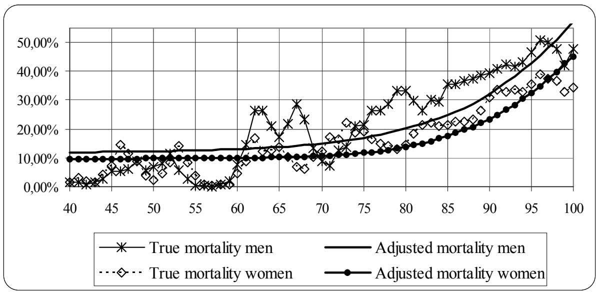

Figure 10: Mortality rates for great dependents. Raw vs. fitted dataJoint linear adjustment

Modified Rickayzen and Walsh correction

The fitting function proposed in Rickayzen and Walsh fits overmortality according to an additive expression of a reciprocal function of one plus an exponential. This fit raises the convergence of overmortality to a maximum value of . This method yields satisfying results if the modelled differentials do not decline after a certain age. With the GK-95 y GR-95 mortality tables there's no declining so the fit is excellent. However the ovemortality differentials respect to the PERMF-00P table decline for both genders after the age of 96. To reflect this effect, we now propose a departure from the Rickayzen and Walsh original formula from a joint correction of the overall mortality to adjust the mortality of dependents. In the present joint correction we consider an additive correction according to the Rickayzen and Walsh expression and a multiplicative correction over the overall mortality rates reflecting the declining of the absolute mortality differentials in the highest ages of the table. Therefore, we get the mortality of dependents from the following formula:

$$q_x^d = q_x \times \beta + \frac{\delta}{1+\lambda^{x_i - x}} \qquad (7)$$

The figure below reflects the OLS fitting of the mortality differential among PERMF-00P tables and the mortality for great dependents considering the Spanish modified HID 98-01 using this joint correction.

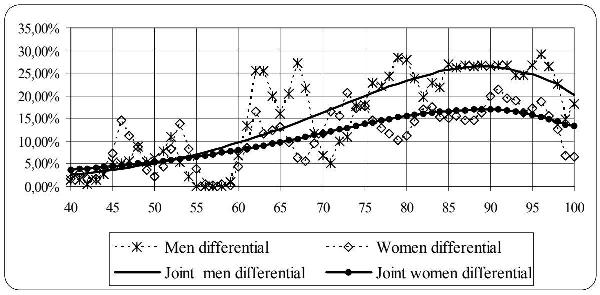

Figure 11: Raw differentials in the mortality rates of dependents and its fitJoint adjustment with additive age-dependent correction

As the figure shows, the fit reflects the decreasing differential among the mortality of healthy individuals and people in state of great dependence after 95 years. The results in the overall mortality with this joint correction, by age and gender, are reflected in the figure below:

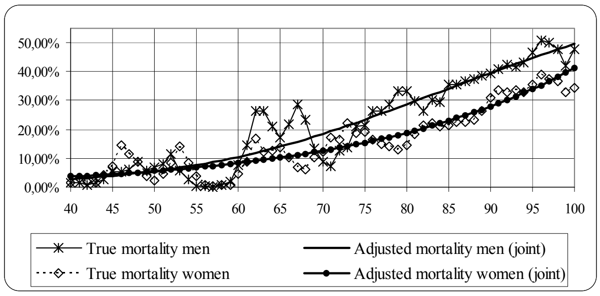

Figure 12: Mortality rates for great dependents. Raw vs. fitted data Joint adjustment with additive age-dependent correction

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