MATHEMATICAL SIMULATION OF THE MAGNETIC FIELD OCCURRED BY THE ARMATURE REACTION OF THE SYNCHRONOUS MACHINE

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ABSTRACT

Using proposed methodology and mathematical simulation parameters of the armature winding, magnetic system geometry is estimated, as well as the saturation's influence on the mentioned parameter is evaluated.
Methods are based on magnetic field mathematical simulation, solving partial derivatives of the differential equation comparatively to the vector magnetic potential A and using one of the most efficient methods – the finite element method (FEM) (Voldek 1978). There is theoretically justified and described method of obtaining the armature winding phase magnetic-flux linkage, i.e., $\Psi(\omega t)$, from the vector potential which, as a function of spatial coordinates $A(x, y)$, is received by mathematical simulation of the stationary magnetic field.
The method is based on solving several stationary magnetic field equations. In equations as a field source are defined the phase currents instantaneous values, which are aligned with the rotor rotation angle $\alpha_i = \omega t_i$.
According to the classical synchronous machine two-reaction theory, synchronous reactances X_d and X_q are determined from the magnetic field's fundamental harmonic in the air gap.

INTRODUCTION

Synchronous machine's inductive reactances and the processes depending on them determine the electromagnetic field, i.e., it's spatial distribution and change in time. This field in operating conditions can be described by Maxwell's equations.
FEM is the most efficient numerical method, which easily allows accurate enough to consider those specific, important factors such as any geometrical shape and size complexity of the magnet system's individual elements, the ferromagnetic material's non-linear characteristic, as well as the field sources (any actual distribution of winding current).

As experience shows while studying the magnetic field, it can be assumed that the machine's magnetic field is plane-parallel.
It is appropriate to substitute the electromagnetic field equations system with one equation which depends on the magnetic vector potential, having only one (axial) component $A = A_z$ in a plane-parallel field. In such equation A is spatial coordinates x, y and time t function, which means that the equation (1) describes space and time alternating magnetic field (Voldek 1978)

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\mu j_a \,, \tag{1}$$

where μ - magnetic permeability; and j_a – the external field source's current density.
Solving this type of equation is rather difficult, therefore, it is appropriate to reduce it to number of simpler tasks.
In this case, the task is based on the fact that in time varying process ($\partial/\partial t \neq 0$) can be viewed as a single fixed process set of different consecutive time points $t_1,\ t_2,\ \ldots,t_i,\ \ldots,t_n$. So, for example, if the field source current density is in time varying sinusoidal function $j_a(t) = j_{am} \sin \omega t$, then equation (1) must be solved n times, each time in the right side of that equation define moment of time t_i that corresponds to current densities moment value $j_a(t_i) = j_{am} \sin \omega t_i$. Solving such a task as the results receive vector potentials, which are essentially a table formed functional dependence $A = f(t)$. A similar approach is used solving non-stationary field equations, when rotor speed $\nu \neq 0$. In this case, it is possible to solve a number of magnetostatic field equations. Each of them corresponds to different consecutive rotor positions.
For the mathematical simulation of the magnetic field and obtaining results the complex multi-functional program *QuickField* (QuickField 2009) is used.
Software provides opportunities for the following actions:

- to describe the geometric model (or topology) of the object under study;

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– to assign the medium characteristics, including various ferromagnetic material magnetization curves $B = f(H)$;

– to assign field sources - a current density in windings as a function of spatial coordinates;

– to assign the Dirichle and/or the Neuman boundary conditions;

– to solve tasks with high precision;

– to get a visual picture of the field;

– to calculate various electromagnetic field differential and integral characteristics.

This paper addresses the following main tasks:

– to apply and to use the available modern software for mathematical simulating of the magnetic field using numerical methods;

– to illustrate with examples the practical use of methods, which quantifiably estimate the magnetic constructive parameters of the system and, above all, the saturation effect on parameters of the machine's electromagnetic field.

DETERMINATION OF THE SYNCHRONOUS REACTANCE USING THE SYNCHRONOUS MACHINE'S MATHEMATICAL SIMULATION RESULTS

As known (Voldek 1978), the magnetic asymmetry of the salient pole synchronous machine's rotor ensures reluctances, which are bigger in quadrature axis (*q* direction) than the reactance in the direct axis (*d* direction). Therefore, for the synchronous machines with rotor's magnetic asymmetry, it is appropriate to use a two-reaction method based on the superposition principle. According to this principle, the direct axis (Φ_d flux) and quadrature axis (Φ_q flux), where the magnetic flux is operating, are mutually independent. It should be noted, that this assumption is correct only for machines with unsaturated magnetic system. However, making additional adjustments based on the magnetic field mathematical simulation results; two-reaction method can also be used for machines with a saturated magnetic system.

According to the two-reaction method theory, the synchronous inductive reactances X_d and X_q, can be determined:

$$X_d = \frac{E_d}{I_d} \tag{2}$$

$$X_q = \frac{E_q}{I_q} \tag{3}$$

In these equations, EMF E_d and E_q are the EMFs induced by the fundamental harmonic armature winding of the direct field and the quadrature field; I_d and I_q - armature current's direct and quadrature components.

The effective value of the EMFs E_d and E_q can be determined using the following formulas

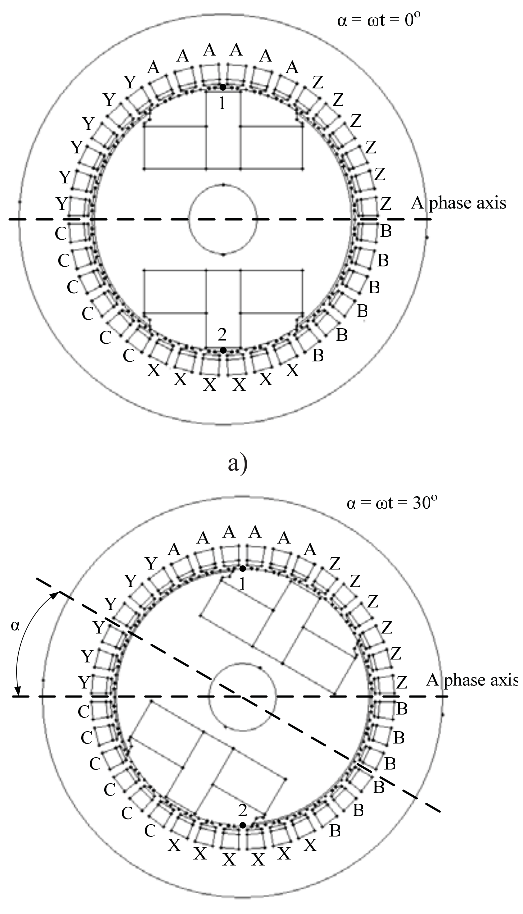
$$E_d = 4{,}44 f \Psi_{d1} k_{w1} \tag{4}$$

$$E_q = 4{,}44 f \Psi_{q1} k_{w1} \tag{5}$$

where Ψ_{d1} and Ψ_{q1} - fundamental harmonic ($\nu = 1$) amplitude values of flux linkages generated by the direct field and the quadrature field , k_{w1} - the fundamental harmonics winding factor.

The necessary sequence of operations is the following: first, determine Ψ_{d1} and Ψ_{q1} (see (4) and (5)), which, in turn, are required for reactance X_d and X_q determination from (2) and (3).

The figure 1 shows the calculation region of the simplified magnetic field model for the three-phase ($m = 3$) two-pole ($2p = 2$) machine with a number of armature slots $Z = 36$.



a)

b)

Figures 1. Distribution of currents in the armature winding phases for determination of the direct axis field's flux linkage $\Psi_{d(\omega t)}$ of the armature in time moments $\omega t_i = 0^\circ$ (a) and $\omega t_i = 30^\circ$ (b), when the rotor rotation angle (electric degrees) is 0° and 30°

The instantaneous value of current in armature windings phases may be determined by the following formulas:

$$\left.\begin{aligned} i_A &= I_m \cdot \cos(\omega t - \alpha) \\ i_B &= I_m \cdot \cos(\omega t - 120^\circ - \alpha) \\ i_C &= I_m \cdot \cos(\omega t - 240^\circ - \alpha) \end{aligned}\right\} \quad (6)$$

where $I_m = \sqrt{2} \cdot I$ - the armature current as the peak value of the field source.

To determine the flux linkage $\Psi_d(\omega t_i)$ with sufficient accuracy, it is preferably that the time step is chosen small enough. As experience of numerical experiments shows, the rotor rotation angle step is appropriate to be chosen so, that $\Delta\alpha = t_z/2$, where t_z - the armature tooth pitch.

The magnetic flux of one pole can be obtained with different armature current instantaneous values according to (7) with the rotor position angle α_i. After a series of magnetic field calculations for time moments t_i

$$\Phi_d(\omega t_i) = \Phi_d(\alpha_i) = (A_{1i} - A_{2i})l \quad (7)$$

and flux linkage with armature winding phase as a function of time

$$\Psi_d(\omega t_i) = (A_{1i})wl \quad (8)$$

where A_{1i} and A_{2i} - the vector potential values on the surface of the armature in points 1 and 2 (fig. 1.), w - number of turns per phase, l - the machine length in axial direction.

Numerical harmonic analysis of the function $\Psi_d(\omega t_i)$ allows to obtain the fundamental harmonic amplitude value Ψ_{d1} of the flux linkage's, which is required according to formula (4) for determination of EMF E_d.

Similarly to the expression (5), the EMF E_q can be obtained if α is replaced with $\alpha - 90^\circ$ (see table 1.) while simulating quadrature axis magnetic field in expression (6).

RESULTS OF THE MAGNETIC FIELDMATHEMATICAL SIMULATION, ITS' ANALYSIS AND USE

The simulation of the synchronous machine magnetic field is made for the experimental three-phase machine with $2p = 2$, in which a single layer full step ($y = \tau$) winding is located in armature 36 slots. Other machine parameters: $f = 50$ Hz, turn number per phase $w = 60$, slots per pole and phase $q = 6$, winding factor $k_w = 0.96$, armature rated current $I_N = 6.56$ A. The ferromagnetic elements of the machine magnetic system are made up from the electrical-sheet steel. The main geometric dimensions are: outer diameter of the armature $D_a = 0.212$ m, the armature's inside diameter $D = 0.136$ m, the air gap $\delta = 0.001$ m; the armature's active length $l_\delta = 0.125$ m; the pole overlapping factor $\alpha_p = 0.604$, the pole pitch $\tau = 0.214$ m m.

Calculations were made for five armature current values $I = (0.4;\ 0.6;\ 0.8;\ 1.0;\ 1.2)I_N$, where the effects of saturation on the magnetic field's character, on parameters X_d and X_q values are evaluated. To obtain the flux linkage with armature winding's phase for a certain current value as a function of time, the instantaneous current value is set according to the rotor rotation angle with step $\Delta\alpha = 5^\circ$ (electric degree), which is matched with the armature current phase's change along the same angle.

Because of the periodicity and symmetry of the function $i(\omega t)$, it is sufficient if the calculations are made in one quarter of the period of current changes, that is $\alpha = \omega t = 90^\circ$ (electric degree).

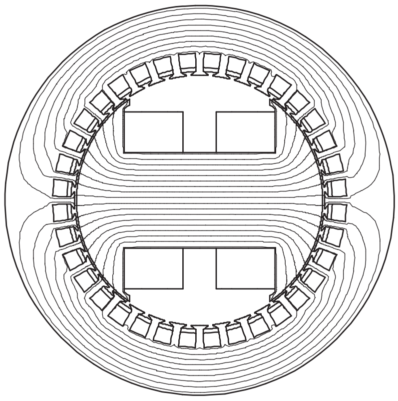
The estimated by the formula (7) armature winding slot's current instantaneous value in relative units are given in table 1. Phase's zone width equals to 6 slots for chosen machine.

Table 1. Phase current's instantaneous value in relative units for different rotor rotation angles (electrical degrees)

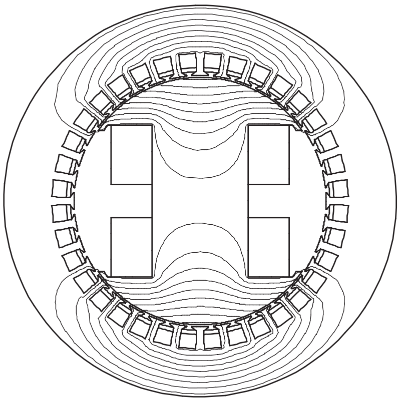
$\alpha = \omega t(^\circ)$	$i^*(A)$	$i^*(B)$	$i^*(C)$
0	1.000	-0.500	-0.500
5	0.966	-0.423	-0.574
10	0.985	-0.342	-0.643
15	0.966	-0.259	-0.707
20	0.940	-0.174	-0.766
25	0.906	-0.087	-0.819
30	0.866	0	-0.866
35	0.819	0.087	-0.906
40	0.766	0.174	-0.940
45	0.707	0.259	-0.966
50	0.643	0.342	-0.985
55	0.574	0.432	-0.996
60	0.500	0.500	-1.000
65	0.423	0.574	-0.996
70	0.342	0.643	-0.985
75	0.259	0.707	-0.966
80	0.174	0.766	-0.940
85	0.087	0.819	-0.906
90	0	0.866	-0.866

In the above mentioned tables angle α values are shown for simulation of direct axis field cases assuming that the A-phase axis coincides with the pole direct-axis (see also fig 1.). The angle $\alpha' = \alpha - 90^\circ$ should be used for simulation of the armature quadrature-axis field.

Figure 2 shows the direct axis and quadrature axis magnetic field's picture, acquired for machine, if the armature current components I_d and I_q in relative units $I_d = I_q = 1$, i.e. equal to the rated current.



a)



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Figures 2. Armature direct reaction (a) and quadrature-axis reaction (b) magnetic field pictures for angle in moment ωt, when the phase *A* current is at maximum value

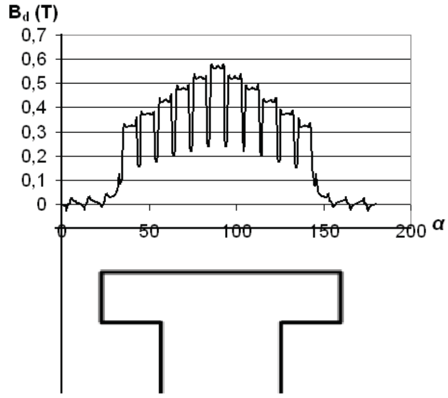
Figures 3 shows distribution curves of the armature direct axis and quadrature axis magnetic flux densities B_d and B_q determined on the teeth and slots middle points on the armature surface.

Flux linkage curves $\Psi_d(\omega t_i)$ and $\Psi_q(\omega t_i)$ are obtained with calculations carried out for different rotor rotation angles, observed with the armature phase currents angle for moments t_i ($\alpha_i = \omega t_i$).Various armature currents I_d^* and I_q^* values for one half of the period are shown on Figure 5.

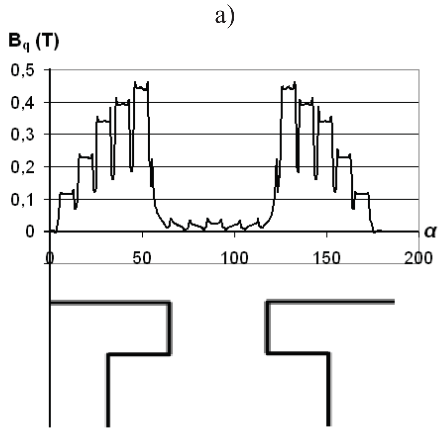
The flux linkage fundamental harmonics Ψ_{d1} and Ψ_{q1} are determined during numerical harmonic analysis of $\Psi_d(\omega t_i)$ and $\Psi_q(\omega t_i)$. The EMFs E_d and E_q values for various armature currents are calculated (see table 2 from formulae (4) and (5)).

Table 2. EMF E_d and E_q fundamental harmonics dependence on the armature currents

I^*	E_d (V)	E_q (V)
0.4	44.44	17.45
0.6	65.09	26.16
0.8	80.94	34.58
1.0	93.29	42.60
1.2	103.49	49.90



a)



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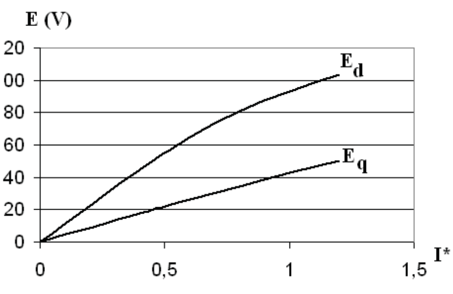
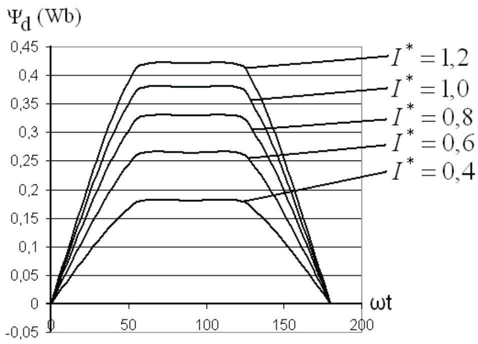
Figures 3. Armatures direct axis (a) and quadrature axis (b) magnetic flux densities distribution in the air gap (on the armature surface); (angle α in electrical degrees)

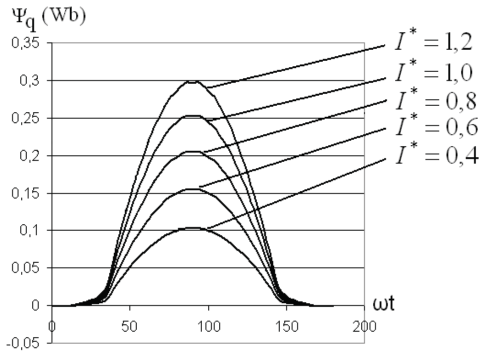
Magnetic system's saturation can be assessed by curves $E_d = f(I^*)$ and $E_q = f(I^*)$ shown on fig. 4.



Figures 4. Armature winding EMF dependence from armature currents in relative units



a)



b)

Figures 5. By armature direct reaction (a) and quadrature reaction (b) induced flux linkage with armature windings phase as a function of time

Reactances X_d and X_q calculated from formulae (2) and (3) are given in table 3. Table 3 also indicates the saturation coefficients $k_{\mu d}$ and $k_{\mu q}$ values corresponding to the rated current $I^* = 1$.

Table 3. Reactances dependence from saturation level induced by armature currents

I^*	X_d (Ω)	X_q (Ω)	X_d/X_q	$k_{\mu d}$	$k_{\mu q}$
0.4	16.93	6.65	2.55		
0.6	16.53	6.64	2.49		
0.8	15.42	6.59	2.34		
1.0	14.22	6.49	2.19	1.22	1.02
1.2	13.14	6.34	2.07		

CONCLUSIONS

The finite element method offers tremendous opportunities for determining the synchronous machine's magnetic field and from it depending characteristics. Synchronous direct axis reactance X_d and quadrature axis reactance X_q can be correctly determined from the magnetic field's mathematical simulating results applying the principle of superposition used in the classical synchronous machine theory. Saturation effect on parameters X_d and X_q can be estimated by the saturation factors $k_{\mu d}$ and $k_{\mu q}$. Saturation factors values are depending on magnetic system geometric dimensions and on level of saturation ($k_{\mu d} = 1.2 \div 1.5$, $k_{\mu q} = 1.0 \div 1.2$). Estimation results are also confirmed by the experimental results of examined machine.

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ALEKSANDRS MESŅAJEVS was born 1985 in Latvia. In 2008 he graduated from Riga Technical University, gaining M.Sc.ing. degree. Presently he is a PhD student.
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After graduating the RPI A.Zviedris has worked as an assistant Professor (1968-1973), and Department of Electrical Machines and Apparatus Head (1973-1984). In 2001 he was elected as Associate Professor in the Department of Electrical Machines and Apparatus, where work to date.

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A. Zviedris is an Expert in Standardization Technical Committee of Electrotechnical Terminology of Electrical Engineering, a member of Latvian Union of Scientists.