

MODELLING AND CONTROL OF HOT-AIR SYSTEM UNDER CONDITIONS OF UNCERTAINTY

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ABSTRACT

The contribution deals with modelling and control of temperature in laboratory model of hot-air system under conditions of parametric uncertainty. In the first instance, the first and second order parametrically uncertain mathematical models of the plant are constructed, and then they are utilized for design of various controllers with conventional structure. The control synthesis exploits general solutions of Diophantine equations in the ring of proper and stable rational functions. Robust stability of final closed control loops is tested using the value set concept and zero exclusion condition.

INTRODUCTION

The uncertainty represents serious problem in many real control applications. One of convenient approaches to uncertain modelling and description supposes no variations in the structure but only in parameters of the controlled system. In such case, one speaks about parametric uncertainty. In spite of the uncertain conditions, the often requirement consists in application of a cheap controller with simple PI or PID structure and fixed coefficients which would ensure stability and desired control behaviour for all expected values of the uncertain parameters.

A potential solution of this task consists in the usage of continuous-time controllers designed through general solutions of Diophantine equations in the ring of proper and stable rational functions (R_{PS}), Youla-Kučera parameterization and divisibility conditions. The principal idea of this approach is adopted from (Vidyasagar 1985; Kučera 1993) while the control design itself is proposed and analysed e.g. in (Prokop and Corriou 1997; Prokop et al. 2002; Matušů et al. 2008). This method brings a single tuning parameter $m > 0$ which can be used for influencing the control response. Later on, closed-loop robust stability can be verified for example with the assistance of the value set concept and zero exclusion condition (Barmish 1994; Bhattacharyya et al. 1995).

This paper aims to present a simple way of constructing a model with parametric uncertainty and also an algebraic approach to continuous-time robust control design. The proposed techniques are applied during control of bulb temperature in laboratory model of hot-air tunnel. In a set of experiments, the controlled system is approximated by first or second order transfer functions with parametric uncertainty, the controllers are designed, the robust stability is verified, and the final control responses are tested and evaluated.

HOT-AIR PLANT DESCRIPTION

The controlled plant has been represented by laboratory model of hot-air tunnel constructed in VŠB – TU of Ostrava (Smutný et al. 2002). Generally, this object can be seen as multi-input multi-output (MIMO) system, however, the experiments have been done on a selected single-input single-output (SISO) loop. The model is composed of the bulb, primary and secondary ventilator and an array of sensors covered by tunnel. The bulb is powered by controllable source of voltage and serves as the source of light and heat energy while the purpose of ventilators is to ensure the flow of air inside the tunnel. All components are connected to the electronic circuits which adjust signals into the voltage levels suitable for CTRL 51 unit. Finally, this control unit is connected with the PC via serial link RS232. The diagram of the plant and the whole control system is shown in fig. 1.

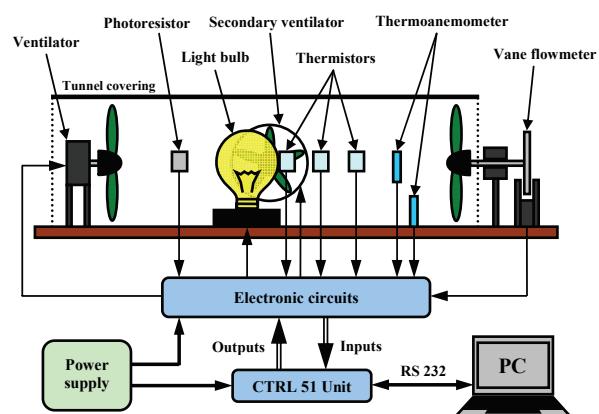


Figure 1: Scheme of Hot-Air Tunnel and Control System

The CTRL 51 unit has been produced in the Academy of Sciences of the Czech Republic (Klán et al. 2003). The tables 1 and 2 denote the meaning of input and output channels of this unit, respectively.

Table 1: Connection of Input Signals of CTRL 51 Unit

Input channel	Sensor
Input 1 (y_1)	Light intensity of the bulb (photoresistor)
Input 2 (y_2)	Temperature a few mm from the bulb (2nd thermistor)
Input 3 (y_3)	Temperature of the bulb (1st thermistor)
Input 4 (y_4)	Temperature at the end of the tunnel (3rd thermistor)
Input 6 (y_6)	Airflow speed (thermoanemometer)
Input 7 (y_7)	Airflow speed (vane flowmeter)

Table 2: Connection of Output Signals of CTRL 51 Unit

Output channel	Actuator
Output 1 (u_1)	Bulb voltage (control of light intensity and bulb temperature)
Output 2 (u_2)	Voltage of the primary ventilator (control of revolutions)
Output 3 (u_3)	Voltage of the secondary ventilator (control of revolutions)

All presented identification and control experiments were performed in MATLAB 6.5.1 environment. The communication between MATLAB and CTRL 51 unit was arranged through four user functions (for initialization, reading and writing of data and for closing) and the synchronization of the program with real time was done via „semaphore“ principle (furthermore, the utilization of MATLAB functions „tic“ and „toc“ as an alternative were tested). To ensure the sufficient emulation of the continuous-time control algorithms, the sampling time 0.1 s was used. The

detailed information about utilization of serial link under MATLAB including mentioned user routines, program synchronization mechanism and several tests can be found in (Dušek and Honc 2002). The discretization of control laws was carried out by left rectangle approximation method.

The considered loop covers bulb voltage u_1 (control signal), which influences temperature of the bulb y_3 (controlled variable). The other actuating signals were preset to constant values – primary ventilator voltage u_2 to 2 V and secondary one u_3 to 0 V.

IDENTIFICATION OF THE SYSTEM

Naturally, the first task was to determine static and dynamic behaviour of the system. The trio of static characteristics measured during 3 different days is plotted in fig. 2.

Note, that the system properties markedly depend on current conditions and that it can be saturated in higher levels of u_1 . Therefore, the value 10 V was excluded from the subsequent process of identification. The fig. 3 shows the set of step responses with the starting point $u_1 = 0$ V while the final value of u_1 is from 1 to 9 V and the fig. 4 depicts the similar responses from $u_1 = 5$ V to 6, 7, 8 and 9 V.

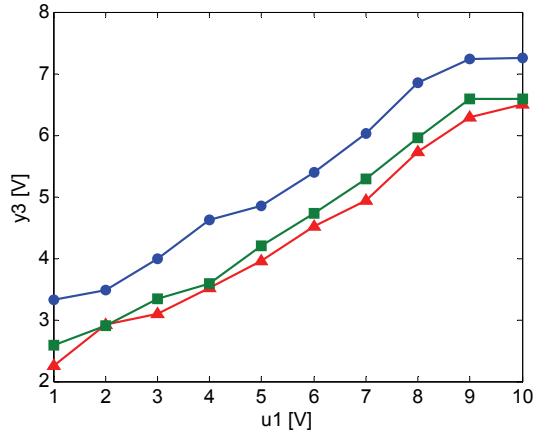


Figure 2: Static Characteristics of the System

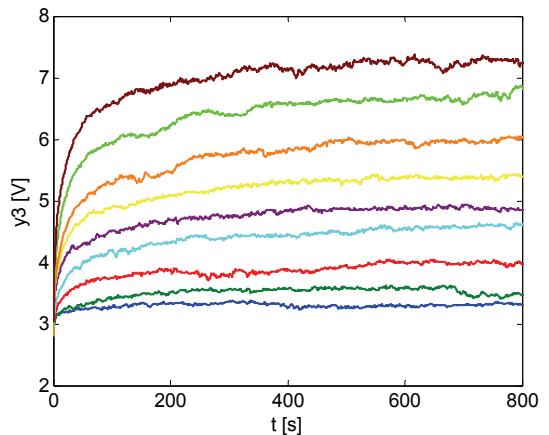


Figure 3: Step Responses Starting from $u_1 = 0$ V

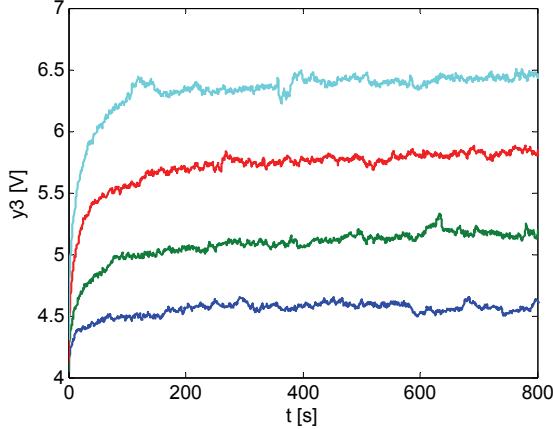


Figure 4: Step Responses Starting from $u_1 = 5\text{ V}$

All measured responses were normalized and approximated by step response of system with selected structure. In the first instance, it has been approximated by first order system:

$$G(s) = \frac{K}{Ts + 1} \quad (1)$$

However, with respect to the character of dynamics which is initially very fast and gradually starts to slow, the first order plant represents simplified solution. On that account, also the second order system:

$$G(s) = \frac{K(\tau s + 1)}{(T_1 s + 1)(T_2 s + 1)} \quad (2)$$

has been assumed. The least squares method was used for identification of time constants. The example of approximation by first (1) and second order system (2) is given in fig. 5. It belongs to u_1 step-change from 0 to 6 V. In this particular case, the transfer functions are:

$$G(s) = \frac{0.4107}{37.2289s + 1} \quad (3)$$

$$G(s) = \frac{0.4107(119.7679s + 1)}{(9.5334s + 1)(186.3907s + 1)} \quad (4)$$

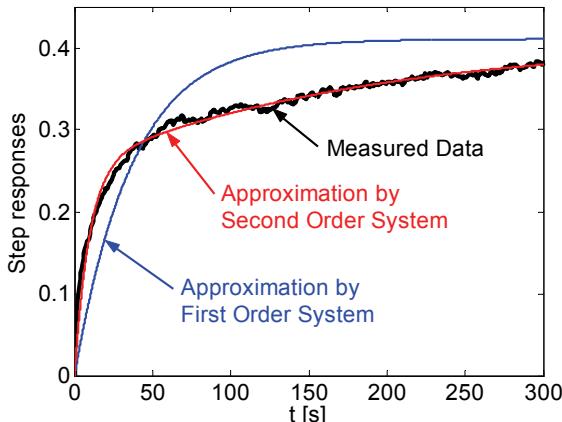


Figure 5: Example of Approximations

In an effort to stress more the initial part of responses with fast dynamics, only first 100 s have been included in optimization of T for first order. The complete results both for first and second order are shown in table 3.

Table 3: Identification Results

u_1 [V]	K [-]	T [s]	τ [s]	T_1 [s]	T_2 [s]
0 - 1	0.2435	41.2259	27.1160	3.6437	72.1970
0 - 2	0.2833	47.2704	33.6512	3.2631	92.8624
0 - 3	0.3594	43.3580	115.9249	11.9881	186.0546
0 - 4	0.4274	48.2019	109.6962	10.8675	195.5283
0 - 5	0.3740	37.2064	76.1006	6.6347	130.6889
0 - 6	0.4107	37.2289	119.7679	9.5334	186.3907
0 - 7	0.4599	41.2868	119.2653	9.5686	194.5886
0 - 8	0.4889	37.9254	114.5048	9.5139	180.0520
0 - 9	0.4680	30.7657	93.7720	9.4876	137.8641
5 - 6	0.5656	35.1384	79.5117	4.9491	137.3248
5 - 7	0.5505	34.5624	90.2150	9.6484	139.7103
5 - 8	0.5676	28.0184	93.1630	8.1383	135.3549
5 - 9	0.5403	26.0924	94.5474	11.5862	123.6040

The set of data from these tables and advisement of substantive properties have led to the construction of models with parametric uncertainty. The lower bound of time constant T in model (5) has been moved down to 5 s because of fast initial dynamics which should also be taken into consideration. Although the intended working point corresponds to reference values of y_3 at 4 and 5 V, the models are going to cover all measured temperature areas:

$$G(s, K, T) = \frac{[0.2; 0.7]}{[5; 50]s + 1} \quad (5)$$

$$G(s, K, \tau, T_1, T_2) = \frac{[0.2; 0.7]([25; 130]s + 1)}{([3; 14]s + 1)([70; 210]s + 1)} \quad (6)$$

ALGEBRAIC SYNTHESIS

The fractional approach developed by Vidyasagar (1985) and Kučera (1993) and discussed in (Prokop and Corriou 1997; Prokop et al. 2002) supposes that transfer functions of continuous-time linear causal systems in R_{PS} are expressed as:

$$G(s) = \frac{b(s)}{a(s)} = \frac{b(s)(s + m)^{-n}}{a(s)(s + m)^{-n}} = \frac{B(s)}{A(s)} \quad (7)$$

where $n = \max \{\deg(a), \deg(b)\}$ and $m > 0$.

Consider a two-degree-of-freedom (2DOF) control system from fig. 6. Take notice that the traditional one-degree-of-freedom (1DOF) system is obtained simply by $R = Q$.

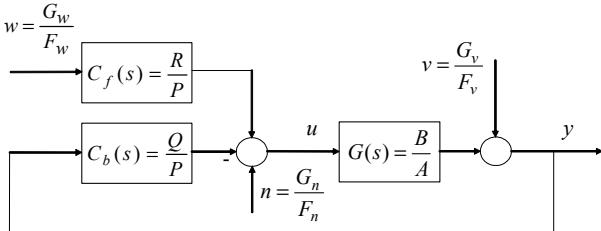


Figure 6: Two-Degree-of-Freedom Control System

$$\text{External signals } w = \frac{G_w}{F_w}, \quad n = \frac{G_n}{F_n} \quad \text{and} \quad v = \frac{G_v}{F_v}$$

represent the reference, load disturbance and disturbance signal, respectively. The most frequent case is a stepwise for reference and load disturbance signal and a harmonic signal for disturbance. Denominators of their transfer functions are then $F_w = F_n = \frac{s}{s+m}$ and $F_v = \frac{s^2 + \omega^2}{(s+m)^2}$, respectively.

Basic relations following from fig. 6 are:

$$y = \frac{B}{A}u + v; \quad u = \frac{R}{P}w - \frac{Q}{P}y + n \quad (8)$$

Further, the following equations hold:

$$y = \frac{BR}{AP+BQ}\frac{G_w}{F_w} + \frac{BP}{AP+BQ}\frac{G_n}{F_n} + \frac{AP}{AP+BQ}\frac{G_v}{F_v} \quad (9)$$

Provided that no disturbances affect the control system, i.e. $n = v = 0$, the control error is given by:

$$e = w - y = \left(1 - \frac{BR}{AP+BQ}\right)\frac{G_w}{F_w} \quad (10)$$

For the structure 1DOF ($R = Q$), the last relation takes the form:

$$e = \frac{AP}{AP+BQ}\frac{G_w}{F_w} \quad (11)$$

The basic task is to ensure stability of the system in fig. 6. All stabilizing feedback controllers are given by all solutions of the linear Diophantine equation:

$$AP + BQ = 1 \quad (12)$$

with a general solution $P = P_0 + BT$, $Q = Q_0 - AT$, where T is free in \mathbb{R}_{PS} and P_0 , Q_0 is a pair of particular solutions (Youla – Kučera parameterization of all stabilizing controllers). Details and proofs can be found e.g. in (Prokop and Corriou 1997; Prokop et al. 2002). Then relations (10) and (11) take the form:

$$e = (1 - BR)\frac{G_w}{F_w} \quad (13)$$

$$e = AP\frac{G_w}{F_w} \quad (14)$$

For asymptotic tracking then follows:

F_w must divide AP for 1DOF

F_w must divide $(1 - BR)$ for 2DOF

The last condition gives the second Diophantine equation for 2DOF structure:

$$F_w S + BR = 1 \quad (15)$$

The design process is demonstrated for first order system. A nominal transfer function is supposed as:

$$G(s) = \frac{b_0}{s + a_0} \quad (16)$$

Further, step-wise reference with $F_w = \frac{s}{s+m}$ and no disturbances are assumed. The Diophantine equation (12) takes the form:

$$\frac{s+a_0}{s+m} p_0 + \frac{b_0}{s+m} q_0 = 1 \quad (17)$$

Multiplying by $(s+m)$ and comparing coefficients give the general stabilizing solution in the form:

$$P(s) = p_0 + \frac{b_0}{s+m} T; \quad Q(s) = q_0 - \frac{s+a_0}{s+m} T \quad (18)$$

where $q_0 = \frac{m-a_0}{b_0}$; $p_0 = 1$ and T is free in \mathbb{R}_{PS} . The asymptotic tracking for a stepwise reference w is given by divisibility of $F_w = \frac{s}{s+m}$ and AP (or only P in this case). It is achieved for $T = t_0 = -\frac{m}{b_0}$ so that P has zero absolute coefficient in the numerator. Then inserting t_0 into (18) gives:

$$P(s) = \frac{s}{s+m}; \quad Q(s) = \frac{\tilde{q}_1 s + \tilde{q}_0}{s+m} \quad (19)$$

$$\tilde{q}_1 = \frac{2m - a_0}{b_0}; \quad \tilde{q}_0 = \frac{m^2}{b_0} \quad (20)$$

Finally, the 1DOF controller has the transfer function:

$$\frac{Q(s)}{P(s)} = \frac{\tilde{q}_1 s + \tilde{q}_0}{s} \quad (21)$$

Note that \tilde{q}_1 , \tilde{q}_0 depend on single tuning parameter $m > 0$. Hence, another topic of interest should be an appropriate choice of m . A potential way of nominal tuning can be found e.g. in (Matusů and Prokop 2008).

CONTROL EXPERIMENTS

First, the uncertain model (5) and nominal system (for controller design):

$$G_N(s) = \frac{0.5}{25s+1} = \frac{0.02}{s+0.04} \quad (22)$$

have been assumed. The tuning parameter $m = 0.0748$, which corresponds to 2 % of first overshoot from (Matusů and Prokop 2008), has been selected. The computed 1DOF PI controller (21), (20) is:

$$C_b(s) = \frac{\tilde{q}_1 s + \tilde{q}_0}{s} = \frac{5.48s + 0.2798}{s} \quad (23)$$

The characteristic polynomial of closed loop with plant (5) and controller (23) can be easily computed:

$$\begin{aligned} p(s, K, T) &= Ts^2 + (1 + K\tilde{q}_1)s + K\tilde{q}_0 = \\ &= [5; 50]s^2 + [2.096; 4.836]s + [0.05596; 0.1959] \end{aligned} \quad (24)$$

This simple polynomial is obviously robustly stable, because it is of second order and all its coefficients are positive, i.e. the whole system is robustly stable. The real closed-loop control behaviour can be seen in fig. 7. The control signal is depicted only in 25 % of its true size because of better perspicuity of controlled variable.

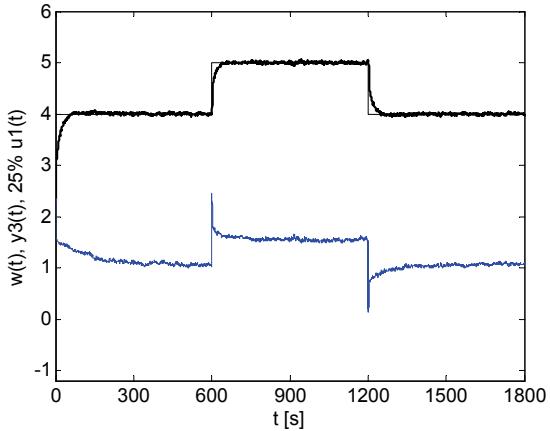


Figure 7: Control of Bulb Temperature by (23)

Next, it has been supposed the system with parametric uncertainty (6) and nominal plant:

$$G_N(s) = \frac{0.5(100s+1)}{(9s+1)(150s+1)} = \frac{0.037s + 0.00037}{s^2 + 0.117s + 0.00074} \quad (25)$$

Unfortunately, the single tuning parameter entails restraint for control design here and it is not easy to find suitable m with “quality” control response. Using synthesis technique from previous section, the chosen value $m = 0.025$ leads to 1DOF PID regulator:

$$\begin{aligned} C_b(s) &= \frac{\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0}{s^2 + \tilde{p}_1 s} = \\ &= \frac{-1.1556s^2 + 0.01324s + 0.001055}{s^2 + 0.02502s} \end{aligned} \quad (26)$$

The plant (6) and controller (26) leads to closed-loop characteristic polynomial:

$$\begin{aligned} p(s, K, \tau, T_1, T_2) &= T_1 T_2 (s^4 + \tilde{p}_1 s^3) + \\ &+ (T_1 + T_2) (s^3 + \tilde{p}_1 s^2) + K \tau (\tilde{q}_2 s^3 + \tilde{q}_1 s^2 + \tilde{q}_0 s) + \\ &+ K (\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0) + (s^2 + \tilde{p}_1 s) \end{aligned} \quad (27)$$

The value sets of this family with multilinear uncertainty structure, plotted via the Polynomial Toolbox for Matlab (Polyx; Šebek et al. 2000) for several non-negative frequencies, are depicted in fig. 8. Unluckily, the zero exclusion condition indicates that polynomial (27) and thus also the whole control system is not robustly stable for assumed range of uncertain parameters, because the value sets include the zero point. The boundaries in (6) are too broad (the requirements are too strong). Margins have to be narrowed to gain the closed loop robustly stable with the controller (26). For details about this very universal and effective technique for graphical testing of robust stability and related topics see e.g. (Barmish 1994; Bhattacharyya et al. 1995). However, the real system has been stable in used working point (with non-minimum phase behaviour), as can be seen in fig. 9.

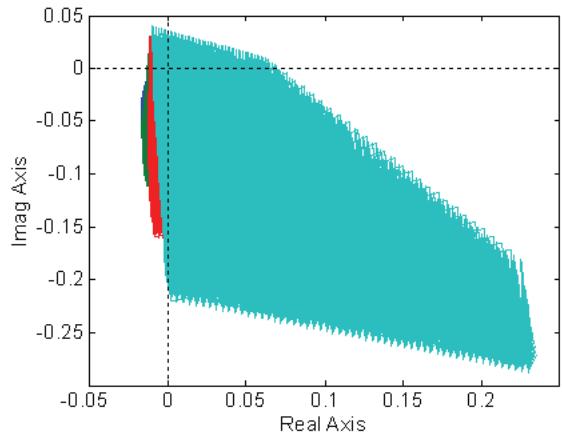


Figure 8: Value Sets for (27) with Parameters (26)

If 2DOF structure and $m = 0.02$ is used, the final controller arises in the form:

$$\begin{aligned} C_b(s) &= \frac{-1.3701s^2 + 0.01727s + 0.000432}{s^2 + 0.01297s} \\ C_f(s) &= \frac{1.08s^2 + 0.0432s + 0.000432}{s^2 + 0.01297s} \end{aligned} \quad (28)$$

The feedforward part does not influence robust stability, i.e. this controller would ensure it under similar conditions as in the previous case. The fig. 10 presents the control results.

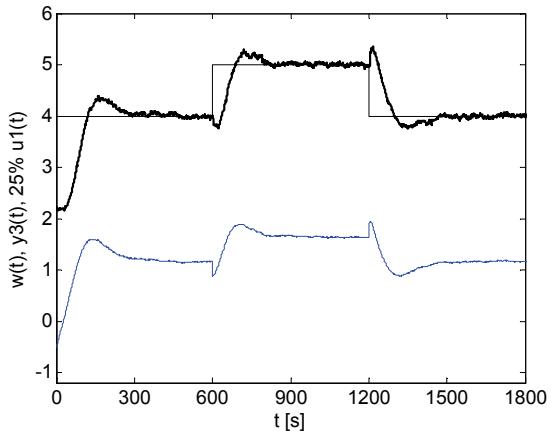


Figure 9: Control of Bulb Temperature by (26)

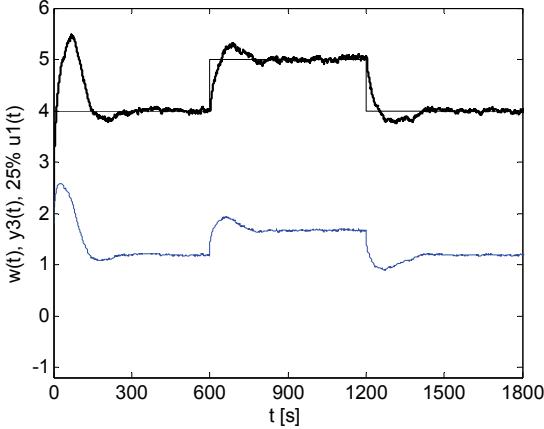


Figure 10: Control of Bulb Temperature by (28)

Another possibility of simplifying (instead of approximation by first order model) can be done via additional order reduction in identified second order system. The order reductions first only in numerator and afterward both in numerator and denominator lead to nominal transfer functions, respectively:

$$G_N(s) = \frac{0.00037}{s^2 + 0.117s + 0.00074} = \frac{0.5}{(9s+1)(150s+1)} \approx \frac{0.5(100s+1)}{(9s+1)(150s+1)} \quad (29)$$

$$G_N(s) = \frac{0.003145}{s + 0.006289} = \frac{0.5}{159s+1} \approx \frac{0.5(100s+1)}{(9s+1)(150s+1)} \quad (30)$$

The former approximation (29) and $m = 0.04$ result in:

$$C_b(s) = \frac{10.4933s^2 + 0.6068s + 0.006912}{s^2 + 0.04222s} \quad (31)$$

while the latter one (30) and $m = 0.0204$ (6 % first overshoot for nominal system) lead to the controller:

$$C_b(s) = \frac{10.9733s + 0.1323}{s} \quad (32)$$

The plant (6) and the controller (31) give, again, the closed-loop characteristic polynomial with structure (27).

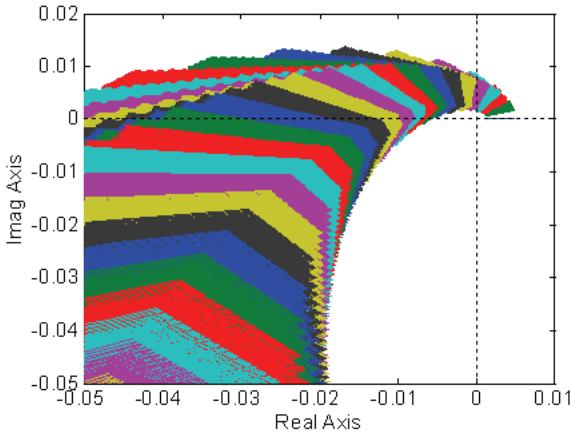


Figure 11: Value Sets for (27) with Parameters (31)

Nevertheless, in this instance, the closed-loop polynomial (27) is robustly stable which results from the fact that the origin of the complex plane is not included in the value sets (see Fig. 11) and (27) has a stable member (Barmish 1994). Thus the whole control system is robustly stable. Furthermore, the controlled system (6) and the regulator (32) yield the polynomial:

$$p(s, K, \tau, T_1, T_2) = T_1 T_2 s^3 + (T_1 + T_2) s^2 + K \tau (\tilde{q}_1 s^2 + \tilde{q}_0 s) + K (\tilde{q}_1 s + \tilde{q}_0) + s \quad (33)$$

which is also robustly stable as follows from Fig. 12. The figs. 13 and 14 show the control responses for both cases.

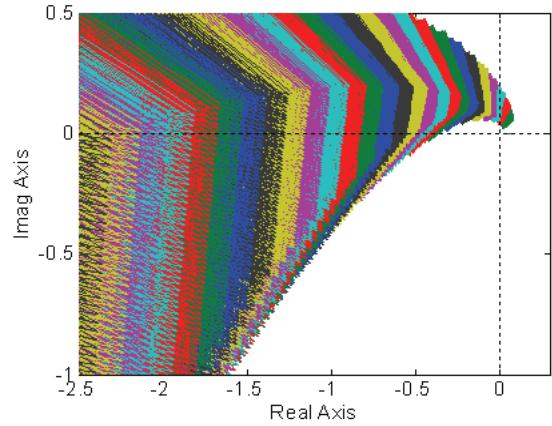


Figure 12: Value Sets for (33) with Parameters (32)

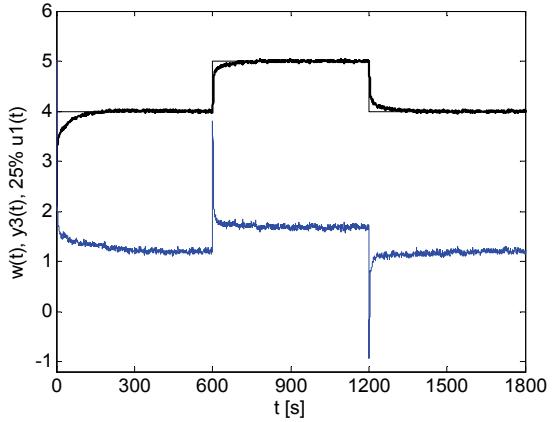


Figure 13: Control of Bulb Temperature by (31)

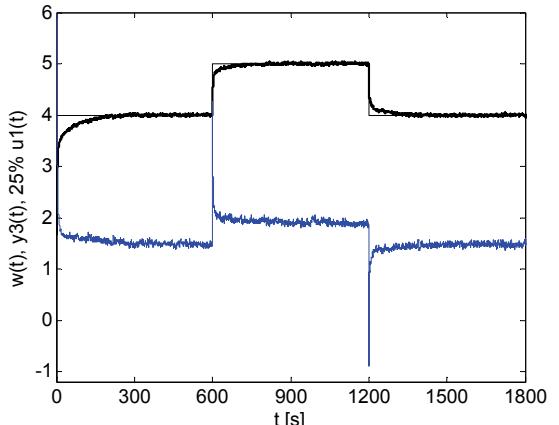


Figure 14: Control of Bulb Temperature by (32)

EVALUATION AND CONCLUSION

The objective evaluation of quality has been performed by meaning of Integrated Squared Error (ISE) criterion. The quantification is expressed in table 4.

Table 4: Outcomes of ISE Calculations

Controller	ISE
(23)	19.2579
(26)	370.7898
(28)	166.0868
(31)	18.8931
(32)	28.4738

The controllers (31) and (23) achieve the best ISE results. However, the regulator (23) generates “less aggressive” actuating signal after step-changes of reference. On the contrary, controller (26) is the worst, moreover with non-minimum phase behaviour. Not an application of 2DOF structure (28) brings about considerable improvement. Problems in control, which paradoxically emerge during use of identified second order model (6) and nominal system (25) are caused by synthesis limitation using single tuning parameter. It arises here as the cost for tuning simplicity.

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