ANALYTICAL MODEL OF EDDY CURRENT-BASED DEVICE MEANT TO THE CONDUCTIVE MATERIALS SORTING

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ABSTRACT

The developed analytical model refers to an electromagnetic device intended for sorting conductive materials, as in the waste management domain. The main component of the device is a probe coil with ferrite open core that has to be placed near the piece of conductive material subject to sorting. When the probe coil is supplied by a mean frequency voltage source, eddy currents are induced into the tested conductive media, the measured impedance depends on its physical parameters. The influence of the physical parameters of the coil probe and of the relative position of the probe coil and material are also analyzed. Based on our new analytical model of the system, many numerical results corresponding to a wide range of working parameters have been investigated, in order to identify most proper possible implementations in engineering practice.

INTRODUCTION

Technical applications of eddy currents are based on revealed changes in the physical properties of materials, e.g. electrical conductivity and magnetic permeability, properties that belong to the object to be controlled. The conductive material is subjected to the alternative magnetic field of a probe coil supplied by mean frequency AC current. The alternative magnetic field induces eddy currents into the conductive material, currents whose magnetic field opposes the inducting magnetic field of the probe coil (Rothwell and Cloud 2001). The resulting magnetic field of the coil depends on the frequency, the material properties, the material's structure or its integrity. From these facts, the eddy currents applications may be applied to measurement of physical properties, material parameters (conductivity, hardness etc.), detection and determination of surface flaws, measurement of covering layers, control of corrosion effects or sorting conductive materials. Eddy currents applications for sorting materials are developed since the management of electrotechnical and electronic waste has become an intense concern for companies in the domain. Therefore, an analytical model of the conductive piece-mean frequency supplied coil ensemble has been developed. The coil impedance modifications due to the induced eddy currents from the conductive material depend on the electrical conductivity and the magnetic permeability of the conductive material, on the frequency domain, on the position of the material relative to the coil and also on the value of the working airgap. Marking the current variations through the probe coil allows separating the metallic conductive materials or separating the ferrous and non-ferrous materials for managing electrotechnical and electronic waste. The analytical model based on the equivalent lumped circuit diagram The probe coil impedance modifications due to the induced eddy currents from the conductive material depend on the electrical conductivity and the magnetic permeability of the conductive material, on the frequency domain, on the position of the material relative to the coil and also on the value of the working airgap. Marking the current variations through the probe coil allows separating the metallic conductive materials or separating the ferrous and non-ferrous materials for managing electrotechnical and electronic waste.

PROBE COIL MODELING

Our study is focused on the particular case of a probe-inductor (fig. 1) manufactured as a coil – 2 wrapped on a half pot-core – 1 of magnetically-soft ferrite. An unavoidable airgap (δ) exists between the core and the tested metallic piece – 3. The magnetic field produced by the current flowing the inductor crosses the low-reluctance domain – 1 and the tested piece – 3 (as the shortest path outside the core), assuming, for simplicity, the leakage magnetic flux as negligible; two field lines are shown in fig. 1. The domain of tested piece crossed by the magnetic field can be approximated by the cylindrical crown of radiuses r_1, r_2, with the field lines oriented radially.

In order to find an equivalent impedance of the inductor, a proper model of the system is required. Assuming that the ferromagnetic core remains unsaturated, it can be modeled as an electric linear resistance, as well as the airgap. Thus, due to the values of the electric resistivities, the eddy currents flowing the ferrite core (it has the electric resistivity up to $10^5 \Omega\text{m}$) are negligible compared to those in the tested piece ($\rho = 10^{-8} \div 10^{-6} \Omega\text{m}$ usually).

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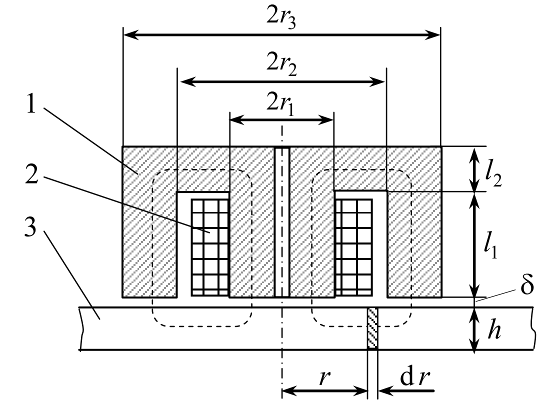

Figure 1: Probe-coil with pot core

Let us consider an infinitesimal slice of the tested piece, as a cylindrical sector of radius r, angle $\mathrm{d}\alpha$ and thickness h (fig. 2a). Its lateral surface can be treated as an infinitesimal rectangle of width $r\,\mathrm{d}\alpha$ (fig. 2b). Such a cross section of the tested piece (fig. 2b) is crossed perpendicularly by an elementary magnetic flux

$$\mathrm{d}\varphi = \varphi\cdot\frac{\mathrm{d}\alpha}{2\pi}, \tag{1}$$

where φ is the total flux produced by the coil of N turns flowed by a current i (both quantities are instantaneous values). According to the Faraday's law, the time-domain evolution of the elementary flux enforces an electric field (see a field line as dashed line in fig. 2b).

It is imperatively to observe that the electric field intensities of two such neighbor cross-sections are canceled along the common border (they are opposed equal vectors). Therefore, the resulting electric field has no component along the thickness, the field lines being circles of radius r, $r\in[r_1, r_2]$.

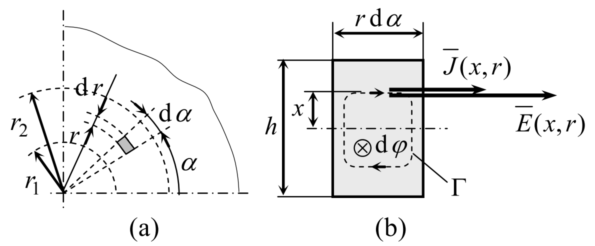

Figure 2: Helpful figure.

The Faraday's law expressed for the path Γ shown in fig. 2b is

$$\oint_{\Gamma}\boldsymbol{E}\,\mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}}{\mathrm{d}t}\left(2x\cdot\frac{\mathrm{d}\varphi}{h}\right), \tag{2}$$

Where $\mathrm{d}\boldsymbol{l}$ is the length element associated to the path Γ? Because the electric field has horizontal components only (as in fig. 2.b), replacing eq. (1) in (2), one obtains:

$$2E(x,r)\cdot r\,\mathrm{d}\alpha = -2\frac{x}{h}\cdot\frac{\mathrm{d}}{\mathrm{d}t}\left(\varphi\frac{\mathrm{d}\alpha}{2\pi}\right)\cdot \tag{3}$$

The absolute value of the electric field is obtained as function of x and r:

$$E(x,r) = \frac{x}{2\pi rh}\cdot\frac{\mathrm{d}\varphi}{\mathrm{d}t};\quad \forall\ 0\le x\le\frac{h}{2} \tag{4}$$

The Ohm's law explains that the electric field enforces (eddy) currents along the same paths as the electric field lines, the current density being

$$J(x,r) = \sigma\cdot E(x,r) = \frac{1}{\rho}\cdot E(x,r) \tag{5}$$

According to the Joule's law, the eddy currents involve the instantaneous power loss in the corresponding domain:

$$p(t) = \int_{Volume}\rho J^2(x,r)\,\mathrm{d}v, \tag{6}$$

where de volume element $\mathrm{d}v$ is chosen conveniently, as the cylindrical crown of radiuses of r and $r+\mathrm{d}r$ respectively and thickness $\mathrm{d}x$:

$$\mathrm{d}v = 2\pi r\,\mathrm{d}x\,\mathrm{d}r \tag{7}$$

The expression (6), computed within the domain crossed by the magnetic field in the tested piece, gives us

$$p(t) = \int_{-\frac{h}{2}}^{\frac{h}{2}}\int_{r_1}^{r_2}\frac{x^2}{4\pi^2r^2h^2\rho}\cdot\left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)\mathrm{d}x\,\mathrm{d}r \tag{8}$$

or

$$p(t) = \frac{h}{24\pi\rho}\cdot\left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)^2\ln\frac{r_2}{r_1}\cdot \tag{9}$$

The total (or equivalent) eddy current can be computed similarly:

$$i_e(t) = \int_{0}^{\frac{h}{2}}\int_{r_1}^{r_2}J(x,r)\,\mathrm{d}x\,\mathrm{d}r = \frac{h\ln\frac{r_2}{r_1}}{16\pi\rho}\cdot\left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right) = K\left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right), \tag{10}$$

where the constant value K depends only on the geometry of the system and the electric resistivity of the tested piece:

$$K = \frac{h}{16\pi\rho}\ln\frac{r_2}{r_1}. \tag{11}$$

The equivalent eddy current corresponds to an equivalent eddy current-loss resistance:

$$R_e = \frac{p(t)}{i_e^2(t)}. \tag{12}$$

Replacing eq. (9) and (10) in (12), one obtains:

$$R_e = \frac{32\pi\rho}{3h\ln\frac{r_2}{r_1}} \tag{13}$$

or

$$R_e = \frac{2}{3K}. \tag{14}$$

If the eddy currents are ignored, the magnetic reluctance of the cylindrical sector of the tested piece, assumed as linear, homogenous and isotropic medium of absolute permeability μ is:

$$R_m = \frac{1}{\mu}\int_{r_1}^{r_2}\frac{\mathrm{d}r}{2\pi rh} = \frac{1}{2\pi\mu h}\cdot\ln\frac{r_2}{r_1}. \tag{15}$$

It can be modeled as an electric linear resistance, numerically equal to R_m. The presence of eddy currents imposes completing this simple model with additional circuit elements, as it was explained in (Mandache and Topan 2009). The model, adapted for harmonic behavior, is shown in fig. 3, surrounded by the dashed line. The element parameters used in the diagram are given by the expressions: (15) – R_m, (11) – K and (14) – R_e. Obviously, since the drop voltage at the terminals AB is numerically equal to the magnetic force, the current is numerically equal to the flux.

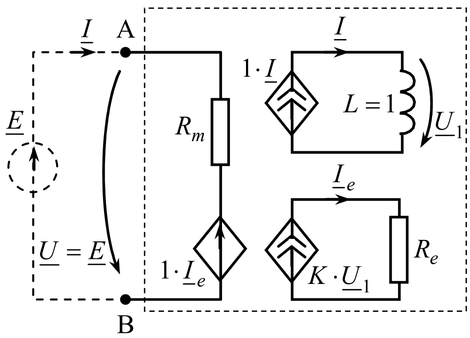

Figure 3: Model of the tested piece crossed by magnetic field.

In order to compute de complex impedance relative to the terminals A-B, the circuit is supplied by an arbitrary chosen independent voltage source $\underline{E}$ and the corresponding current $\underline{I}$ is computed using circuit analysis methods (Wilson and Riedel 2001).
A convenient mathematical model of this circuit can be written as follows:

$$\begin{cases} R_m\underline{I} = \underline{E} - \underline{I}_e \\ \underline{I}_e = K\underline{U}_1 \\ \underline{U}_1 = j\omega\underline{I} \end{cases} \tag{16}$$

Reducing the variables $\underline{U}_1$, $\underline{I}_e$, one obtains:

$$\left(R_m + j\omega K\right)\cdot\underline{I} = \underline{E} \tag{17}$$

It results:

$$\underline{Z}_{AB} = \frac{\underline{E}}{\underline{I}} = R_m + j\omega K. \tag{18}$$

This impedance is enclosed in the model of the whole inductor, which is built as in (Mandache and Topan 2009) and adapted for our application (fig. 4). Other parameters of the diagram are: the electric resistance of the coil (R_L), the number of turns (N) and the magnetic reluctance of one airgap, assuming that both airgaps have the same reluctance (R_δ). The reluctance of the ferromagnetic core has been deliberately neglected. The terminals of the probe inductor are noted M, N and the impedance $\underline{Z}_{MN}$ will be computed. Using the arbitrary chosen independent voltage source $\underline{E}_L$ and applying circuit analysis methods (Wilson and Riedel 2001), a convenient mathematical model is built:

$$\begin{cases} R_L\underline{I}_L = \underline{E}_L - N\underline{U}_1 \\ \left(2R_\delta + \underline{Z}_{AB}\right)\cdot\underline{\Phi} = N\underline{I}_L \\ \underline{U}_1 = j\omega\underline{\Phi} \end{cases} \tag{19}$$

Reducing the variables $\underline{U}_1$ and $\underline{\Phi}$, the mathematical model (19) becomes:

$$\left(R_L + \frac{j\omega N^2}{2R_\delta + \underline{Z}_{AB}}\right)\cdot\underline{I}_L = \underline{E}_L. \tag{20}$$

Consequently, the impedance of the inductor is obtained:

$$\underline{Z}_{MN} = \frac{\underline{E}_L}{\underline{I}_L} = R_L + \frac{j\omega N^2}{2R_\delta + \underline{Z}_{AB}}. \tag{21}$$

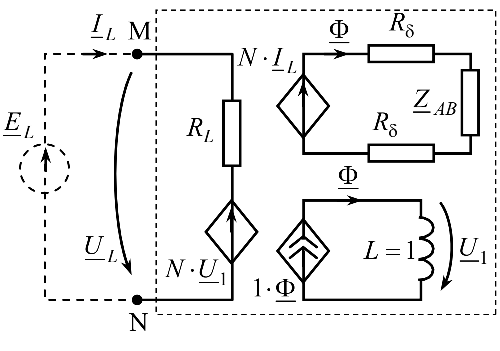

Figure 4: Model of the probe-inductor.

It could be useful to bring the expression (21) under the form

$$\underline{Z}_{MN} = R_{MN} + jX_{MN}. \tag{22}$$

where, using (18), the real and imaginary parts are:

$$R_{MN} = R_L + \frac{K N^2}{\left(\frac{2R_\delta + R_m}{\omega}\right)^2 + K^2}, \qquad (23)$$

$$X_{MN} = \frac{\frac{2R_\delta + R_m}{\omega} \cdot N^2}{\left(\frac{2R_\delta + R_m}{\omega}\right)^2 + K^2}. \qquad (24)$$

RESULTS

Using the analytical relations that describe the probe coil impedance variation, a Matlab application has been developed in order to obtain graphical and numerical results. The numerical results reveals, in case of powering the coil from a mean frequency generator, the influence of frequency, material parameters and piece-probe coil mutual position over the coil impedance and the current, respectively. The Matlab application has been applied for non-ferrous materials like copper, aluminum and their alloys (bronze aluminum and brass), as well as for ferrous metals like steel and nickel. For all these materials, the values of electrical resistivity and magnetic permeability are known. The application may be used for other materials whose parameters are known. For a 1...20kHz frequency range and a 0.1mm air gap between the material and the coil, the model results are graphically presented in figure 5 for non-ferrous materials and figure 6 for ferrous materials.

As figure 5 depicts, as frequency increases, so does the coil impedance and hence, the value of the current decreases. For frequency values lower than 1 kHz, the difference between the current values are minimal, so sorting becomes difficult. Optimal working frequency values are set around 10 kHz.

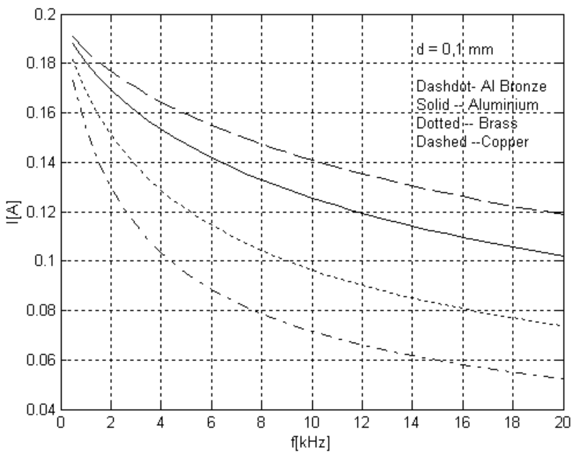

Figure 5: The current variation vs frequencies - non ferrous materials

As one can expect, for a given frequency, the highest current value is obtained for bronze aluminum, a material that has the lowest conductivity, and the lowest current value is obtained for copper, a material having the highest conductivity value.

Figure 6 present similar results, but for ferrous materials. For the same frequency value, the highest current value is obtained for nickel. For the ferrous materials the magnetic permeability has a great influence over the current value. It can be seen that 10 kHz is a good frequency value for sorting.

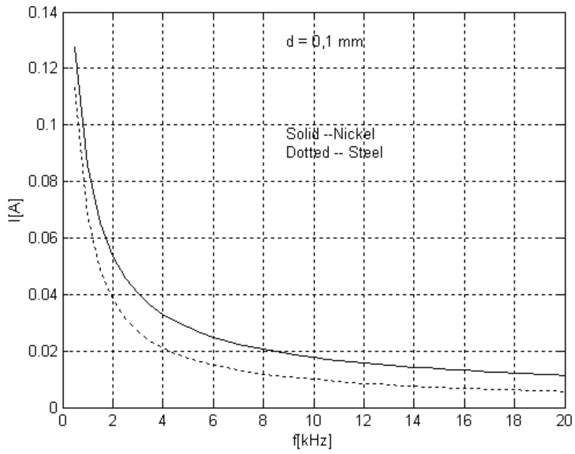

Figure 6: The current variation vs frequencies - ferrous materials

Figure 7 for non-ferrous materials and figure 8 for ferrous materials, were drawn in order to analyze the influence of airgap over the current value.

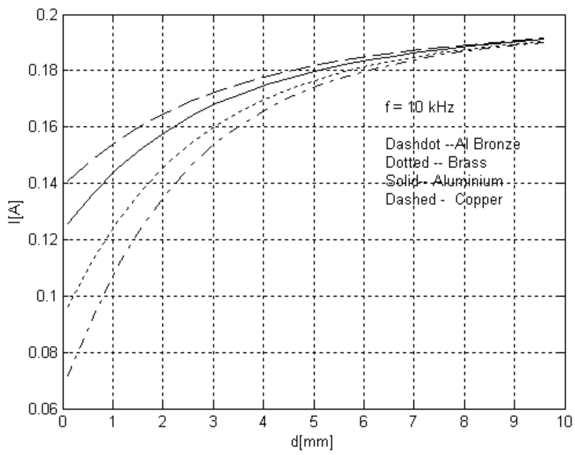

Figure 7: The current variation vs airgap - non ferrous materials

For the same 10 kHz working frequency value, important changes in the value of the current are obtained for values of the air gap lower than 1 mm. For higher values of the air gap, differences are insignificant, sorting becomes almost impossible and current values are close to those of the current absorbed by the coil in absence of the metallic material. The same conclusions are drawn in case of ferrous pieces, as shown in figure 8.

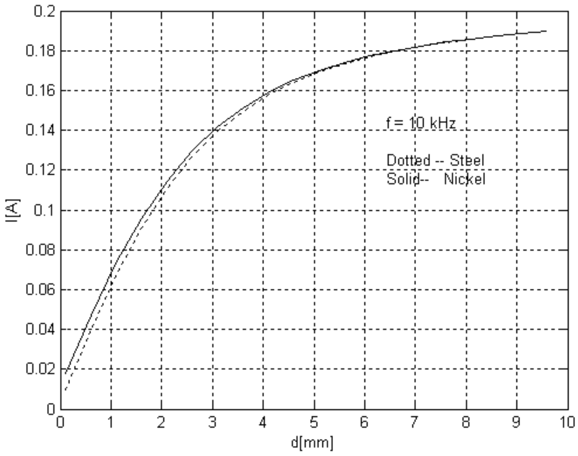

Figure 8: The current variation vs airgap - non ferrous materials

To emphasize the above mentioned conclusions, figures 9 and 10 were drawn. The graphical families of curves, for 3 frequency values (5, 10 and 15 kHz) and for 2 types of materials (copper and aluminum in figure 9 and steel and nickel in figure 10) are also presented.

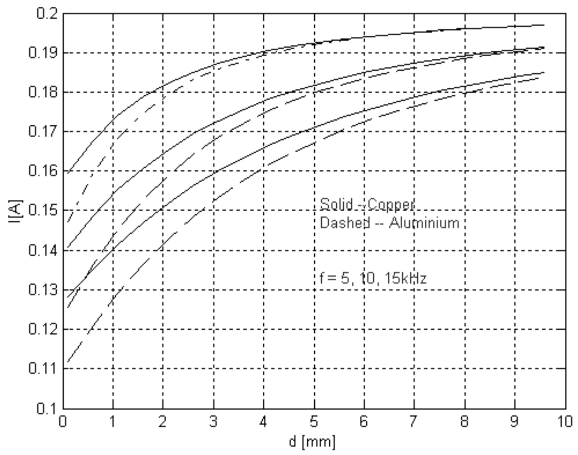

Figure 9: The current variation vs airgap – cooper and aluminum, f=5, 10, 15kHz

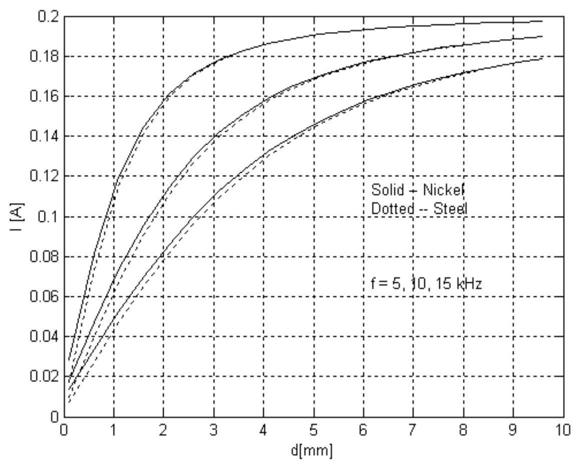

Figure 10: The current variation vs airgap – nickel and steel, f=5, 10, 15kHz

Sorting ferrous and non-ferrous materials using a device based on the presented analytical model can be obtained by comparing the values of the currents to the value of the current absorbed by the coil in absence of a metallic material. This is depicted in figure 11.

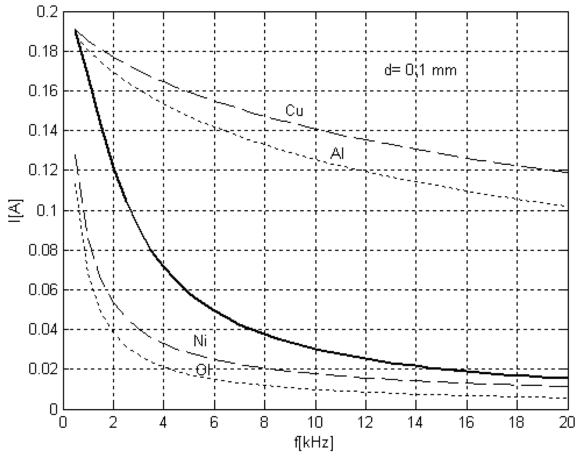

Figure 11: Comparison of the currents for non ferrous material or ferrous materials and the current of coil in absence of the metallic piece

Hence, one can observe that the value of the current absorbed by the coil when a non-ferrous material is present is higher than the values of the currents through the coil when a metallic material is absent, while the values of the current through the coil when a ferrous material is present are smaller than those obtained when a metallic material is absent.

Hence, one can observe that, as expected, that the currents values through the coil in the presence of non ferrous materials are above the current values in the absence of metallic piece, because of the higher equivalent impedance of the coil, while the currents values in presence of the ferrous materials are under the current values in the absence of metallic piece.

CONCLUSIONS

The analytical model of the probe coil-metallic piece ensamble, presented in this paper, based on eddy currents can be used for sorting conductive materials or for separating ferrous metals from non-ferrous metals, while supplying the probe coil with frequencies around 10 kHz and values of the working air gap lower than 1mm. The analytical model based on the equivalent lumped circuit diagram is quite simple and it allows the detailed study of the sorting device. The model has a good accuracy for the specified applications.

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