

DIGITAL SMITH PREDICTORS – DESIGN AND SIMULATION STUDY

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ABSTRACT

Time-delays (dead times) are found in many processes in industry. Time-delays are mainly caused by the time required to transport mass, energy or information, but they can also be caused by processing time or accumulation. The contribution is focused on a design of algorithms for digital control for processes with time-delay. The algorithms are based on the some modifications of the Smith Predictor (SP). One modification of the SP based on the digital PID controller was applied and it was compared with two designed modifications based on polynomial approach. The program system MATLAB/SIMULINK was used for simulation verification of these algorithms.

INTRODUCTION

Time-delays appear in many processes in industry and other fields, including economical and biological systems (see Normey-Rico and Camacho 2007). They are caused by some of the following phenomena:

- the time needed to transport mass, energy or information,
- the accumulation of time lags in a great numbers of low order systems connected in series,
- the required processing time for sensors, such as analyzers; controllers that need some time to implement a complicated control algorithms or process.

Consider a continuous time dynamical linear SISO (single input $u(t)$ – single output $y(t)$) system with time-delay T_d . The transfer function of a pure transportation lag is $e^{-T_d s}$ where s is complex variable. Overall transfer function with time-delay is in the form

$$G_d(s) = G(s)e^{-T_d s} \quad (1)$$

where $G(s)$ is the transfer function without time-delay.

Processes with significant time-delay are difficult to control using standard feedback controllers. When a

high performance of the control process is desired or the relative time-delay is very large, a predictive control strategy must be used. The predictive control strategy includes a model of the process in the structure of the controller. The first time-delay compensation algorithm was proposed by (Smith 1957). This control algorithm known as the Smith Predictor (SP) contained a dynamic model of the time-delay process and it can be considered as the first model predictive algorithm. Historically first modifications of time-delay algorithms were proposed for continuous-time (analogue) controllers. On the score of implementation problems, only the discrete versions are used in practice in this time. Some modifications of the digital Smith Predictors are designed and verified by simulation in this paper.

DIGITAL SMITH PREDICTORS

Although time-delay compensators appeared in the mid 1950s, their implementation with analogue technique was very difficult and these were not used in industry. Since 1980s digital time-delay compensators can be implemented. In spite of the fact that all these algorithms are implemented on digital platforms, most works analyze only the continuous case. The digital time-delay compensators are presented e.g. in (Palmor and Halevi 1990, Normey-Rico and Camacho 1998). The discrete versions of the SP and its modifications are suitable for time-delay compensation in industrial practice.

Structure of Digital SP

The block diagram of a digital SP (see Vogel and Edgar 1980, Hang et al. 1986, Hang et al. 1989, Hang et al. 1993) is shown in Fig. 1. The function of the digital version is similar to the classical analogue version. The block $G_m(z^{-1})$ represents process dynamics without the time-delay and is used to compute an open-loop prediction. The difference between the output of the process y and the model including time-delay \hat{y} is the predicted error \hat{e}_p as shown in Fig. 1 where u , w and e are the control signal, the reference signal and the error. If there are no

modelling errors or disturbances, the error between the current process output and the model output will be null and the predictor output signal \hat{y}_p will be the time-delay-free output of the process. Under these conditions, the controller $G_c(s)$ can be tuned, at least in the nominal case, as if the process had no time-delay. The primary (main) controller $G_c(z^{-1})$ can be designed by the different approaches (for example digital PID control or methods based on algebraic approach). The outward feedback-loop through the block $G_d(z^{-1})$ in Fig. 1 is used to compensate for load disturbances and modelling errors. The dash arrows indicate the tuned parts of the Smith Predictor.

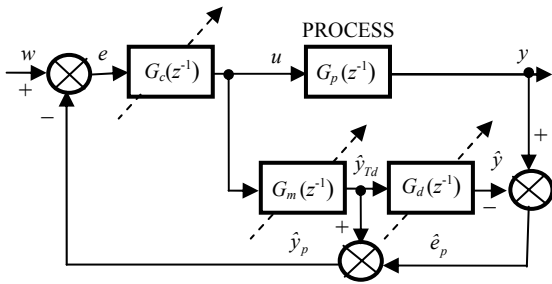


Figure 1: Block Diagram of a Digital Smith Predictor

Most industrial processes can be approximated by a reduced order model with some pure time-delay. Consider the following second order linear model with a time-delay

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} z^{-d} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-d} \quad (2)$$

for demonstration of some approaches to the design of the adaptive Smith Predictor. The term z^{-d} represents the pure discrete time-delay. The time-delay is equal to dT_0 where T_0 is the sampling period.

Identification of Time-delay

In this paper, the time-delay is assumed approximately known or possible to be obtained separately from an off-line identification using the least squares method

$$\hat{\theta} = (F^T F)^{-1} F^T y \quad (3)$$

where the matrix F has dimension $(N-n-d, 2n)$, the vector y $(N-n-d)$ and the vector of parameter model estimates $\hat{\theta}$ $(2n)$. N is the number of samples of measured input and output data, n is the model order. Equation (3) serves for a one-off calculation of the vector of parameter estimates $\hat{\theta}$ using N samples of measured data. The individual vectors and matrix in equation (3) have the form

$$F = \begin{bmatrix} -y(n+d) & -y(n+d-1) & \cdots & -y(d+1) \\ -y(n+d+1) & -y(n+d) & \cdots & -y(d+2) \\ \vdots & \vdots & \cdots & \vdots \\ -y(N-1) & -y(N-2) & \cdots & -y(N-n) \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} u(n) & u(n-1) & \cdots & u(1) \\ u(n+1) & u(n) & \cdots & u(2) \\ \vdots & \vdots & \cdots & \vdots \\ u(N-d-1) & u(N-d-2) & \cdots & u(N-d-n) \end{bmatrix} \quad (4)$$

$$y^T = [y(n+d+1) \quad y(n+d+2) \quad \cdots \quad y(N)] \quad (5)$$

$$\hat{\theta}^T = [\hat{a}_1 \quad \hat{a}_2 \quad \cdots \quad \hat{a}_n \quad \hat{b}_1 \quad \hat{b}_2 \quad \cdots \quad \hat{b}_n] \quad (6)$$

Consider that model (2) is the deterministic part of the stochastic process described by the ARX (regression) model

$$y(k) = -a_1 y(k-1) - a_2 y(k-2) + b_1 y(k-1-d) + b_2 y(k-2-d) + e_s(k) \quad (7)$$

where $e_s(k)$ is the random nonmeasurable component.

The vector of parameter model estimates is computed by solving equation (3)

$$\hat{\theta}^T(k) = [\hat{a}_1 \quad \hat{a}_2 \quad \hat{b}_1 \quad \hat{b}_2] \quad (8)$$

and is used for computation of the prediction output.

$$\hat{y}(k) = -\hat{a}_1 y(k-1) - \hat{a}_2 y(k-2) + \hat{b}_1 u(k-1-d) + \hat{b}_2 u(k-2-d) \quad (9)$$

The quality of ARX model can be judged by the error, i.e. the deviation

$$\hat{e}(k) = y(k) - \hat{y}(k) \quad (10)$$

In this paper, the error was used for suitable choice of the time-delay dT_0 . The LSM algorithm (3) – (6) is computed for several time-delays dT_0 and the suitable time-delay is chosen according to quality of identification based on the prediction error (10).

Consider the following fifth order linear system

$$G(s) = \frac{2}{(s+1)^5} = \frac{2}{s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + 1} \quad (11)$$

System (11) was identified by discrete model (2) using off-line LSM (3) – (6) for different time-delay dT_0 ; $T_0 = 0.5$ s. Criterion of identification quality is based on sum of squares of error

$$J_{\hat{e}^2} = \frac{1}{N} \sum_{k=1}^N \hat{e}^2(k) \quad (12)$$

This criterion represents accuracy of process identification. From Fig. 2, it is obvious that criterion (12) has minimum value for time-delay $d = 2$.

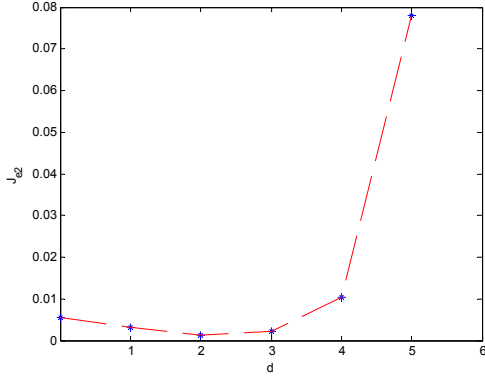


Figure 2: Criterion of Quality Identification

The suitable discrete model (2) which best approximates model (11) is in the form

$$G(z^{-1}) = \frac{0.0246z^{-1} + 0.0378z^{-2}}{1 - 1.7508z^{-1} + 0.7857z^{-2}} z^{-2} \quad (13)$$

ALGORITHMS OF DIGITAL SMITH PREDICTORS

Digital PID Smith Predictor (PIDSP)

Hang *et al.* (1989, 1993) used to design of the main controller $G_c(z^{-1})$ the Dahlin PID algorithm (Dahlin 1968). This algorithm is based on the desired close-loop transfer function in the form

$$G_e(z^{-1}) = \frac{1 - e^{-\alpha}}{1 - z^{-1}} \quad (14)$$

where $\alpha = \frac{T_0}{T_m}$ and T_m is desired time constant of the first order closed-loop response. It is not practical to set T_m to be small since it will demand a large control signal $u(k)$ which may easily exceed the saturation limit of the actuator. Then the individual parts of the controller are described by the transfer functions

$$G_c(z^{-1}) = \frac{(1 - e^{-\alpha}) \hat{A}(z^{-1})}{(1 - z^{-1}) \hat{B}(1)}; \quad G_m(z^{-1}) = \frac{z^{-1} \hat{B}(1)}{\hat{A}(z^{-1})}$$

$$G_d(z^{-1}) = \frac{z^{-d} \hat{B}(z^{-1})}{z^{-1} \hat{B}(1)} \quad (15)$$

where $B(1) = \hat{B}(z^{-1})|_{z=1} = \hat{b}_1 + \hat{b}_2$.

Since $G_m(z^{-1})$ is the second order transfer function, the main controller $G_c(z^{-1})$ becomes a digital PID controller having the following form:

$$G_c(z^{-1}) = \frac{Y(z)}{E(z)} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}} \quad (16)$$

where $q_0 = \gamma$, $q_1 = \hat{a}_1 \gamma$, $q_2 = \hat{a}_2 \gamma$ using by the substitution $\gamma = (1 - e^{-\alpha}) / \hat{B}(1)$. The PID controller output is given by

$$u(k) = q_0 e(k) + q_1 e(k-1) + q_2 e(k-2) + u(k-1) \quad (17)$$

Digital Pole Assignment Smith Predictor (PASP)

The second controller applied in this paper was designed using a polynomial approach. Polynomial control theory is based on the apparatus and methods of a linear algebra (see e.g. Kučera 1991, Kučera 1993). The polynomials are the basic tool for a description of the transfer functions. They are expressed as the finite sequence of figures – the coefficients of a polynomial. Thus, the signals are expressed as infinite sequence of figures. The controller synthesis consists in the solving of linear polynomial (Diophantine) equations. The design of the controller algorithm is based on the general block scheme of a closed-loop with two degrees of freedom (2DOF) according to Fig. 3.

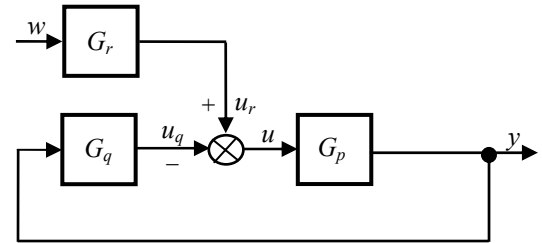


Figure 3: Block Diagram of a Closed Loop 2DOF Control System

The controlled process is given by the transfer function in the form

$$G_p(z^{-1}) = \frac{Y(z)}{U(z)} = \frac{B(z^{-1})}{A(z^{-1})} \quad (18)$$

where A and B are the second order polynomials. The controller contains the feedback part G_q and the feedforward part G_r . Then the digital controllers can be expressed in the form of a discrete transfer functions

$$G_r(z^{-1}) = \frac{R(z^{-1})}{P(z^{-1})} = \frac{r_0}{1 + p_1 z^{-1}} \quad (20)$$

$$G_q(z^{-1}) = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{(1 + p_1 z^{-1})(1 - z^{-1})} \quad (21)$$

According to the scheme presented in Fig. 3 (for $e_s = 0$), the output y can be expressed as

$$Y(z^{-1}) = \frac{G_p(z)G_r(z)}{1 + G_p(z)G_q(z)} W(z^{-1}) \quad (22)$$

Upon substituting from Equation (18) - (21) into Equation (22) it yields

$$Y(z^{-1}) = \frac{B(z^{-1})R(z^{-1})}{A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1})} W(z^{-1}) \quad (23)$$

where

$$A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1}) = D(z^{-1}) \quad (24)$$

is the characteristic polynomial.

The procedure leading to determination of polynomials Q , R and P in (18) and (19) is (Bobál et al. 2005). A feedback part of the controller is given by a solution of the polynomial Diophantine equation (24). An asymptotic tracking is provided by a feedforward part of the controller given by a solution of the polynomial Diophantine equation

$$S(z^{-1})D_w(z^{-1}) + B(z^{-1})R(z^{-1}) = D(z^{-1}) \quad (25)$$

For a step-changing reference signal value $D_w(z^{-1}) = 1 - z^{-1}$ holds and S is an auxiliary polynomial which does not enter into controller design. A feedback controller to control a second-order system without time-delay will be derived from Equation (22), where the characteristic polynomial is chosen as

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3} + d_4 z^{-4} \quad (26)$$

For a step-changing reference signal value it is possible to solve Equation (25) by substituting $z = 1$

$$R = r_0 = \frac{D(1)}{B(1)} = \frac{1 + d_1 + d_2 + d_3 + d_4}{b_1 + b_2} \quad (27)$$

The 2DOF controller output is given by

$$u(k) = r_0 w(k) - q_0 y(k) - q_1 y(k-1) - q_2 y(k-2) + (1 + p_1)u(k-1) + p_1 u(k-2) \quad (28)$$

Digital Linear Quadratic Smith Predictor (LQSP)

The linear quadratic control methods try to minimize the quadratic criterion with the penalization of the controller output:

$$J = \sum_{k=0}^{\infty} \{ [w(k) - y(k)]^2 + \lambda [u(k)]^2 \} \quad (29)$$

where λ is the so-called penalization constant which gives the rate of the controller output on the value of

the criterion (where the constant at the first element of the criterion is considered equal to one). In this paper, criterion minimization will be realized through the spectral factorization for an input-output description of the system. Spectral factorization of polynomials of the first and second order degree can be computed simply; the procedure for higher degrees must be performed iteratively. For the coefficients of the second order characteristic polynomial $D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2}$ of the closed loop are derived in (Bobál et al. 2005).

The LQ controller of 2DOF structure has the same form as controller (28), only $d_3 = d_4 = 0$ in (26) and (27).

SIMULATION VERIFICATION ADAPTIVE DIGITAL SP CONTROLLER ALGORITHMS

The above mentioned SP controllers are not suitable for the control of unstable processes. Therefore, four types of processes were chosen for simulation verification of digital adaptive SP controller algorithms.

Consider the following continuous-time transfer functions:

$$1) \text{ Stable non-oscillatory } G_1(s) = \frac{2}{(s+1)(4s+1)} e^{-4s}$$

$$2) \text{ Stable oscillatory } G_2(s) = \frac{2}{4s^2 + 2s + 1} e^{-4s}$$

3) With non-minimum

$$\text{phase } G_3(s) = \frac{-5s+1}{(s+1)(4s+1)} e^{-4s}$$

Let us now discretize them a sampling period $T_0 = 2$ s.

The discrete forms of these transfer functions are (see Equation (2))

$$G_1(z^{-1}) = \frac{0.4728z^{-1} + 0.2076z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-2}$$

$$G_2(z^{-1}) = \frac{0.6806z^{-1} + 0.4834z^{-2}}{1 - 0.7859z^{-1} + 0.3679z^{-2}} z^{-2}$$

$$G_3(z^{-1}) = \frac{-0.5489z^{-1} + 0.8897z^{-2}}{1 - 0.7419z^{-1} + 0.0821z^{-2}} z^{-2}$$

4) Stable fifth-order transfer function (11) $G_4(s)$ with discrete model (13) $G_4(z^{-1})$.

A simulation verification of proposed design was performed in MATLAB/Simulink environment. A typical control scheme used is depicted in Fig. 4.

This scheme is used for systems with time-delay of two sample steps. Individual blocks of the Simulink scheme correspond to blocks of the general control scheme presented in Fig. 1. Blocks Compensator 1 and Compensator 2 are parts of the Smith Predictor and they correspond to $G_m(z^{-1})$ and $G_d(z^{-1})$ blocks of

Fig. 1 respectively. The control algorithm is encapsulated in Main Pole Assignment Controller which corresponds to $G_c(z^{-1})$ Fig. 1 block.

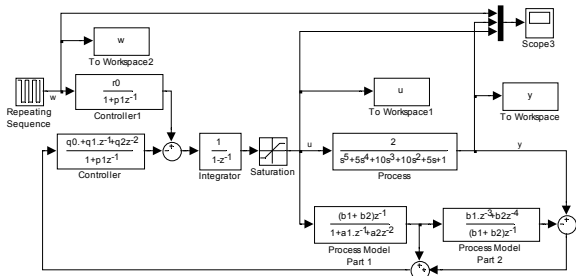


Figure 4: Simulink Control Scheme

SIMULATION RESULTS

Simulation Verification of Digital PIDSP

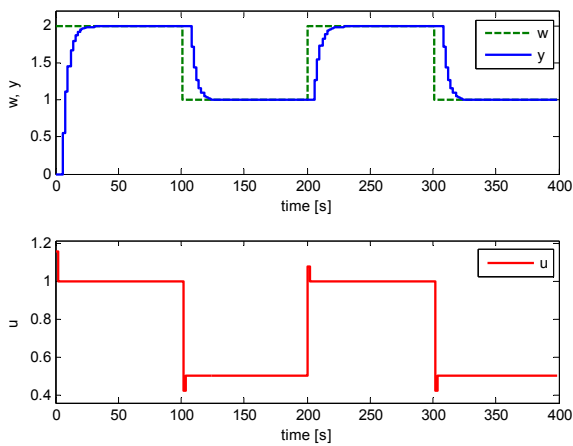


Figure 5 Control of the Model $G_1(z^{-1})$, Controller PIDSP

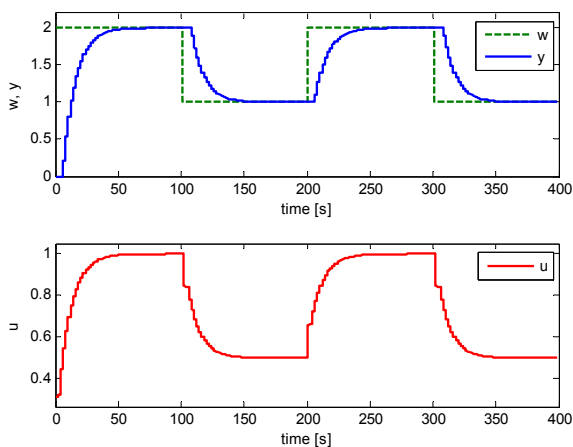


Figure 6 Control of the Model $G_2(z^{-1})$, Controller PIDSP

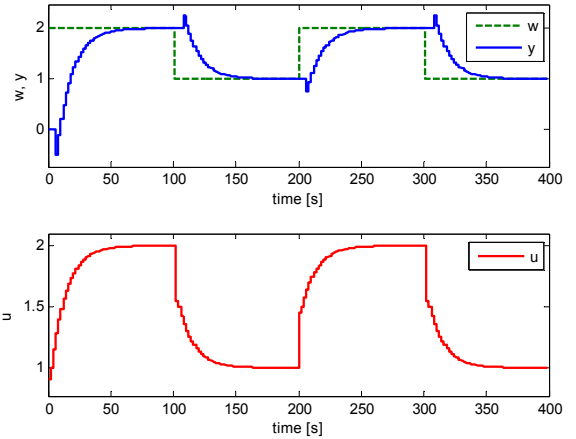


Figure: 7 Control of the Model $G_3(z^{-1})$, Controller PIDSP

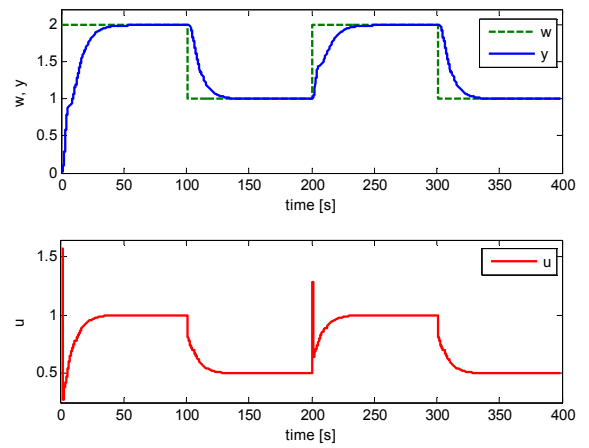


Figure 8 Control of the Model $G_4(z^{-1})$, Controller PIDSP

Simulation Verification of Digital PASP

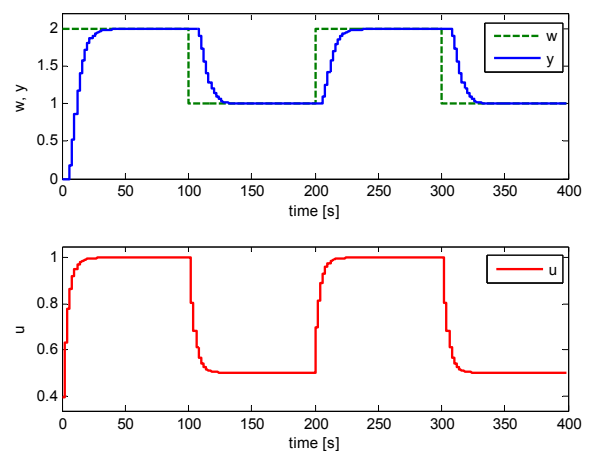


Figure: 9 Control of the Model $G_1(z^{-1})$, Controller PASP

Figs. 5 - 8 illustrate the simulation control performance using PIDSP controller (15) - (17). A suitable time constant T_m was chosen so as the process output y was without overshoot. The control quality is very good.

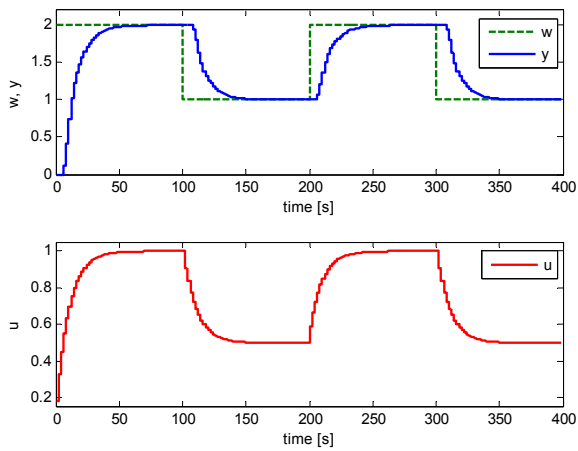


Figure: 10 Control of the Model $G_2(z^{-1})$, Controller PASP

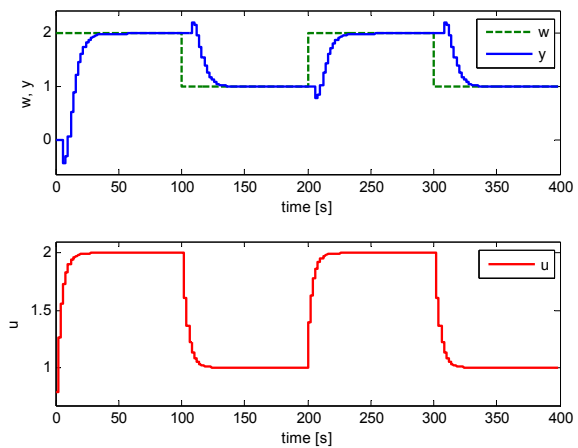


Figure: 11 Control of the Model $G_3(z^{-1})$, Controller PASP

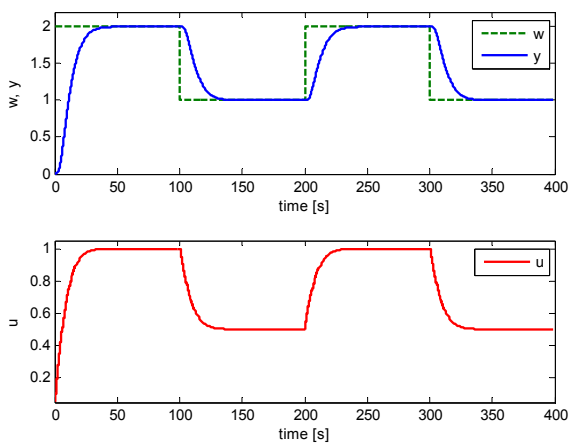


Figure: 12 Control of the Model $G_4(z^{-1})$, Controller PASP

Figs. 9 - 12 illustrate the simulation control performance using PASP controller (20), (21), (28). A suitable pole assignment was chosen on the basis of previous experiments. The control quality is very good (the process output y is without overshoot and controller output u has non-oscillatory course).

Simulation Verification of Digital LQSP

Figs. 13 and 14 illustrate the simulation control performance using LQSP controller (28). It is obvious from these Figs. that courses of the process outputs y have very good quality (without or with only small overshoot and with short settling time), however the controller outputs oscillates when the reference signal changes.

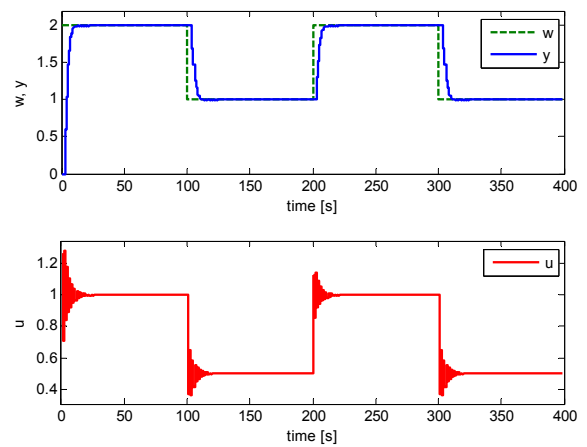


Figure: 13 Control of the Model $G_1(z^{-1})$, Controller LQSP

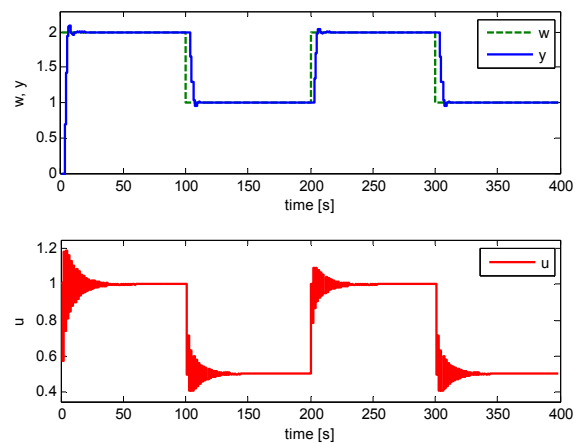


Figure: 14 Control of the Model $G_2(z^{-1})$, Controller LQSP

The control process of the model $G_1(z^{-1})$ is similar as that one using the controller PIDSP. The model $G_2(z^{-1})$ is controlled with small overshoot of y in the initial part (see Fig. 14). The control of the non-minimum phase $G_3(z^{-1})$ was unstable. When

simulating the control of the fifth-order model $G_4(z^{-1})$ the controller output u oscillates between constraints. The relative low-quality control using the controller LQSP could be caused by choosing a second degree characteristic polynomial $D(z^{-1})$ (optimal polynomial is a fourth degree with four poles). The procedure for higher degrees than two must be performed iteratively.

CONCLUSION

Digital Smith Predictor algorithms for control of processes with time-delay based on polynomial design (pole assignment and linear quadratic control) were proposed. The polynomial controllers were derived purposely by analytical way (without utilization of numerical methods) to obtain algorithms with easy implementability in industrial practice. Both pole assignment and linear quadratic control algorithms were compared by simulation with adaptive digital Smith PID Predictor. Four models of control processes were used for simulation verification (the stable non-oscillatory, the stable oscillatory, the non-minimum phase and high (fifth)-order). Results of simulation verification demonstrated advantages and disadvantages of individual approaches to control of above mentioned processes with time-delay. The proposed digital Smith Predictors will be verified in real-time laboratory conditions. Adaptive versions of digital Smith Predictors are designed in (Bobál et al., 2011).

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