CORPORATE VALUATION MODEL IN A STOCHASTIC FRAMEWORK

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KEYWORDS

Corporate valuation, Business modeling, Financial simulation

ABSTRACT[1]

In this paper we present a corporate valuation model, which integrates the different approaches of various fields of finance in a single stochastic framework. We construct a stylized company, which has its explicit production, inventory, export and import activity. In our simulation we analyze the affect of the future volatility of three stochastic factors (production, inflation and exchange rate) and their correlations on the distribution of the current value of the firm. Furthermore we investigate the added value of the possibility of early termination of the project.

MOTIVATION

In corporate finance the immanent uncertainty of financial processes is treated generally by taking the expected value of a random variable – the elements of the future cash-flow – and the risk of the realization appears in the increased discount factor.[2]. As a consequence of this solution, on one hand the cash-flow of a firm is predicted on the base of the expected value of its projects and its financing, on the other hand the stock price movement is described with a distribution, pretending as if the risk of the equity was generated by the capital markets (the trading) only, and not even partly by the corporate operation itself. Moreover the classical present value calculation neglects the stochastics of the applied financial processes like inflation, money market returns and currency rates.

In the following analysis we present a corporate valuation model, which uses the distribution instead of the expected value of the stochastic processes. In our calculations the values of the stochastic factors are generated as a realization of correlated Ito processes. So the input parameters of the valuation model come from the random walk model used in derivative asset pricing in the financial markets.

Nevertheless this Monte Carlo simulation is not an alternative of the sensitivity, or scenario analysis. In case of sensitivity analysis our aim is to identify the factors the changes of which have a critical effect on the results (being either a distribution or an expected value), and which because of this need to be forecasted very carefully. The scenario analysis however investigates some special joint outcomes of the random variables. This kind of analysis can be carried out in our framework as well.

The structure of the paper is as follows. The next session introduces the theoretical background and then we present the assumptions of our model. The second part of the paper contains the results of the simulations: first we analyze the effect of the volatility of one stochastic factor on the distribution of the current value of the firm, then we model the corporate value taking all 3 factors volatile and finally the real option of terminating the project early is valued.

THEORETICAL BACKGROUND

Financial modeling uses several models to describe the time depending stochastic variables. The classic model assumes that the change in a stochastic parameter (market price) is independent of its past price movements (Markov property, due to market efficiency). The change in the price during a time of *dt* adds up from a deterministic and a stochastic part, where the random part is normally distributed. If this change is continuous both in time and value, the process is called Ito-process:

$$dS_t = x(S,t)dt + \sigma(S,t)dw_t \qquad (1)$$

Where dS_t is the change of process S during a period of dt, the length of which approaches to zero and x and σ are variables depending on S and the time (t); dw stands for the change of a Wiener process[3] during the same period, that is a normally distributed random variable with a mean of zero and variance of t.

In case of market traded assets, like foreign exchange rates, normality is a consequence of the market

1 The paper was supported by the European Union and co-financed by the European Social Fund in the framework of TÁMOP-4.2.2/B-10/1-2010-0023 project.

2 However the Gaussian distribution emerges when modeling the stock returns, and consequently the lognormality of stock prices is accepted, the corporate finance deals with constant figures.

3 See details in Hull (2009) and Medvegyev, Száz (2010).

Proceedings 26th European Conference on Modelling and Simulation ©ECMS Klaus G. Troitzsch, Michael Möhring, Ulf Lotzmann (Editors)
ISBN: 978-0-9564944-4-3 / ISBN: 978-0-9564944-5-0 (CD)

efficiency, namely that only a per definition random new information can affect the actual price.
We agreed to use the same process (with a slight modification) for the process of inflation and production. In the simulation the price can change only in discrete time, that's why we used the discrete version of equation (1):

$$\Delta S_t = x(S,t)\Delta t + \sigma(S,t)\Delta w_t \qquad (2)$$

This one-letter difference (the change in time is considered not in limit) has severe mathematical consequences, which exceeds the content of our paper. The most important consequence of the dicretization of the continuous model is the fact, that the risk cannot be perfectly eliminated – except for the binomial approach. In all other discrete cases (trinomial model, etc) the market is incomplete (Medvegyev, Száz 2007).
The random figures used for the simulation are calculated by weighting independent normal variables, where the weights are generated from the Cholesky factorization of the correlation matrix. The method is detailed among others in Bau and Trefethen (1997).

THE MODEL

The model we used for our calculations is not very much different to the standard present value calculus. In order to be able to analyze more complex problems in the future we built an MS Excel model that is more detailed and its parameters can be set more flexibly. So beside balance sheets and income statements it also contains a business cash flow and a discounted cash flow based business valuation model.
The explicit period is 20 years long, in the terminal value the invested capital can not generate a return in excess of the required level, so the value of the firm equals the invested capital. The firm produces only one type of product that is sold both locally and for export.
The invested assets and working capital needed for the production are invested at time zero. The useful lifetime of the invested assets is 10 years, the firm uses a linear depreciation and amortization model. In year 10 we need to replace the invested assets and the model extends until this replacement is completely used up again. In this paper – as some financial theories do - we assume, that financing is always available for good projects, so our model-company is financed exclusively from equity, we disregard the question of capital structure.
Both selling and purchasing prices and the cost of replacement for invested assets are affected by the inflation rate of the corresponding currency. Yet there is no difference made between consumer and industrial price index.
Among the expenses of the firm we have both fix (independent of the quantity produced) and variable items. The turnover days of inventory, receivables and payables can be set year-by-year individually, though in the models presented here those are all set to 30 days meaning those depend only upon the sales.
Our model calculates the tax due always based on the given years earnings so no deferred tax exists. We also took care of money not needed for the operation. If the extra money can not be redrawn as dividend because the maximum of that is the annual after tax earning, owners decrease capital (buy back shares).
Based on the cash flow of the 20 years predicted we determine the value of the firm (project) for the start of each year. For that we apply the APV method where the sum of the free cash flow (FCFF) and the tax shield (TS) generated by the annual interest payments on debt is to be discounted by the operative cost of capital (r_A) With a formula:

$$V_0 = \sum_{i=1}^{20} \frac{FCFF_i + TS_i}{\prod_{j=1}^{i}(1+r_{Aj})} + \frac{IC_{TV}}{\prod_{j=1}^{20}(1+r_{Aj})} \qquad (3)$$

where V stands for the value of the firm. As terminal value we used the invested capital value for the end of the 20^{th} year in line with our assumption that after that year achieved and required return will equal. (For a detailed description of this model see Koller, Goedhart and Wessels (2010).)

In our model three stochastic processes appear. Probability variable describes for each of the 20 years the demand (and so the produced quantity) of the good (Q), the local inflation (Π) and the foreign exchange rate (S). The variables in contrast to the classic Monte Carlo simulation are not independent from each other but rather the correlation between them can be set by parameters.
The expected amount of annual quantity sold is given. The source of uncertainty here is a random variable following a normal distribution with an expected value of zero and a set standard deviation. This variable gives the difference in percentages between the actual and the expected annual quantity. The risk originating from this is somewhat limited: because of long term contracts the realized value may not be lower than *b* percent (60%) of the expected and due to the maximum capacity of the machines it may not exceed *a* percent (140%). (The expected value is 1 million pieces.)

$$Q_t = \max(\min(1+\sigma_q \Delta w_q ; a); b) \qquad (4)$$

For the focus of our investigation we picked the uncertainty emerging associated with the local inflation whilst the change in the foreign price level is dealt with as an exogenous, well predictable variable. The changes of the local interest rates follow exactly the pattern of the local inflation (in other words the local real interest rate is constant) while the exchange rate (starting form 1 euro = 300 HUF) is linked to the inflation differences only as a trend with some volatility around that. We assume the foreign inflation to be constant and modest (2 percentage) compared to the local level.

$$\Delta\Pi_t = \sigma_I \Delta w_I \quad (5)$$

So while in case of the inflation and the production the expected value has an unchanged mean though the full time horizon of the model in case of the exchange rate we have a non-constant mean. This is because the mean of the later for a given year is the theoretical rate calculated from the previous year's rate based on the inflation differences.

$$\Delta S_t = (\Pi_0^{domestic} - \Pi_0^{foreign})S_t \Delta t + \sigma_S S_t \Delta w_S \quad (6)$$

The correlation matrix of the random parts of the stochastic processes (Δw_i) is predetermined and constant in time.

First our model calculates five thousand realizations for each combinations of starting parameters and then determines the base statistics for the distribution. We recorded not only the net present value of the project (that is the difference between the intrinsic value and the invested capital) but also the value of the firm and the present value of tax paid to the state and the probability of bankruptcy.

The starting point of our calculus is very similar to a formal paper of Száz [2007] on the Ho-Lee model. That model examined the long term equilibrium ratios using a stochastic interest rate. In our current model the stochastic inflation determines the change in interest rates and it explicitly shows at the changing risk involved in the exchange rates.

ANALYSIS OF THE DISTRIBUTION OF CORPORATE VALUE

First, we calculated the corporate value by switching all risks off by setting all standard deviations to zero. In this case there is no uncertainty, each and every parameter can be foreseen. In our theoretical example the value of the company is 1.497,62 million HUF, and it requires 1.360 million HUF invested capital. Therefore entering into that business has 137,62 million HUF net present value (Market Value Added), meaning the project is worth to finance.

We applied the same real operative (unlevered) cost of capital (r_A=12%) in every scenario, which is a constraint of our analysis, as the cost of capital may vary according to the fact whether the sales revenue derives from export or domestic market, or whether it is ensured by fixed quantity sold or it is random.

On the other hand the cost of capital is a function of numerous other factors (like macroeconomic environment, industry or labor force needs), so the systematic risk is considered identical in all cases. We assume that the volatility of the analyzed factors stem from individual and diversifiable risk, and consequently do not enhance the risk premium of the project.

Effect of the volatility of production

The increasing volatility of the production – corresponding to the relevant theory – increases the extent of the distribution, and so the probability of negative outcomes, but it reduces the expected value as well. Figure 1 depicts the probability distributions of the net corporate value in time 0, assuming different volatility of the annually produced quantity. Table 1 summarizes the statistics of the distributions.

Figure 1: Distribution of corporate value as a function of production volatility (HUF million)

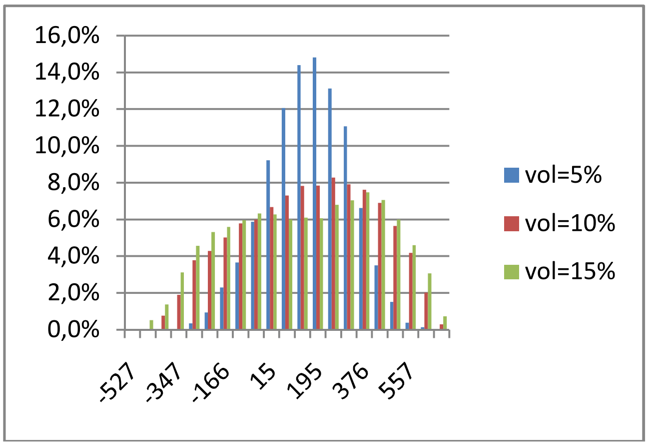

Table 1: Statistics of the distribution

HUF million	Volatility		
	5%	**10%**	**15%**
Mean	134	125	110
Standard deviation	157	254	282
St. Dev/Mean	117%	203%	255%
Maximum	598	659	677
Minimum	-439	-464	-527
Median	140	140	122
Quartile 1	28	-69	-120
Quartile 2	140	140	122
Quartile 3	247	330	348
Probability of default	20%	32%	37%

A slight volatility (5%) of the production results a negative net present value in 20% of the cases, meaning that the generated cash flow is not enough to cover the required cost of capital. In the worst case (1 realization out of 5000) the discounted value of the cash flow during the 20 years is only 68% of the invested capital. The increasing volatility causes trivially higher upside and downside deviations.

As the produced quantity can fluctuate in a +/-40% range around the mean, in the theoretical worst case the production is the fixed minimum (600 thousand pieces annually). In that case the firm value is -667 million HUF. The term "probability of default" refers not to a real default in our analysis, but the probability that the return fails to exceed the cost of capital, as we assume that financing is always available, the project is to be continued in case of negative cash flow as well.

Exchange rate volatility

The other source of the dispersion of the corporate profit and so the corporate value derives from payments in foreign currency. In the example shown in this paper only the sales revenue (export) depends on foreign exchange rate, but of course any other items of the cash flow could contain similar risk. The sensitivity toward currency risk is determined by the net position in foreign currencies that can be reduced or even perfectly eliminated by natural hedge (matching foreign currency position of incoming and outgoing items: e.g. export revenue and foreign exchange loan) or financial derivatives.

According to our assumptions the expected change of the foreign exchange rate equals the difference of the inflation – the uncovered interest rate parity is held –, and the sales price follows the change of the price level, therefore if the exchange rate volatility is 0 percent, the denomination of the sales revenue has no affect on the results. Table 2 shows the statistics of the distribution of the corporate value by 50% and 100% export ratio (denominated and settled in euro) and different EUR/HUF exchange rate volatility. The expected value is the same almost in all cases, except for the scenario of full export and 15% currency rate volatility. Here we can see a drastic drop (by 33%) of the mean of the corporate value, which is to be explained by the fact that in contrast to the process of the production, we do not apply any constraint on the exchange rate fluctuation, so the appearance of extreme values has impressive effect. An exchange rate volatility of 15% – without any currency hedge – causes for an exporter company the risk of failing the project in half of the cases, and almost 150% of the invested capital can be lost (here we neglect the possibility of getting out of the project, so the owners meet their payment obligation even if the net present value of the project at the end of the year 10 is negative). It is remarkable, that the drift of the EUR/HUF exchange rate during the first decade of the 2000s, was not only less than the interest rate difference, but it was in most of the years negative, resulting a forint-appreciation.

Table 2: Corporate value in case of FX-risk

Export ratio	50%			100%		
HUF million	Volatility			Volatility		
	5%	10%	15%	5%	10%	15%
Mean	136	137	133	133	133	91
Standard deviation	125	252	386	251	526	838
St. Dev/Mean	92%	185%	289%	188%	396%	916%
Maximum	660	1 245	2 482	1 230	2 484	5 470
Minimum	-244	-600	-754	-812	-1 598	-2 052
Median	131	114	74	121	100	-1
Quartile 1	49	-43	-136	-36	-220	-487
Quartile 2	131	114	74	121	100	-1
Quartile 3	219	285	346	293	456	570
Probability of default	14%	31%	40%	30%	43%	50%

Stochastic inflation

In our model exclusively the domestic inflation is stochastic, we handle the foreign inflation to be constant. The price level affects both the revenue and the cost side, so the change of the inflation has a balanced influence on the profitability of the company. The inflation takes effect on the corporate value principally through the financing costs, as it is included in the nominal cost of capital. At a moderate inflation level (3%), volatility of inflation can be neglected, as even an extreme (50%) volatility causes just a modest range of corporate value fluctuation (table 3).

Volatility of inflation can cause negative net present value if the inflation exceeds 10%, with a larger volatility of 50%. In this extreme case the probability of default is 5%, but it is due to the higher cost of capital, and accordingly low expected value, the affect of the volatility is less significant.

Table 3: Affect of the price level and volatility of inflation on corporate value

Inflation	**3%**				**10%**			
HUF million	**Volatility**				**Volatility**			
	5%	**10%**	**15%**	**50%**	**5%**	**10%**	**15%**	**50%**
Mean	138	138	138	137	18	18	19	22
Standard deviation	1	1	2	7	1	3	4	14
St. Dev/Mean	0%	1%	2%	5%	8%	15%	22%	64%
Maximum	140	143	145	164	23	28	34	78
Minimum	135	132	128	105	13	7	4	-21
Median	138	138	138	137	18	18	18	21
Quartile 1	137	137	136	133	17	16	16	12
Quartile 2	138	138	138	137	18	18	18	21
Quartile 3	138	139	139	142	19	20	21	31
Probability of default	0%	0%	0%	0%	0%	0%	0%	5%

Three stochastic factors

Each of the simulations above present the effect of one stochastic factor on corporate value. Supposing all the three parameters to be stochastic, we have to model the joint distribution of the three risk factors. The factor volatilities used in the model are: 5% (production); 10% (inflation)[4]; 10% (exchange rate)[5].

We applied three correlation structure: independent factors (correlation coefficient: 0); perfect positive correlation – meaning an increase in the production is followed by increasing inflation and exchange rate; and a “real” correlation structure, which we intended to construct to be close to the real market circumstances. Consequently the correlation between inflation and exchange rate is strong (δ=0.8), as weakening of the forint usually causes higher inflation. We assume a weak, almost zero correlation between the production and the inflation (δ=0.1), because inflation has a two-sided effect on the produced quantity. The correlation of exchange rate and production is assumed to be positive, but not too strong (δ=0.4). Table 4 contains our results.

The higher correlation (economic predictability) enhances the corporate value. The same result was achieved by Csányi, Juhász and Megyik (1997) in their simulation of a corporate population. On the other hand the deterministic relationship excludes the possibility that the unfavorable market movements of the factors are balanced, the risks are less diversified, and so the confidence of the estimation.

Table 4: Affect of correlations of the stochastic factors

Export ratio	**50%**		
HUF million	**correlation**		
	real	**0**	**1**
Mean	145	131	165
Standard deviation	352	300	422
St. Dev/Mean	244%	229%	256%
Maximum	1 991	1 576	2 118
Minimum	-805	-767	-1 077
Median	111	105	122
Quartile 1	-98	-78	-125
Quartile 2	111	105	122
Quartile 3	342	313	412
Probability of default	37%	36%	38%

According to that the intervention of the state into the economy (for example the home currency is under pressure because of an extreme foreign debt) is value destroying. However the bias caused by the state can lead to less correlated affects of the economic factors.

As in such cases the link between different risk factors may be shifted some negative tendencies in the real economy may be counterbalanced by the weakening of the correlations. But as soon the distortions disappear (the increase of foreign debt slows) and the basic connections come again to dominate and the effect of the real economic tendencies (global and euro crisis) may influence the economy undimmed. The value of the projects would change in line with the basic tendencies end their risk will surely grow.

[4] The standard deviation of the annual consumer price indices between 2000-2011 was 2,2%, according to the data of the Hungarian Central Statistical Office.

[5] The annual EUR/HUF exchange rate volatility is 9,2% since 2000; the volatility of the last 5 years is 12,2% , based on the daily ECB fixings.

TERMINATION OF THE PRODUCTION

It is an important assumption of the above analysis, that the company will operate for 20 years after starting the project (entering into a contract about the annual delivery) and there is no chance to cease production even if the discounted cash-flow of the further years is negative.

The possibility of termination the project earlier can be considered as a real option. In case of arising financing need (negative cash flow), it is always worth to examine, whether the discounted cash flow of the further years can cover the new investments and otherwise abandon it.

We build this option in our model allowing the project owners to deny the additional capital investment at the end of the 10^{th} year. In the absence of risky factors (all processes are deterministic), this option has of course no value. Using the volatility figures and the "real" correlation matrix of the previous section, the option of early termination enhances the initial corporate value by 11 million HUF, 7% of the total project value (table 5.).

The real option has no influence on the probability of default, but it moderates the extent of the losses in the negative outcomes.

Table 5: Distribution of the corporate value, with and without the possibility of early termination

Export ratio	**50%**	
HUF million	**Real option**	
	not available	**available**
Mean	145	156
Standard deviation	352	343
St. Dev/Mean	244%	220%
Maximum	1 991	1 962
Minimum	-805	-576
Median	111	110
Quartile 1	-98	-97
Quartile 2	111	110
Quartile 3	342	360
Probability of default	37%	37%

CONCLUSION

In this paper we presented the consequences of the uncertainty deriving from stochastic production and inflation in addition to the market risk of currency rates on the corporate value. We constructed a simplified corporate model, which includes production, investment, inventory, export and import. Our calculations confirm the more significant role of the fluctuation of foreign exchange rates on the corporate value volatility of an exporter company, than the volatility of production itself. The reason for this is the fact, that the lower income of a decreasing production is counterbalanced partly by the parallel reduction of the costs. The volatility of inflation proved to be less determining, as we provided, that inflation affects the similarly both the incoming and outgoing items of the profit and loss. Furthermore early termination of the project as a real option enhances the corporate value considerably.

The extent of these effects can be critical in corporate operation. We offered a framework that expands the usual corporate valuation models with the methods used in risk management and derivatives' pricing. Monte Carlo simulation is the most adequate tool for that.

We plan to develop further our research by releasing the assumption of perfect market liquidity (financing sources being always available), in order to investigate the effect of the capital structure, and the different types of loan.

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