LINEAR MODELLING AND SIMULATION OF AN ENDOGENOUS GROWTH MODEL WITH HETEROGENEOUS ENTREPRENEURS

Félix-Fernando Muñoz[1], F. Javier Otamendi[2]
[1]Universidad Autónoma de Madrid
Departamento de Análisis Económico: Teoría Económica e Historia Económica
Francisco Tomás y Valiente 5
28049 Madrid, Spain
E-mail: felix.munoz@uam.es
[2]Universidad Rey Juan Carlos, Campus Vicálvaro
Departamento Economía Aplicada I
Paseo Artilleros s/n
28032 Madrid, Spain
E-mail: franciscojavier.otamendi@urjc.es

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ABSTRACT

The lack of 'true' entrepreneurs is often blamed for the failure of international aid and expansionary internal economic policy. Experience attempts to solve this question by responding that "we must create entrepreneurs", or "it is necessary to stimulate entrepreneurial activity." In this paper, we present an economic model of endogenous growth based on the Rivera-Batiz & Romer (1991) model that can explain how changes in the structure of payoffs of the economy can affect the rate of growth of output via the rate of change of the number of innovations. Different characterizations of heterogeneous entrepreneurs in an economy are modelled as well as their intention to innovate. We solve the entrepreneur problem using both analytical equations and simulation, and compare the solutions of a simplified linear problem in terms of future developments and applications.

INTRODUCTION

Why is it that over economies with apparently similar growth possibilities we observe large differences in growth rates? Why is it that the lack of entrepreneurs (or in the negative case, the abundance of speculators) is often blamed for the failure of international aid and expansionary internal economic policy? Why is it more difficult to organise production in some countries than in others? Why are some economies more innovative than others? These are all questions present in public debates, and that frequently appear in relation to economic policy. Experience attempts to solve these questions by responding that "we must create entrepreneurs", "it is necessary to stimulate entrepreneurial activity".

These questions, and their alleged solutions, are evidence of the relationship that necessarily exists between the actions or decisions of entrepreneurs and the economic performance of a society as measured, for example, by the rate of change of *per capita* output, γ_y (Figure 1) in terms of the innovation quantities.

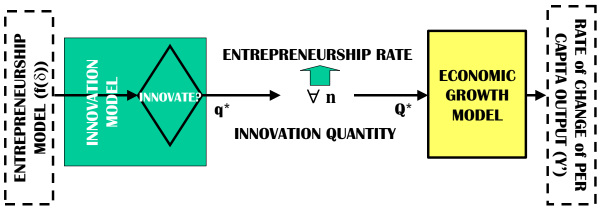

Figure 1. Entrepreneurship and per capita output

This relationship has been studied by economists, with greater or lesser intensity, ever since Economics was born as an independent science. The answers that modern economic theory has given have been rather varied, but it is only recently that they have been formally integrated in the type of mainstream model that attempts to explain economic processes; models of endogenous growth that formally incorporate the intentional actions of entrepreneurs in the explanation of how the rate of growth is determined (Romer, 1994).

However, a closer examination of these models reveals that the above mentioned intuition is only partial. One of the reasons for this type of limitation in these models can be found in the definition of entrepreneur that is used, since the models only take into account a particular case, that of the entrepreneur as an innovator. The models understand *entrepreneurship* as a unique and homogeneous function, and therefore it is rather difficult to apply the theory to questions of endogenous growth that were mentioned above (unless it is done with *ad hoc* models).

The paper explores how an extension of the concept of entrepreneur within a model of endogenous growth, can generate a wider range of explained phenomena than can be obtained in traditional models. Section 2 is devoted to model the entrepreneurs while Section 3 and Section 4 are used to explain the proposed models of innovation and economic growth. In Section 5, all the

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models are integrated into a full model, which might be solved both analytically and via simulation. A linear example is solved in Section 6. Section 7 compares both solution techniques and discusses about further research possibilities.

MODELLING ENTREPRENEURS

Conceptualisation of entrepreneurs.

In a letter to F.A. Walker, Léon Walras recognised that the definition of entrepreneur was, under his point of view, «le nœud de totut l'économique» (Jaffé, 1965) Indeed, this concept plays a fundamental role in the explanation of many economic facts —like economic growth—, but at the same time there must exist few central economic theory concepts that have been understood and applied in such a diverse manner (Casson 1982). The main function of the entrepreneur within the economy, from the point of view of the contribution to economic growth, is to increase the productivity of the system, or to create added value, through a -generally understood- process of innovation. The immediate consequence of innovation is the introduction of certain new aspects of the productive process, or of (new) types of more productive capital goods, or consumption goods with greater added value, and the corresponding resource reallocations that this implies. The introduction of innovations in the productive process increases the productivity of the system, thereby providing the impulse for economic growth.

From mainstream economics we can identify at least three different kinds of entrepreneurship: (1) the entrepreneur-producer, (2) the entrepreneur-innovator and (3) the entrepreneur-rent seeker. Obviously, real-life entrepreneurs share characteristics of these three ideal types, although with different degrees of intensity of each type. In this paper we are primarily interested in considering the two most opposing stereotypes as far as their consequences for economic growth are concerned: innovative entrepreneurial activity and rent seeking. We shall leave the entrepreneur-producer type aside, since in endogenous growth models this type is secondary (although important).

Given this simplification of entrepreneurial activity, we hypothesise that: (a) the maximum potential growth is associated with the use of the greatest part of the existing entrepreneurial capacity, and consequentially, of available resources, in activities that result in an increase in productivity; while on the other hand, (b) the allocation of all of the existing entrepreneurial capacity to rent seeking activities generates a lower economic performance in terms of *per capita* output growth -and, as a limit case, may degenerate into a contractive economy).

In short, the behaviour of the economy depends on how entrepreneurs carry out their entrepreneurial functions, which in turn is set by the internal evolution of the social dynamics that determine the distribution of entrepreneurs between the two extreme types. So, different social structures of payoffs (Baumol, 1993) will result in different economic performances (in terms of growth of output, for example) through different distributions of heterogeneous entrepreneurs between innovators and rent seekers.

The entrepreneurial density function.

A simple way to determine the distribution of entrepreneurs between the two extreme types –the innovator and the rent seeker entrepreneur- consists of assuming that each entrepreneur in the economy establishes the relative value of each of the two extreme types together with the restrictions that impinge upon these values; thus we can define an *entrepreneurial density function* between the two extremes. Moreover, since each extreme type implies different resource allocations, depending on whether there is a greater density of one type or the other in the economy we observe different rates of growth due to the fundamental link that determines γ_y in endogenous growth models. This entrepreneurial density function reflects the *hypothesis of heterogeneity* of entrepreneurs.

We characterise entrepreneurs as follows: we distribute the objectives and activities of economic agents between two extreme types, denoted by type 0 — associated with pure innovative activities — and type 1 — agents whose underlying (and unique) objective is the search for the maximum possible source of earning in the economy. Any given type of entrepreneur that is characterised by some type of mixture of the two extremes can be defined by his relative position, denoted by δ. Assuming a continuum of entrepreneurial types, $\delta\in[0,1]$, by construction we can interpret δ as the "psychological distance" of any given entrepreneur from the extreme type 0, and consequentially, $(1-\delta)$ is the "psychological distance" from type 1.

With each type of entrepreneur, we associate a "density" and a "distribution", respectively denoted by $f(\delta)$ and $F(\delta)$. In an abstract model, this functions could be *any* random variable at all, and so in principle, each economy is characterised by a given "density" of entrepreneurs of each feasible type.

MODELLING INNOVATION

The objective of a given entrepreneur δ will be defined in terms of a *subjective utility function* that defines the degree to which he will take part in activities of type 0 and of type 1, depending on his own δ. We denote by $q(\delta)$ the amount of innovation that an entrepreneur of type δ will generate under the assumption that an invention is available to him. The entrepreneur can decide between:

(a) producing a positive quantity of that good, $q(\delta) > 0$
(b) obtaining a "guaranteed" income, denoted by E (expressed in units of account), by carrying out other activities in some other sector of the economy thereby generating $q(\delta) = 0$.

We assume that the utility gains from each alternative are compared using the values $q(\delta)$; that is, the entrepreneur first decides how much of the innovative good he would like to produce should he decide to produce it at all, and then decides which activity he will finally opt for. Of course, even if he decides in favour of the innovative alternative (which we denote as E), he will always search for an earning that at least is equal to the cost of innovating, α_0, should he decide to repeat his plan in future periods. The entrepreneur will then compare this earning, from his position δ, with his other option, E.

The objective function of an entrepreneur δ.
In this context, the objective function of a heterogeneous entrepreneur might be defined as:

$$J(\delta,q)=(1-\delta)\ u(q)+\delta\ v(\pi(q)) \qquad (1)$$

It can really be thought of as a special type of utility function in which the first term, u(q), represents the *average earnings in utility terms* that the entrepreneur can obtain by innovating, which depends directly on the amount of the innovative good that is introduced into the economy. On the other hand, the second term, $\pi(q)$, is the profit associated with a given amount of the innovative good *q* and $v(\pi(q))$ is the utility gains that this profit produces. It is this second term that allows entrepreneur δ to compare the profits associated with this activity with the profit that can be obtained by alternative uses of his entrepreneurial capacity; the first term, however, always refers to the utility obtained from innovating. In this way, each entrepreneur has two ways of obtaining "subjective gains": via pure innovation; and via economic profit, which may be obtained either innovating or by alternative short run means. How each entrepreneur weighs exactly each method of improving his personal situation is described in the model by a relative position (δ) in the entrepreneurial density function, $f(\delta)$.

The entrepreneur's restriction.
No entrepreneur would want to become bankrupt by carrying out the activity that he values most; rather he would attempt to obtain the best possible position from among those available that are associated with his plan: he will attempt to obtain the maximum profit (or yield) associated with his decision. However, we are assuming that this *profit is subjective* — and so it is written in terms of a *utility* function — and is only a *minimal condition* that must be satisfied; that is, a restriction on a more general entrepreneur.

If the *economic profit* is defined as:
$\pi(q)\equiv(p(q)-c)q \quad (2)$

where $\pi(q)$ is the profit associated with the innovative good, *q*; *p*(*q*) is the "demand function" for the innovative good; and *c* is the average cost associated with the production of an additional unit of *q*, then we can define this minimal condition as follows:
$(p(q)-c)q\geq\alpha(\bullet) \quad (3)$

where $\alpha(\bullet)$ is a function that sets the minimum yield that the plan of each type δ entrepreneur must obtain.

The function $\alpha(\bullet)$ has the following arguments: (1) the position of the entrepreneur in $f(\delta)$, which determines the subjective evaluation of the yield associated with the decision to innovate in relation to (2) the cost of introducing an innovation α_0; and (3) the yield that can be obtained in other sectors of the economy, E.

The (necessary) condition to innovate.
Each type δ entrepreneur compares the (potential) earnings associated with each innovative plan — the amount that he would prefer to produce of the innovative good — with E. Then, for $q(\delta) > 0$, it must happen that: $\pi_\delta(q)\geq\alpha(\delta)$, $\forall\delta\in[0,1]$, for certain values of α_0 and E. If this *minimum condition* is not satisfied, the entrepreneur opts for the activity that produces the result E, thereby producing $q(\delta) = 0$ new goods.

The entrepreneur's problem (EP).
Therefore, we may set-up the entrepreneur's problem (EP) at a given moment of time, as follows:

$$\text{(EP)}\quad \begin{cases} \operatorname*{Max}_{q\geq 0}\ J(\delta,q)=(1-\delta)\ u(q)+\delta\ v(\pi(q)) \\ \text{s.t.:}\ \ \pi(q)\equiv(p(q)-c)q\geq\alpha(\delta,\alpha_0,\$) \end{cases} \qquad (4)$$

For greater simplicity, we set α_0 and E to equal constants for each entrepreneur, that are identical for all types of inventions/innovations, and $E\geq\alpha_0$. The same assumptions can be applied to the average costs of production of an additional unit of the new good, *c*.[1 2]

[1] A very common assumption in the endogenous growth literature (as well as the assumptions of symmetry and a constant initial cost of innovating -η in Romer's (1990) model-, etc.) In a certain way, these assumptions replace the functions of production of new goods in the cost functions.
[2] A very different problem, although of significant theoretical interest, is the consideration of the demand function for the new goods, p(q); on this point, we simply adopt a construction of the Grossman-Helpman (1991) type.

Solution of (EP). The continuity of the utility function $J(\delta,q)$ and the fact that the feasible set is closed and bounded guarantees that there *exists a global maximum* (Weierstrass Theorem). Moreover, since $J(\delta,q)$ is strictly concave and the feasible set is convex, the Fundamental Theorem of Convex Programming guarantees that the global maximum of (EP) is unique. We shall denote this maximum by q_δ^*.

Therefore, the amount of the innovative good that each type δ entrepreneur, q_δ, will be a function of the following variables:

$q_\delta^*=q(\alpha_0, E, c, p, \delta)$ (5)

The quantity of innovation goods will be higher:

- the lower the minimum profit claimed by entrepreneurs in innovative activities, α_0;
- the lower the alternative sources of gains, E;
- the lower the cost of (re)producing innovations, *c*;
- the higher the price of innovations, *p*;
- the higher the propensity of society to innovation — the "higher" the density of entrepreneur capacity towards innovation, $f(\delta)$.

The shape of this last variable will drastically differ between economies — between different economies at the same time or inside the "same" economy at different times.

Aggregate solution of (EP).

Since the "*average"* value of q_ε for the economy as a whole must be determined, and using the concept of the density function of entrepreneurial capacity, $f(\delta)$, we add the individual quantities across entrepreneurs:

$$q_\varepsilon = \int_0^1 q(\delta) f(\delta) d\delta \qquad (6)$$

The rate of entrepreneurs (δ_0)

An additional output indicator of the (EP) is the rate of entrepreneurs of a given economy. Referred as δ_0, its value is calculated after solving the following equation:

$\pi_{max} = \alpha(E,\delta_0)$ (7)

Those entrepreneurs with $0 \leq \delta \leq \delta_0$ will innovate and those with higher values of δ will not.

MODELLING ECONOMIC GROWTH

Due to its simplicity, we use an adapted version of Rivera-Batiz & Romer, endogenous growth model for evaluating the evolution of γ_y over time. Thus, we assume the following definition of capital:

$K(t) = A(t)\, q_\varepsilon(t)$ (8)

where *K*(*t*) is the accumulated amount of goods that have been incorporated in the production process, and *A*(*t*) is the number of varieties of new production goods that have been generated up to the moment of time *t*. That is to say, at each moment *t*, the amount of varieties that can be used in the production of output is given by the amount (and number) of innovative goods of previous periods, as well as those that are introduced at moment *t*. Hence, the introduction of new goods will imply an increase (change) in *A*(*t*) equal to *A'*(*t*); and *A*(*t*) will depend on both the dynamics of inventions and the dynamics of innovations.

In the Rivera-Batiz and Romer model, changes in *A* are denoted as: $A' = \tau HA$; that is, the rate of growth of *A* is proportional to the human capital in the system, *H*, and a parameter that measures the productivity of this capital, τ. In these types of model, it also turns out that all inventions are introduced into the economic system as innovations. In our case, things do not happen with such a high degree of automation, since the entrepreneurs, from the problem (EP) and their relative types, δ, will determine the amount of each invention (variety) that will be produced, where a possible solution is that no positive amount at all is produced, i.e.: an invention is not transformed into an innovation.

The rate of change of innovations according to the constant growth RBR model is:

$$\gamma_A(t) \equiv \frac{\dot{A}(t)}{A(t)} = \frac{(1-\mu)\beta q_\varepsilon(t) - \dot{q}_\varepsilon(t)c(t)}{q_\varepsilon(t)c(t) + \alpha_0(t)} \qquad (9)$$

where we also assume a consumption function with a constant marginal propensity to consume $C=\mu Y$, with $0<\mu<1$, and β is the productivity of the innovation.

It is easy to show that this rate of change is increasing in q_ε and decreasing in $\dot{q}_\varepsilon$. That is to say, the rate of growth of the number of new goods is directly proportional to investment in the introduction of new goods — and the proportion is the productivity of new goods —, and is inversely proportional to the cost of developing and producing them, and in the production of existing goods.

If we use a constant growth model independent of time, then, the rate of change of innovations is:

$$\gamma_y = \frac{A'}{A} = \frac{(1-\mu)\beta q_\varepsilon}{q_\varepsilon c + \alpha_0} \qquad (10)$$

MODELLING ECONOMIC GROWTH BASED ON ENTREPRENEUR ACTIVITY

Figure 2 depicts the combination of the models into an endogenous growth model based on the new categorization of entrepreneurs. Given $f(\delta)$, it is possible to compute innovation quantities for each entrepreneur, and then for the Economy. This aggregate quantity $Q^*=q_\varepsilon$ is used then as the input to the RBR constant growth model.

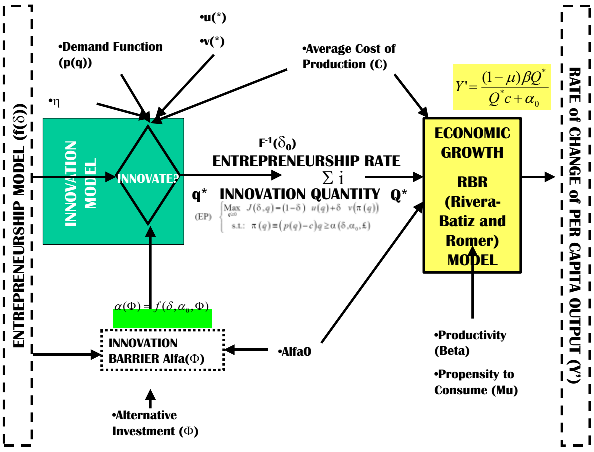

Figure 2. The proposed integrated model

In terms of computation, the model is usually difficult to solve analytically. The steps of calculating the quantities using the (EP) and the aggregate q_ε using integration could take too long to compute. That is why a simulation approach might be preferred.

SIMULATING ECONOMIC GROWTH BASED ON ENTREPRENEUR ACTIVITY

Instead of integrating over δ to calculate the quantity q_ε, it is possible to simulate for any $f(\delta)$. What follows is the pseudocode that has been designed to summarize the process of estimating γ_y:

```
Input data: f(δ), E, α0
Input data: p(q), c, u(q), v(•), π(•)
Input data: μ, β
For each simulation s
        For each entrepreneur i
            Generate δ
            Compute α(E), π(•)
            If π(•)>α(E), calculate q* by solving EP
        Compute indicators
            Q as the sum of q*'s
            δ0
            Rate of entrepreneurship as F-1(δ0)
        Compute
```

This type of modelling facilitates the use of higher-order functions as well as modifications over time if needed.

SIMPLIFIED EXAMPLE: LINEAR FUNCTIONS

In this simple example, the functions that we use are linear so that the problem is readily solvable both analytically and via simulation:

- $p(q)=a-bq$ with $a,b>0$, so $\pi(q)=(a-bq-c)q$
- $u(q)=\eta q$; $v(x)=x$
- $\alpha(\delta, \alpha_0, E) = (1-\delta)\,\alpha_0 + \delta E$
- $f(\delta)$ follows a Uniform distribution, $U(0,1)$

Therefore, the entrepreneur problem is:

$$(EP)\quad \begin{cases} \max\limits_{q\geq 0} J(\delta,q)=(1-\delta)u(q)+\delta v(\pi(q)) \\ \quad \text{s.t.}: \pi(q)\geq \alpha(\delta,\alpha_0,E) \\ \quad \alpha(\delta,\alpha_0,E)=(1-\delta)\alpha_0+\delta E \end{cases} \qquad (12)$$

Analytically, the solution for each individual entrepreneur is given by:

$$q^*(\delta,\alpha_0,E)=\begin{cases} \min\{q_1(\delta,E),q_2(\delta,E)\} & \text{if } \alpha(\delta,E)\leq \frac{(a-c)^2}{4b} \\ 0 & \text{if } \alpha(\delta,E)> \frac{(a-c)^2}{4b} \end{cases} \qquad (13)$$

where:

$$q_1=\frac{(a-c)}{2b}+\frac{\sqrt{(a-c)^2-4b\alpha(\Phi)}}{2b}$$
$$q_2=\frac{(a-c-\eta)}{2b}+\frac{\eta}{2b\delta} \qquad (14)$$

The rate of entrepreneurs (δ_0) follows:

$$\delta_0=\frac{\frac{(a-c)^2}{4b}-\alpha_0}{\Phi-\alpha_0} \qquad (15)$$

However, and even in this simple case, it is complicated to calculate q_ε; it will be very complex to integrate q^* due to the existence (at least a priori) of non-linearities and the shape of $f(\delta)$.

For the sake of validating the simulation model, the following example has been used:

- $f(\delta)$: Uniform (0,1); $E = 0.5$; $\alpha_0 = 0.01$
- $p(q) = 1\text{-}1\ q$; $c = 0.5$; $u(q) = 1\ q$
- $\mu = 0.8$; $\beta = 1.5$

The results for N = 100 entrepreneurs after theoretically calculating the analytical values are $Q^* = 4.3152$ and $\delta_0 = 10.7143\%$. If S = 50 simulations are run:

- The goods in the Economy, Q*, lie between 2.3307 and 7.0562 with an average of 4.2390
- The rate of entrepreneurship, δ_0, lies between 6% and 18% with and average of 10.4694%.
- The rate of change of *per capita* output, lies between 59.4895% and 59.8304% with an average of 59.7009%.

CONCLUSIONS

The model analysed in this paper is a simple example of how widening the content of the concept of entrepreneur –allowing him to carry out heterogeneous tasks within his entrepreneurial capacity formalised in this model by (EP), allow us to give more meaningful answers to the questions posed at the very beginning of this paper.

From (EP) we have been able of establish the relationship between variables such as minimum profit threshold, opportunity of gains in other sectors of the economy, the cost of innovation, etc., and the quantity of innovation finally produced depending on the attitude of entrepreneurs themselves and society in general towards innovation effort — defined by f(□)—, and the consequences of all this in the rate of growth of output.

The innovation-growth model is solvable using simulation techniques. Therefore, it is our aim to keep on improving the simulation model to be able to increase its applicability. In particular, we are already addressing the possibility of incorporating non-linearities in the functions as well as incorporating other growth models. Moreover, we are trying to develop rules to vary all the parameters and functions over time.

The potential of the model for policy making must also be investigated. On that regard, we are aware of the limitations of the proposed macroeconomic growth model "as-of-today", limitations that would also be easier to avoid if simulation modelling is used to solve the entrepreneur's problem (EP).

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REFERENCES

Baumol, W. (1993). Entrepreneurship, Management and the Structure of Payoffs. MIT Press, Cambridge, Mass.

Billari Francesco C., Thomas Fent, Alexia Prskawetz & Jürgen Scheffran (eds.) Agent-Based Computational Modelling. Applications in Demography, Social, Economic and Environmental Sciences. Physica-Verlag, Heidelberg, 2006.

Grossman, G. and E. Helpman (1991). Innovation and Growth in the Global Economy. MIT Press.

Casson, M. (1982). The Entrepreneur: an Economic Theory. Oxford, Martin Robertson.

Dosi, G., Fagiolo, G. Roventini, A. (2010) Schumpeter me etnia Keynes. A policy-friendly model of endogenous growth and business cycles. Journal of Economic Dynamics and Control, 34, pp. 1748-1767

Jaffé, W. (ed.) (1965) The correspondence of Léon Walras. L 800. North Holland.

Rivera-Batiz, L.A. and P.M. Romer (1991). "Economic Integration and Endogenous Growth." Quarterly Journal of Economics, 106(2) May, 531-555.

Romer, P.M. (1994). "The Origins of Endogenous Growth". The Journal of Economic Perspectives, Vol. 8, No. 1. (Winter), pp. 3-22.

AUTHOR BIOGRAPHIES

FÉLIX-FERNANDO MUÑOZ is PhD. in Economics (Economic Theory) at Universidad Autónoma de Madrid (UAM). His research areas of interest are Endogenous Growth Theory, Evolutionary Economics and Economic Methodology. He is director of the Master in International Economics program at UAM.

JAVIER OTAMENDI received the B.S. and M.S. degrees in Industrial Engineering at Oklahoma State University, where he developed his interests in Simulation and Total Quality Management. Back in his home country of Spain, he received a B.S. in Business Administration and a Ph.D. in Industrial Engineering. He is currently a simulation and statistics consultant and university professor at the Rey Juan Carlos University in Madrid.