NOVEL MULTIVARIABLE LABORATORY PLANT

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ABSTRACT

A novel multivariable laboratory plant is presented. The process is a variation of known "four interconnected water tanks". The novelty of the process lies in a way how to ensure multi-variability. Pneumatic volumes above the water levels are connected together and orifices are placed into those circuits. Cross interactions exists only in transient states so the working area is not reduced. First principle nonlinear mathematical model is derived and presented together with its linearized form and experimental identification of unknown parameters.

1. INTRODUCTION

Importance of laboratory experiments in control engineering education is evident. Industrial interest in use of multivariable control techniques (Shinskey 1981; Goodwin et al. 2000; Skogestad and Postlethwaite 2005) is topical problem too. Multivariable laboratory processes are not very common. There exist some commercial products as e.g. Helicopter body on support, Ball&Plate model produced by TecQuipment Ltd, 2 DOF (Degrees of Freedom) Helicopter, Rotary Inverted Pendulum modules from QUANSER Innovative Educate, Four axis Control Moment Gyroscope or Torsional Plant from Educational Control Products, Twin Rotor MIMO from Feedback plc. Some of the multivariable processes are constructed at universities for their own use as various water tank processes, combinations of temperature and flow control of liquids or air, distillation columns, chemical reactors, heat exchangers and other apparatuses. Disadvantage of such systems is usually higher complexity so the systems are more time and money consuming to operate and service. From this point of view water tanks are quite simple – water pumps are cheap and reliable, water level measurement by use of pressure sensors is easy too. Very elegant multivariable process called "quadruple-tank process with an adjustable zero" was developed at Department of Automatic Control in Lund (Johanson and Nunes 1998). We have decided to design and construct our variation of "four tank system". Instead of using three-path valves or connecting water levels we have closed and connected air spaces above water levels. This caused decreasing of original working area so we added air orifices into the air circuits. Novel multivariable process with interesting features arises.

The outline of the paper is as follows. Laboratory plant is presented in section 2. Its nonlinear first principle model is derived in section 3. This model is linearized and state-space representation is given in section 4. In section 5 experimental identification is carried out. Model is verified in section 6. Some conclusions are given in section 7.

2. LABORATORY PROCESS

Laboratory Hydraulic-Pneumatic System (HPS) was designed and realized at Department of Process Control University of Pardubice (Klán et al. 2005; Macháček et al. 2005). It includes a combination of hydraulic and pneumatic components. The pneumatic circuits create cross coupling between both classical double tank sections (Åström and Lundh 1992) and form a multivariable system with non-typical feature. Four cylindrical water tanks are the main parts - see Fig. 1 and 2. Water is pumped by two pumps into upper tanks LH and RH, flows into lower tanks LL and RL and from here back into the reservoir. Water flow rates are controlled by input signal of the pumps u_L, u_R - voltage in the range 0÷10 V, which is amplified and changed into 4÷10 V in pump unit. The levels in lower tanks are measured indirectly by difference pressure sensors.

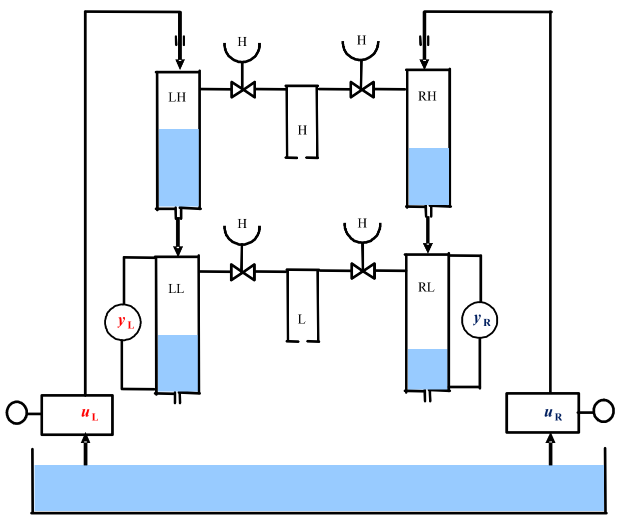

Figure 1: Scheme of HPS

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Output of the pressure sensors y_L, y_R is given in a form of electric voltage in the range 0÷10 V. Air spaces above the water levels are connected together by pneumatic circuits H and L with manually configurable valves. Orifices in air chambers serve as a connection between pneumatic volumes and atmosphere. The system structure and its behaviour may be changed by the size of orifices and by valves setting in the pneumatic circuits. There is atmospheric pressure in pneumatic circuits in the steady state. If water level is changing pressure in pneumatic volumes changes too and influences adjacent water levels. Air flows into or from air chamber and gradually equilibrates with atmospheric pressure. That means that multivariable cross effect has only dynamic character and after some time disappears – see step responses in Fig. 5 and 6. Diameter of the air orifice influences gain and time life of the effect (effect is stronger but slower for smaller orifice).

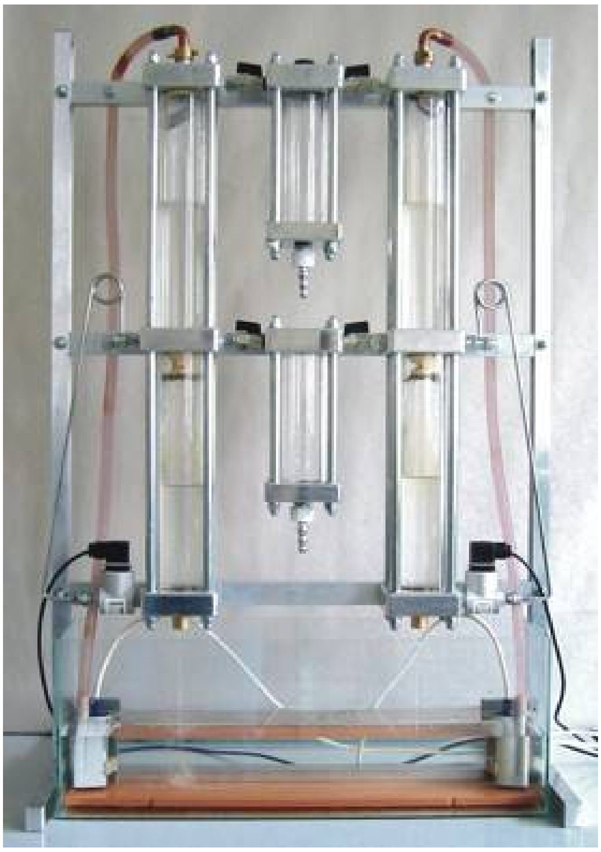

Figure 2: Hydraulic-Pneumatic System

3. NONLINEAR MODEL

Nonlinear model of HPS is derived on the basis of physical laws and system construction. Only the lower pneumatic circuit is considered in this model (upper pneumatic volume C_H is open into atmosphere).

Models of the water tanks can be described by

a) mass balance based on law of mass conservation

$$Q_1 = Q_2 + \rho S \frac{dh}{dt} \quad (1)$$

where Q_1 is water inlet mass flow rate [kg s^{-1}], Q_2 is water outlet mass flow rate [kg s^{-1}], ρ is water density [kg m^{-3}], S is cross-section area of tank [m^2] and h is water level [m].

b) Bernoulli equation - water outlet mass flow rate is given as

$$Q_2 = k\, s \sqrt{2\rho} \sqrt{h\rho g + p_1 - p_2} \quad (2)$$

where k is discharge coefficient [-], s is cross-section area of orifice [m^2], g is acceleration of gravity [m s^{-2}], p_1 is pressure above the water level [Pa] and p_2 is pressure under the orifice [Pa].

In Table 1 variables used in Equations (1) and (2) are specified according to Fig. 1.

Table 1: Notation of variables for tanks nonlinear model

Tank	Q_1	Q_2	S	k	s	h	p_1	p_2
LH	Q_L	Q_{LH}	S_L	k_L	s_L	h_{LH}	p_A	p_L
RH	Q_R	Q_{RH}	S_R	k_R	s_R	h_{RH}	p_A	p_L
LL	Q_{LH}	Q_{LL}	S_L	k_L	s_L	h_{LL}	p_L	p_A
RL	Q_{RH}	Q_{RL}	S_R	k_R	s_R	h_{RL}	p_L	p_A

The lower pneumatic circuit was modelled on the basis of

a) mass balance equivalent of Equation (1)

$$0 = Q_{CL} + \frac{d}{dt}(V_L \rho_L) \quad (3)$$

where Q_{CL} is air outlet mass flow rate [kg s^{-1}], V_L is pneumatic volume (air chamber volume plus volume above water levels) [m^3] and ρ_L is air density [kg m^{-3}].

b) equivalent of Equation (2), which was simplified into the following form

$$Q_{CL} = k_{CL}\, s_{CL} (p_L - p_A) \quad (4)$$

where k_{CL} is air discharge coefficient [s m^{-1}], s_{CL} is cross-section area of air orifice [m^2], p_L is pressure in the pneumatic loop [Pa] and p_A is atmospheric pressure [Pa].

c) equation of gas state

$$p_L = \rho_L r T \quad (5)$$

where r is air specific gas constant [J K^{-1} kg^{-1}] and T is air temperature [K].

Equations (3), (4) and (5) may be combined together into relationship

$$\frac{d(p_{\mathrm{L}}V_{\mathrm{L}})}{dt} = -k_{\mathrm{CL}}s_{\mathrm{CL}}rT(p_{\mathrm{L}} - p_{\mathrm{A}}) \qquad (6)$$

For pneumatic volume holds

$$V_{\mathrm{L}} = S_{\mathrm{L}}(H - h_{\mathrm{LL}}) + V_{\mathrm{CL}} + S_{\mathrm{R}}(H - h_{\mathrm{RL}}) \qquad (7)$$

where H is tanks height and V_{CL} is volume of the air chamber.

Static characteristic of the pump is considered in the form $Q = a(u - \tilde{u})^b$, where Q is water mass flow rate [kg s^{-1}], u is input signal for the pump unit [V], $\tilde{u}$ is the input signal [V] corresponding to zero mass flow rate, a and b are the pump specific coefficients.

Pressure sensor static characteristic is in the form $y = c \cdot h + d$, where y is output signal from the pressure sensor [V], h is water level [m] c and d are the pressure sensor specific coefficients.

The model has five state variables - four water levels and pressure in the lower pneumatic circuit, two inputs - input signals for pump unit u_{L} and u_{R} and two outputs - output signals from pressure sensors y_{L} and y_{R}.

4. LINEARIZED MODEL

Nonlinear model is analytically linearized for control design purposes. The linearization is realized by the Taylor expansion of the original nonlinear equations - the second and higher order terms are omitted. Symbol Δ denotes deviation of variable from steady state, *e.g.* $\Delta h = h - h_0$, where subscript 0 denotes steady state. The steady state of pressure in lower pneumatic circuit is atmospheric pressure p_{A}.

By linearization of Equation (2) we get

$$\Delta Q_2 = \frac{k^2 s^2 \rho^2 g}{Q_0}\Delta h + \frac{k^2 s^2 \rho}{Q_0}\Delta p_1 - \frac{k^2 s^2 \rho}{Q_0}\Delta p_2 \qquad (8)$$

where Q_0 steady-state mass flow rate.

We can rewrite Equation (1) in deviation form as

$$\Delta Q_1 = \Delta Q_2 + \rho S \frac{d\Delta h}{dt} \qquad (9)$$

If we substitute Equations (8) and (9) and by respecting notation from Table 1 we get two linear differential equations for upper and lower tanks

$$T\frac{d\Delta h_{\mathrm{H}}}{dt} = -\Delta h_{\mathrm{H}} + Z\Delta p_{\mathrm{L}} + Z_{\mathrm{Q}}\Delta Q \qquad (10)$$

$$T\frac{d\Delta h_{\mathrm{L}}}{dt} = \Delta h_{\mathrm{H}} - \Delta h_{\mathrm{L}} - 2Z\Delta p_{\mathrm{L}} \qquad (11)$$

where

$$T = \frac{SQ_0}{k^2 s^2 \rho g}, \quad Z = \frac{1}{\rho g}, \quad Z_{\mathrm{Q}} = \frac{T}{\rho S}.$$

General terms are given. Particular variables for Equations (10) and (11) have to be filled from Tables 1 and 2.

Table 2: Notation of variables for tanks linear model

Tank	Δh_{H}	Δh_{L}	Q_0	ΔQ	T	Z_{Q}
LH	Δh_{LH}	-	Q_{L0}	ΔQ_{L}	T_{L}	Z_{QL}
RH	Δh_{RH}	-	Q_{R0}	ΔQ_{R}	T_{R}	Z_{QR}
LL	Δh_{LH}	Δh_{LL}	Q_{L0}	-	T_{L}	-
RL	Δh_{RH}	Δh_{RL}	Q_{R0}	-	T_{R}	-

Model of the air circuit is nonlinear because of $p_{\mathrm{L}}V_{\mathrm{L}}$ term in Equation (6). We can write its linearization in point [p_{A}, V_{L0}] as

$$p_{\mathrm{L}}V_{\mathrm{L}} = p_{\mathrm{A}}V_{\mathrm{L0}} + p_{\mathrm{A}}\Delta V_{\mathrm{L}} + V_{\mathrm{L0}}\Delta p_L \qquad (12)$$

and calculate left term of Equation (6) as

$$\frac{d(p_{\mathrm{L}}V_{\mathrm{L}})}{dt} = p_{\mathrm{A}}\frac{d\Delta V_{\mathrm{L}}}{dt} + V_{\mathrm{L0}}\frac{d\Delta p_{\mathrm{L}}}{dt} \qquad (13)$$

where V_{L0} is steady-state value of the lower pneumatic volume.

By derivation of (7) and rewriting to deviations we get

$$\frac{d\Delta V_{\mathrm{L}}}{dt} = -S_L\frac{d\Delta h_{\mathrm{LL}}}{dt} - S_R\frac{d\Delta h_{\mathrm{RL}}}{dt} \qquad (14)$$

If we substitute (13) and (14) into (6) (term p_{L}-p_{A} equals to Δp_{L}) we get for dynamic of the lower pneumatic circuit following equation

$$T_{\mathrm{p}}\frac{d\Delta p_{\mathrm{L}}}{dt} + \Delta p_{\mathrm{L}} = Z_{\mathrm{hL}}(\Delta h_{\mathrm{LH}} - \Delta h_{\mathrm{LL}}) + Z_{\mathrm{hR}}(\Delta h_{\mathrm{RH}} - \Delta h_{\mathrm{RL}}) \qquad (15)$$

where

$$T_{\mathrm{p}} = \frac{V_{\mathrm{L0}}Q_{\mathrm{L0}}Q_{\mathrm{R0}}}{2p_{\mathrm{A}}\left(k_{\mathrm{L}}^2 s_{\mathrm{L}}^2 Q_{\mathrm{R0}} + k_{\mathrm{R}}^2 s_{\mathrm{R}}^2 Q_{\mathrm{L0}}\right) + Q_{\mathrm{L0}}Q_{\mathrm{R0}}rTk_{\mathrm{CL}}s_{\mathrm{CL}}},$$

$$Z_{\mathrm{hL}} = \frac{p_{\mathrm{A}}S_{\mathrm{L}}T_{\mathrm{p}}}{V_{\mathrm{L0}}T_{\mathrm{L}}}, \quad Z_{\mathrm{hR}} = \frac{p_{\mathrm{A}}S_{\mathrm{R}}T_{\mathrm{p}}}{V_{\mathrm{L0}}T_{\mathrm{R}}}$$

By linearization of static pump characteristic we get deviation form

$$\Delta Q = ab\left(u_0 - \tilde{u}\right)^{b-1}\Delta u = Z_u \Delta u \qquad (16)$$

We can write static characteristic of the pressure sensors in deviation form as

$$\Delta y = c\Delta h \qquad (17)$$

Particular variables used in Equations (16) a (17) have to be filled from Tables 1, 2 and 3.

Table 3: Notation of variables for pumps and pressure sensors linear model

Position	ΔQ	Δu	u_0	Z_u	Δy	Δh
L	ΔQ_L	Δu_L	u_{L0}	Z_{uL}	Δy_L	Δh_{LL}
R	ΔQ_R	Δu_R	u_{R0}	Z_{uR}	Δy_R	Δh_{RL}

If we compose appropriate state vector we are able to rewrite linear model into following state space form

$$\underbrace{\begin{bmatrix} \frac{d\Delta h_{LH}}{dt} \\ \frac{d\Delta h_{RH}}{dt} \\ \frac{d\Delta p_L}{dt} \\ \frac{d\Delta h_{LL}}{dt} \\ \frac{d\Delta h_{RL}}{dt} \end{bmatrix}}_{\frac{d\mathbf{x}}{dt}} = \underbrace{\begin{bmatrix} -\frac{1}{T_L} & 0 & \frac{Z}{T_L} & 0 & 0 \\ 0 & -\frac{1}{T_R} & \frac{Z}{T_R} & 0 & 0 \\ \frac{Z_{hL}}{T_p} & \frac{Z_{hR}}{T_p} & -\frac{1}{T_p} & -\frac{Z_{hL}}{T_p} & -\frac{Z_{hR}}{T_p} \\ \frac{1}{T_L} & 0 & -\frac{2Z}{T_L} & -\frac{1}{T_L} & 0 \\ 0 & \frac{1}{T_R} & -\frac{2Z}{T_R} & 0 & -\frac{1}{T_R} \end{bmatrix}}_{\mathbf{A}} \cdot \underbrace{\begin{bmatrix} \Delta h_{LH} \\ \Delta h_{RH} \\ \Delta p_L \\ \Delta h_{LL} \\ \Delta h_{RL} \end{bmatrix}}_{\mathbf{x}}$$

$$+ \underbrace{\begin{bmatrix} \frac{Z_{QL}}{T_L} & 0 \\ 0 & \frac{Z_{QR}}{T_R} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{\mathbf{B}} \cdot \underbrace{\begin{bmatrix} \Delta u_L \\ \Delta u_R \end{bmatrix}}_{\mathbf{u}} \qquad (18)$$

$$\underbrace{\begin{bmatrix} \Delta y_L \\ \Delta y_R \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & c_L & 0 \\ 0 & 0 & 0 & 0 & c_R \end{bmatrix}}_{\mathbf{C}} \cdot \underbrace{\begin{bmatrix} \Delta h_{LH} \\ \Delta h_{RH} \\ \Delta p_L \\ \Delta h_{LL} \\ \Delta h_{RL} \end{bmatrix}}_{\mathbf{x}} \qquad (19)$$

5. EXPERIMENTAL IDENTIFICATION

The model has 13 unknown parameters beside parameters connected with construction of the plant. Water orifice discharge coefficients k_L, k_R, air orifice discharge coefficient k_{CL} and coefficients for pump and pressure sensor static characteristics (a, b, u_0, c, d) must be estimated experimentally. Except air orifice discharge coefficient k_{CL} all other parameters can be estimated from one experiment. Both air circuits are open to the atmosphere and control voltages for both pumps are changed stepwise gradually. Water flow rate, water levels and pressure sensors outputs are measured after getting into a steady-state. Parameters are calculated by numerical optimization method.

Parameters a and b (for pump static characteristic) are calculated from input voltages for pump unit u and mass flow rates Q – see Table 4.

Table 4: Parameters of pump static characteristics

Position	a	b	$\tilde{u}$
L	7.98e10^{-3}	0.552	0.8
R	8.53e10^{-3}	0.495	0.9

Parameter $\tilde{u}$ corresponds to a voltage when the liquid starts to flow to the tanks. Parameters k (water orifice discharge coefficients) are calculated from mass flow rates Q and water levels h – see Table 5. Pressures p_1 and p_2 equals to atmospheric pressure p_A.

Table 5: Parameters of tank static characteristics

Position	k
L	0.737
R	0.735

Parameters c a d (for pressure sensors static characteristics) are calculated from water levels h and output from the pressure sensors y – see Table 6.

Table 6: Parameters of sensors static characteristics

Position	c	d
L	32.5	0.12
R	32.2	0.03

It is not possible to estimate air orifice discharge coefficient from static data. Dynamic experiment had to be carried out – to measure response of the water levels for varying pump powers – see solid line in Figure 3. Unknown parameter is estimated by numerical optimization – least square error cost function is used together with simulated response of the dynamic nonlinear model – see dotted line in Figure 3. For the air orifice discharge coefficient see Table 7.

Table 7: Air orifice discharge coefficient

k_{CL}	0.094

Geometric dimensions and physical constants are given in Tables 8 and 9 – this information can be seen as a part of the experimental identification too.

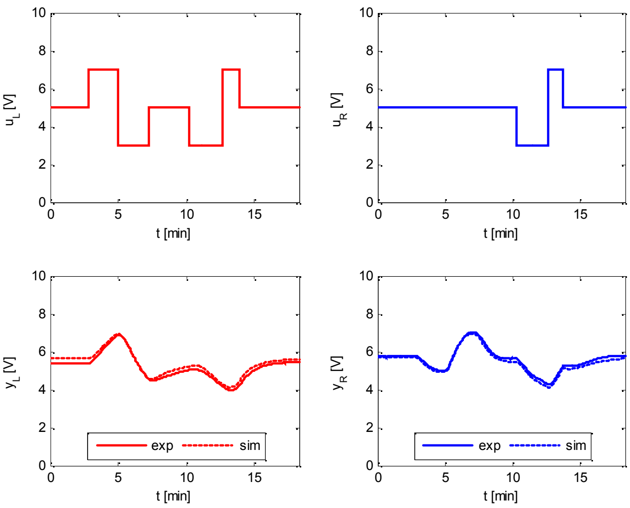

Figure 3: Dynamic experiment for experimental identification

Table 8: Geometric dimensions

Notation	Value	Unit	Meaning
D_L	0.05	m	left tanks diameter
D_R	0.04	m	right tanks diameter
H	0.28	m	tanks height
V_{CL}	0.45e10^{-3}	m^3	volume of the lower air chamber
d_L	0.004	m	diameter of the left tanks orifice
d_R	0.004	m	diameter of the right tanks orifice
d_{CL}	2e10^{-4}	m	diameter of the lower air orifice

Table 9: Physical constants

Notation	Value	Unit	Meaning
ρ	1000	kg m^{-3}	water density
r	287	J K^{-1} kg^{-1}	air specific gas constant
g	9.81	m s^{-2}	acceleration of gravity
T	293	K	air temperature
p_A	101325	Pa	atmospheric pressure

We can study nonlinearity of the plant by using identified nonlinear process model. If we substitute equations of static characteristics of pumps, water tanks and pressure sensors we get total static characteristic of HPS in a form

$$\Delta y = c\underbrace{\frac{Q_0}{k^2 s^2 \rho^2 g}}_{Z_Q}\underbrace{ab\left(u_0-\tilde{u}\right)^{b-1}}_{Z_u}\Delta u \qquad (20)$$

The plant is almost linear in term of steady-state gain. Static characteristic of the tanks is a square function and static characteristic of the pump is close to a square root function. Steady-state gains for whole working area are plotted in upper axes in Figure 4. Time constants of the water tanks in equation (11) are plotted in lower axes in Figure 4. System has smallest time constants for smaller water levels. Dynamic of the process is second order with multiple time constant. Settling time is changing approx. eight times. This holds for the case that the both pneumatic circuits are open.

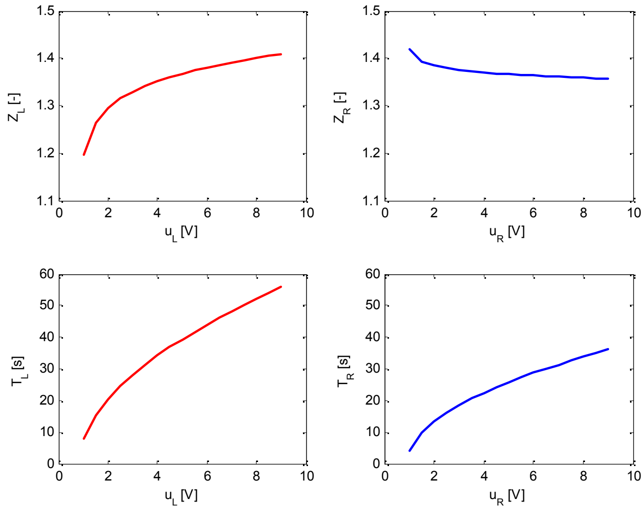

Figure 4: Gains (HPS) and time constants (water tanks)

Situation is different with closed pneumatic circuits. The dynamic of the plant is slowed down and nonlinearity in term of time constants is smaller if the lower pneumatic circuit is closed. This can be observed from step responses. Step responses are calculated for two ultimate cases – for working point close to minimal and maximal water levels – see Fig. 5 and 6.

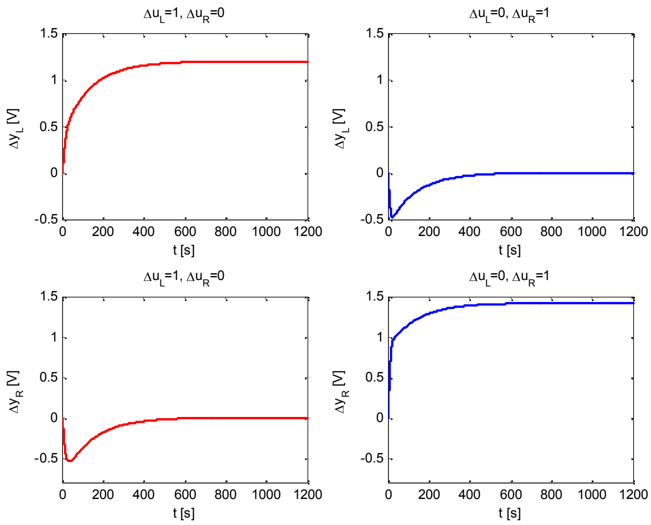

Figure 5: Step responses – u_L = 1 V, u_R = 1 V

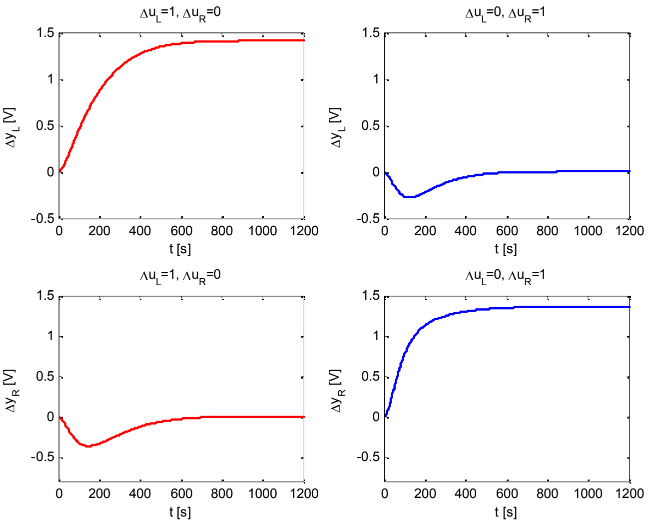

Figure 6: Step responses – u_L = 9 V, u_R = 9 V

Settling time between these two cases is approx. doubled.

6. MODEL VERIFICATION

Step response of HPS is measured – response to a step change in left pump power from 4 to 6 V. Right pump power was kept constant at 5 V. Measured and calculated responses are shown in Figure 7 – measured with the blue and calculated with the red line. Responses are calculated from nonlinear process model. For this particular working point and step change size the linear model would give very similar response.

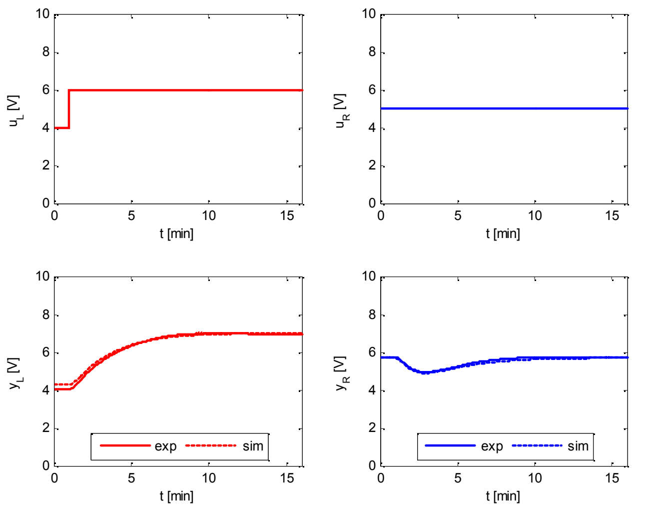

Figure 7: Step response

7. CONCLUSIONS

Novel multivariable laboratory process is introduced. It is a combination of known quadruple water tanks process and pneumatic circuits. Pneumatic volumes are closed and connected instead of water volumes. Air orifice is added to pneumatic circuits and gives arise to a system with interesting features. First principle model is derived. Its linearized state space form can be used for control design. Controllers can be tried and simulated with nonlinear model first and consequently applied to the real laboratory process. From the control point of view the system is interesting because upper water levels are not measured and can easily under- or over-flow. This is hardly to solve with classical controllers but for example state-space predictive controller could be a good choice. State observer would solve the problem of unmeasured upper water levels and controller with respecting state constraints would guarantee that the levels do not cross their limits.

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