SOCIODYNAMIC DISCRETE CHOICE APPLIED TO TRAVEL DEMAND: MULTI-AGENT BASED SIMULATION AND ISSUES IN ESTIMATION

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ABSTRACT

This paper discusses a multi-agent based model of binary choice behavior with interdependence of decision-makers' choices. Analytical results established by other authors are briefly summarized where agent heterogeneity is not explicitly treated. Next the well-known Erdős-Rényi network class is considered to introduce agent heterogeneity via an explicit local interaction structure. Then the model is applied in an example of intercity travel demand using empirical data to introduce individual agent heterogeneity beyond that induced by the local interaction structure. Studying the long run behavior of more than 120,000 multi-agent based simulation runs reveals that the initial estimation process can be highly sensitive to small variations in network instantiations. We show that this is an artifact of two issues in estimation, and highlight particular attention that is due at low network density and at high network density. Limitations in the present work are summarized and suggestions for future research efforts are outlined.

1 INTRODUCTION

Pioneered in the domain of travel demand by Ben-Akiva (1973), Domencich and McFadden (1975), and others, discrete choice analysis has become an industry standard in land use and transportation planning models. An outstanding methodological challenge remains however in the treatment of the interdependence of various decision-makers' choices. There is growing awareness and interest in the influence that social factors have on transportation and land use behaviors (Dugundji, Páez and Arentze 2008; Dugundji et al. 2011).

While there exists a substantial stream of research in *identifiable* intra-household interactions and explicit inter-household interactions of extended family, friends and colleagues in travel demand modeling such as coordination of individual daily activity patterns, joint participation in activities and travel, mechanisms for allocation of maintenance activities, and activity location and residential location choice behavior, the topic of *aggregate* or collective social interactions between individuals in different households at a market level in travel demand has only recently begun to attract attention. Some examples of the empirical estimation of a discrete choice model with aggregate social interactions with application to transportation include Dugundji and Walker (2005), Goetzke (2008), Goetzke and Andrade (2009), Goetzke and Rave (2010), Goetzke and Weinberger (2012). Some explorations of the dynamical behavior of such a model with application to transportation include Fukuda and Morichi (2007), Páez and Scott (2007), Páez, Scott and Volz (2008), Arentze and Timmermans (2008), and Dugundji and Gulyás (2008). This paper continues this line of research, exploring a multi-agent based model of binary choice behavior with interdependence of decision-makers' choices.

Discrete-choice estimation results controlling overall mechanisms related to individual heterogeneous preferences are embedded in a multi-agent based model to be able to observe the simulated evolution of choice behavior over time with socio-dynamic feedback due to network effects. Studying the long run behavior of more than 120,000 multi-agent based simulation runs reveals that the initial estimation process can be highly sensitive to small variations in network instantiations. We show that this is an artifact of two issues in estimation, and highlight particular attention that is due at low network density and at high network density. This finding is an important warning with respect to empirical application of agent-based models.

2 MODEL

Discrete choice theory allows prediction based on computed individual choice probabilities for heterogeneous agents' evaluation of alternatives. In accordance with the notation and convention in Ben-Akiva and Lerman (1985), the so-called binary logit model is specified as follows. Assume a population of N decision-making entities indexed $(1,\ldots,n,\ldots,N)$ each faced with a choice among two alternatives of some universal choice set $C = \{i,j\}$, say, choice of travel by car versus by railway, which we assume to be available to all agents. The choice alternatives are further assumed to be mutually exclusive (a choice for one alternative excludes the simultaneous choice for another alternative, that is, an agent cannot choose two

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alternatives at the same moment in time) and collectively exhaustive within C (an agent must make a choice for one of the options in the choice set).

Let $U_{in} = V_{in} + \varepsilon_{in}$ be the utility that a given decision-making entity n is presumed to associate with elemental alternative i in its choice set, where V_{in} is the deterministic (to the modeler) or so-called "systematic" utility and ε_{in} is an error term. The error term represents unobserved heterogeneity. Such unobserved heterogeneity may arise due to unobserved attributes of the choice alternatives, unobserved characteristics of the decision-making entities or simply measurement errors in observed attributes and/or characteristics. Also in the case where instrumental variables are used as a proxy for variables which are not observable, the error term is relevant for capturing unobserved heterogeneity. Under the assumption of Gumbel distributed disturbances ε_{in}, the probability P_{in} that agent n chooses alternative i has a convenient closed form expression, given by:

$$P_{in} = \frac{e^{\mu V_{in}}}{e^{\mu V_{in}} + e^{\mu V_{jn}}} = \frac{e^{\mu\left(V_{in}-V_{jn}\right)}}{e^{\mu\left(V_{in}-V_{jn}\right)}+1} \tag{1}$$

where μ is a strictly positive scale parameter which we generally normalize to 1. The assumption that the disturbances are Gumbel distributed can be defended as an approximation to the normal density.

2.1 Global Social Influence: The Field Effect Model

The classical discrete choice framework discussed so far assumes independent individuals. Aoki (1995), Brock and Durlauf (2001) and Blume and Durlauf (2003) relax this assumption. Their approach is to assume that the otherwise independent individuals are influenced by an aggregate of all other choices in the community. There is an inherent dynamic because each individual re-evaluates its choice based on the choices made by other individuals. This implies an implicit time-trajectory of repeated choices that defines the dynamics of the system. It is in this sense that we call this a "socio-dynamic" model: the dynamics are driven by social influence, albeit global social influence at this stage. The steady states of this dynamic process are reviewed below.

Let N_i and N_j be the total numbers of decision-making entities who have chosen respectively alternative i and alternative j at time t. Since we assume the choice set to be mutually exclusive and collectively exhaustive, for the binary case we have $N = N_i + N_j$. Now let $x_i = N_i / N$ and $x_j = N_j / N = (1 - x_i)$ be the global proportions of decision-making entities who have made each choice, and define the field variable:

$$x \equiv x_i - x_j = x_i - (1 - x_i) = 2x_i - 1 \tag{2}$$

Note that the field variable x varies on the range -1 to 1. In the limit where x = -1, none of the decision-making entities in the sample have chosen alternative i, that is, all have chosen alternative j. In the limiting case where x = 1, all of the decision-making entities in the sample have chosen alternative i, and none have chosen alternative j. In the case where x = 0, half of the decision-making entities in the sample have chosen alternative i, and half have chosen alternative j.

Global social dynamics are introduced by allowing the term $V_{in} - V_{jn}$ in equation (1) to be a linear-in-parameter β function of the proportions x_i and x_j of decision-making entities who have made each choice:

$$V_{in} - V_{jn} \equiv \beta f\left(x_i - x_j\right) = \beta f\left(x\right) \tag{3}$$

The function $f(x)$ is an arbitrary function of x. In our application we consider $f(x)$ linear in x, however the analytical results apply more generally. Substituting equation (3) into (1) and normalizing the scale parameter μ =1, we have:

$$P_{in}\left(x\right) = \frac{e^{\beta f(x)}}{e^{\beta f(x)}+1} \tag{4}$$

Aoki (1995) shows that the mean φ of the field variable x is governed by the deterministic differential equation:

$$\frac{d\varphi}{dt} = \kappa\frac{1-\varphi}{2}P_{in}\left(\varphi\right) - \lambda\frac{1+\varphi}{2}P_{jn}\left(\varphi\right) \tag{5}$$

Substituting (4) into (5) and normalizing κ =1 and λ =1, we have:

$$\frac{d\varphi}{dt} = \frac{1}{2}\left(\tanh\frac{1}{2}\beta f\left(\varphi\right)\right) - \frac{\varphi}{2} \tag{6}$$

Stationary points are zeros of $d\varphi /dt$. Thus the key equation to determine local equilibria is:

$$\frac{d\varphi}{dt} = 0: \quad \varphi = \tanh\frac{1}{2}\beta f\left(\varphi\right) \tag{7}$$

This equation can be solved conveniently by plotting the left-hand-side and the right-hand-side on a graph, and finding their intersection. Depending on the specification of $f(\varphi)$ and the value of β, this equation may have more than one solution.

2.2 Local Social Influence: Erdős-Rényi Networks

The model described in the previous section assumes uniform, global and perfect information access. The very fact that certain influences are transferred via social interactions, and thus via social networks implies heterogeneous local information. Therefore, in the following we extend the model to explicitly model interaction networks.

Each decision-making entity n is assigned a set of "reference" decision-making entities influencing its choice. At each time step during the iteration phase, the decision-making entities look at the choices their particular reference entities made in the previous round, plus their own choice, and calculate *localized* values of the difference in systematic utility between the alternatives:

$$V_{in} - V_{jn} \equiv \beta f\left(x_{in} - x_{jn}\right) = \beta f\left(x_n\right) \qquad (8)$$

The critical difference between equations (8) and (3) is that subscript n now becomes important in determining $x_n = x_{in} - x_{jn}$. The "reference" relationships introduced here define a graph or network.

It is hypothesized that different network structures yield different system behavior. In practice however, it can be difficult to reveal the exact details of the relevant network(s) of reference entities influencing the choice of each decision-making entity. Moreover, the actual reference entities for a given decision-making entity may not be among those in the data sample. One way to test the above hypothesis theoretically even without reliable empirical information about the social influence network is by studying abstract classes of networks in the hope of identifying classes of networks that yield similar results.

An early abstract model of social interaction is due to Erdős and Rényi (1959). Their random network consists of a number of nodes and set of random edges between them, such that the probability of the existence of a given link is uniform across all possible edges. The actual number of the links is determined by the density p of the network, which is usually perceived as a parameter of the Erdős-Rényi graph. Here network density p is defined as the ratio of the number of actual existing links to the number of all theoretically possible links in a fully connected network with the given number of nodes. Otherwise said, p is the "link probability," the probability that a link exists.

One advantage of studying random networks is that they are perhaps the simplest possible networks that are general enough to describe a wide range of graphs, from unconnected nodes to a fully connected network (ie. a graph that contains all possible links). In addition, they accomplish this without introducing any explicit bias into the structure of the network. Moreover, results are known about important properties such as at approximately what value of p will the network become connected (ie. when each node is "reachable" along the edges from any other node), or otherwise said, when a so-called "giant component" will emerge. Finally, an important feature of random networks which is observed in real-life social networks is the so-called "small-world" property: the average path length l (the average number of "hops" between an arbitrary pair of nodes) is less than or of the order $\ln(N)$, where N is the number of nodes.

3 EMPIRICAL DEMONSTRATION

In the next step of our model development process we now turn our attention to an empirical application of intercity transportation mode choice behavior. Here we include individual level heterogeneity in two ways. We use revealed preference survey data collected by the Hague Consulting Group for the Netherlands Railways to assess factors which influence the binary choice between car versus rail for intercity travel (Ben-Akiva and Morikawa, 1990). We also test the role of the social influence, modeled as an Erdős-Rényi graph over a full sweep of link probabilities from zero to one. In the limit that the link probability approaches zero, we have a classical binary logit model without social interaction. In the limit that the link probability approaches one, we recover a fully-connected network. For the special case of very high link probabilities, we can therefore apply approximate theoretical benchmark results in section 2.1 to verify our agent-based model implementation.

At the outset of section 2, we discussed the notion of a "systematic" utility V_{in} that a given decision-making entity n is presumed to associate with a particular alternative i. We have considered until now the interaction effect as the only term in the systematic utility. In typical transportation applications, the systematic utility is commonly assumed to be defined by a function of observable characteristics $\mathbf{S}_n$ of the decision-making entity and observable attributes $\mathbf{z}_{in}$ of the choice alternative for a given decision-making entity. We will consider the term $V_{in} - V_{jn}$ in equation (1) to have the general form:

$$\begin{aligned} V_{in} - V_{jn} &\equiv \beta f\left(x_{in} - x_{jn}\right) = \beta f\left(x_n\right) \\ &= \beta x_n + h + \boldsymbol{\gamma}'\mathbf{S}_n + \boldsymbol{\zeta}_i'\mathbf{z}_{in} - \boldsymbol{\zeta}_j'\mathbf{z}_{jn} \end{aligned} \qquad (9)$$

where $\boldsymbol{\gamma} = [\gamma_1, \gamma_2, \ldots]'$, $\boldsymbol{\zeta}_i = [\zeta_{i1}, \zeta_{i2}, \ldots]'$ and $\boldsymbol{\zeta}_j = [\zeta_{j1}, \zeta_{j2}, \ldots]'$ are vectors of unknown utility parameters respectively corresponding to the relevant observable agent characteristics $\mathbf{S}_n$, and observable agent-specific attributes $\mathbf{z}_{in}$ of the choice alternative such that:

$$\begin{aligned} \boldsymbol{\gamma}'\mathbf{S}_n &= \gamma_1 S_{n1} + \gamma_2 S_{n2} + \ldots \\ \boldsymbol{\zeta}_i'\mathbf{z}_{in} &= \zeta_{i1} z_{in1} + \zeta_{i2} z_{in2} + \ldots \\ \boldsymbol{\zeta}_j'\mathbf{z}_{jn} &= \zeta_{j1} z_{jn1} + \zeta_{j2} z_{jn2} + \ldots \end{aligned} \qquad (10)$$

The term h is a so-called "alternative specific constant" (ASC); it is included as good practice to explicitly account for any underlying bias for one alternative over another alternative. In other words, h reflects the mean of ε_{jn} - ε_{in}, that is, the difference in the utility of alternative i from that of j when all else is equal. In general the utility parameters $\boldsymbol{\zeta}_i$ and $\boldsymbol{\zeta}_j$ may take alternative specific values, however in this paper we will consider only "generic" values of the utility parameters $\boldsymbol{\zeta} \equiv \boldsymbol{\zeta}_j = \boldsymbol{\zeta}_j$ and define $\mathbf{z}_n \equiv \mathbf{z}_{in} - \mathbf{z}_{jn}$ so that we have the further simplification:

$$V_{in} - V_{jn} = \beta x_n + h + \boldsymbol{\gamma}'\mathbf{S}_n + \boldsymbol{\zeta}'\mathbf{z}_n \qquad (11)$$

The travel behavior data available to us is cross-sectional. Due to the expense and logistical aspects of data collection, it is common that a transportation agency or other commissioning party will use cross-sectional data to estimate a model, and then make forecasts about how variables will change over time and use the revised variables to make forecasts (Ben-Akiva and Lerman, 1985). This is the approach taken in this paper. This said, it would be better to have estimated a panel model and use the panel estimates within our agent-based model if we had had panel data available. We hope this paper may serve as a call to the community for the relevance of network panel data in improving modelling efforts. Descriptions of the survey variables available for use in our modeling endeavor are given in Table 1. There are no reported missing values.

Table 1: Description of Variables in Survey Data

Name	Type of variable	Description
choice	y_n	Travel mode choice indicator: 1 if rail; 0 if car
gender	S_n	Gender of the respondent: 1 if female; 0 if male
business	S_n	Business trip indicator
shoprec	S_n	Shopping/recreation trip indicator
ttcar	z_{in}, i = car	In-vehicle travel time for the car alternative (minutes)
tccar	z_{in}, i = car	Travel cost for the car alternative (NLG)
ovtcar	z_{in}, i = car	Out-of-vehicle time to walk from parking place to destination (minutes)
ttrail	z_{jn}, j = rail	In-vehicle travel time for the rail alternative (minutes)
tcrail	z_{jn}, j = rail	Travel cost for the rail alternative (NLG)
ovtrail	z_{jn}, j = rail	Out-of-vehicle time for access and egress for rail (minutes)

As in section 2.2, in our agent-based model each decision-making entity n is assigned a set of "reference" decision-making entities influencing its choice. At each time step, the decision-making entities look at the choices their particular reference entities made in the previous round, plus their own choice, and calculate localized values of the difference in systematic utility between the alternatives:

$$\begin{aligned} V_{in} - V_{jn} &= \beta x_n + h + \gamma_1 * \text{gender}_n \\ &+ \gamma_2 * \text{business}_n + \gamma_3 * \text{shoprec}_n \\ &+ \zeta_1 * (\text{tt}_{\text{car}} - \text{tt}_{\text{rail}})_n + \zeta_2 * (\text{tc}_{\text{car}} - \text{tc}_{\text{rail}})_n \\ &+ \zeta_3 * (\text{ovt}_{\text{car}} - \text{ovt}_{\text{rail}})_n \end{aligned} \qquad (12)$$

We are interested in how the dynamics of the discrete choices of these heterogeneous individuals depend on the structure of the underlying social influence network. We vary the network density p on the parameter range (0,1) ranging from a non-connected to a fully-connected graph, excluding endpoints. We select the following 30 values of p to sample: 0.005 to 0.100 at increment 0.005 (20 network density values), 0.200 to 0.600 at increment 0.100 (5 network density values), 0.700 to 0.900 at increment 0.050 (5 network density values). In each case, we repeatedly situate the agents in 20 distinct instantiations of an Erdős-Rényi graph per network density value.

For these 600 networks (30 network density values times 20 network instantiations per density value), we repeatedly compute the local interaction field variable x_n for each of the 235 agents in the sample in two ways, one with counting the agent's own choice in its reference group (that is, with so-called "self loops"), and one without counting the agent's own choice in its reference group (that is, without self loops). From a theoretical perspective, the model with self-loops is interesting because in the limiting case of network density p = 1 (a fully-connected network), we recover Aoki's original model if there were no other explanatory variables in the utility function. Likewise from a theoretical perspective, the model without self-loops is interesting because in the limiting case of network density p = 0 (a non-connected network), we have pure random behavior if there were no other explanatory variables in the utility function. From a multi-agent based simulation perspective, the model with self-loops at low network density might logically provide inertia in the behavior, damping down the volatility of switching from one choice to another. At high network density, when the number of agents is large, there is not likely to be discernible difference in the multi-agent based simulation behavior between the model with self-loops and without self-loops.

After preliminary model specification testing, we proceed to repeatedly estimate sets of the utility parameters β, h, γ_1, γ_2, γ_3, ζ_1, ζ_2, ζ_3 in equation (12) via maximum likelihood estimation for each of the 1200 network scenarios described above, with two different binary logit utility specifications: with the alternative specific constant h freely estimated, and with the alternative specific constant constrained to zero. Using the distinct sets of coefficients for each of the 2400 estimated models, we then run 50 multi-agent based simulations with distinct pseudo-random number sequences for 2000 iterations per each run, per each model. The value of x representing the difference between the aggregate mode shares x_i and x_j in the sample at the last time step of each run is counted in histograms, one per each network instantiation. We group these histograms by network density in each of the four experimental settings (with or without self-loops and with and without an alternative specific constant). Figure 1 shows aggregate histograms from each experimental setting at low and high network density values.

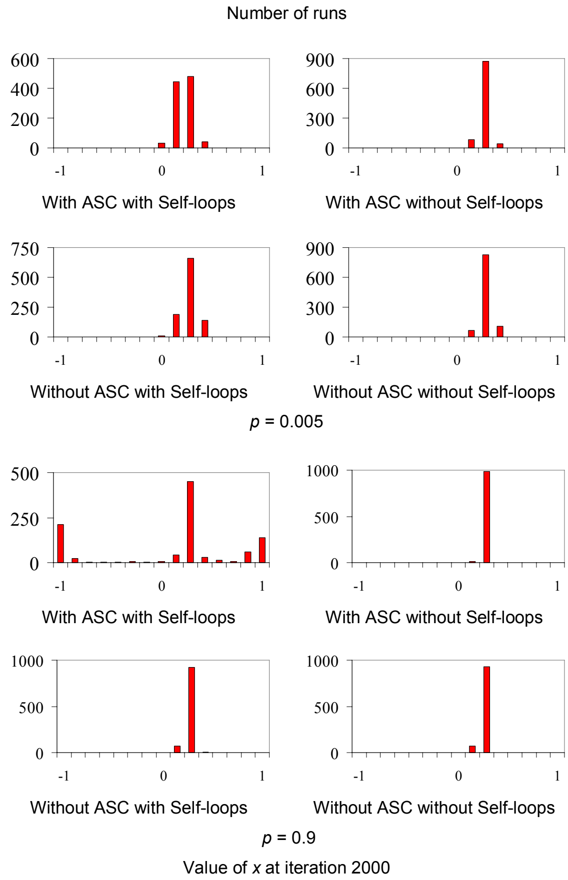

Figure 1: Aggregate histograms for four model specifications with and without an alternative specific constant and with and without self loops, at network density p = 0.005 and p = 0.9; for each histogram there are 20 Erdős-Rényi network instantiations per density value, 50 multi-agent based simulation runs per network instantiation, and 2000 iterations per simulation run; the values of the set of utility parameters are re-estimated per each network instantiation; the bins of the histogram encompass a range from -1 to 1

Molloy and Reed (1998) have shown that the critical point when a giant component emerges, occurs around $p = 1/N + \varepsilon$, where N is the number of nodes and $\varepsilon > 0$ is a small value. In our case of 235 agents this formula gives $p \sim 0.005$ as a critical point when, in practical terms, the graph becomes connected. We therefore might potentially expect to see behavioral transitions occurring somewhere in the low network densities. However, the most striking result is that only with the model with the alternative specific constant and with self loops do we ever get the signature bimodal histogram. Thus we can conclude that adding additional agent-specific heterogeneity in our model beyond the heterogeneity automatically induced by the localized interactions does indeed seem to matter.

A closer analysis of the histograms reveals a significant insight: not all of the network instantiations for the models with the alternative specific constant and with self loops give the signature bimodal histogram. In fact at higher network densities we see both the single sharply-peaked distribution and the signature bimodal histogram co-existing as outcomes with the alternative specific constant and with self loops, giving a hint at some kind of instability in the model. See Figure 2. This insight motivates a further analysis of the estimated coefficient values which we will undertake in section 4.

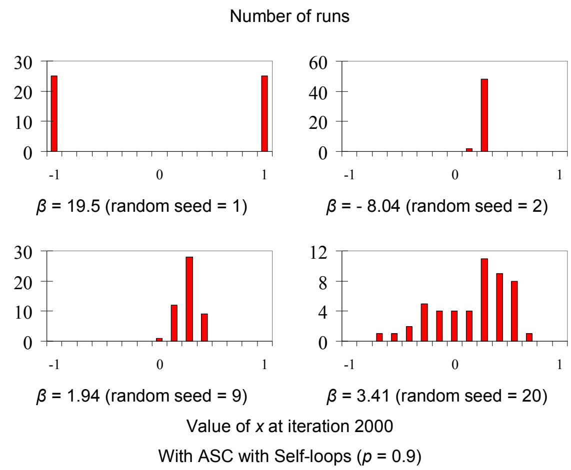

Figure 2: Individual histograms for four Erdős-Rényi network instantiations with different network generator random seeds for the model specification with an alternative specific constant and with self loops at network density $p = 0.9$; there are 50 multi-agent based simulation runs per network instantiation, and 2000 iterations per run; the value of the model coefficients are re-estimated per each network instantiation; the bins of the histogram encompass a range from -1 to 1

For the case shown in Figure 2 with $p = 0.9$, we may suppose that the network density is close enough to approaching unity that the mean field analytical results in section 2.1 may be relevant as an approximate guidepost. Using the mean values of variables given in Table 1 and the estimated utility parameters β, h, γ_1, γ_2, γ_3, ζ_1, ζ_2, ζ_3 for each of the network instantiations with random seeds shown in Figure 2, we plot the left-hand-side and the right-hand-side of equation (7) on a graph, and find their intersection for $\beta f(x_n) = V_{in} - V_{jn} = \beta xn + h + \boldsymbol{\gamma}'\mathbf{S}n + \boldsymbol{\zeta}'\mathbf{z}n$ as given in equation (12). In Figure 3, we can see that the effect of adding the alternative specific constant, the agent characteristics and the agent-specific attributes of choice alternatives to the model is to shift the tanh curve horizontally so that the curve no longer crosses the line $y = \varphi$ at $\varphi = 0$. A larger value of the certainty parameter β is accordingly necessary to achieve the signature bimodal behavior.

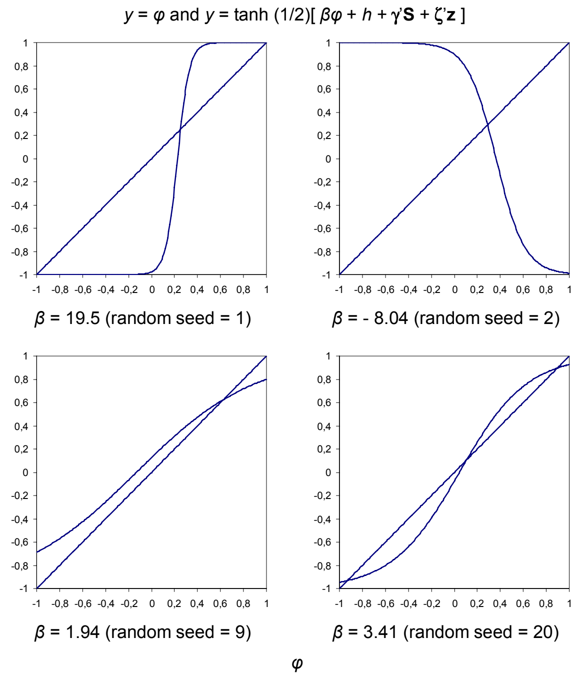

Figure 3: Plots of $y = \tanh (1/2)\ [\beta x_n + h + \boldsymbol{\gamma}'\mathbf{S}_n + \boldsymbol{\zeta}'\mathbf{z}_n]$ and $y = \varphi$ versus φ for values of estimated utility parameters β, h, γ_1, γ_2, γ_3, ζ_1, ζ_2, ζ_3 in the individual network instantiations with the same network generator random seeds shown in Figure 2; stable equilibria are seen where the tanh curves the line y = φ "from above," and unstable equilibrium are seen where the tanh curves the line y = φ "from below"; both the y-axis and φ are shown in the range from -1 to 1

4 ISSUES IN ESTIMATION

We plot the sets of estimated coefficient values for the same 30 network density values swept in section 3. Figure 4 shows the four model specifications with and without an alternative specific constant and with and without self loops (20 network instantiations per density value). Analyzing the plots, the clue to our puzzling behavior in section 3 becomes obvious in light of the analytical results in section 2.1. Far from being constant across all estimated models, we see instead systematic variation in the estimated coefficient values. From the analytical benchmark in section 2.1, we know that the coefficient on the interaction variable must be sufficiently large and positive relative to the other contributions in the utility in order to trigger the signature bimodal histogram long-run behavior. What we see is that for many of the models, this coefficient on the local interaction variable is in fact negative. In such case we can never expect to see the signature bimodal histogram.

There are two subtle issues to understand about the estimation. One issue has to do with correlation of explanatory variables. The other issue has to do with an explanatory variable or linear combination thereof being (almost) a perfect predictor for the dependent variable.

A key aspect to recognize about the local interaction variable is that in networks with a large number of agents and at high network density, the "local" interaction variable will become effectively "global", ie. constant across agents in the network - whereby this variable will become highly correlated with the value of unity included in the model when estimating an alternative specific constant. This leads to a violation in the estimation process. In fact the local interaction variable will be perfectly correlated with unity at network density $p = 1$ (a fully-connected network) when the model includes self loops. In Figure 4, we can visually track the increasing correlation as the network density increases, between the coefficient on the local interaction variable and the alternative specific constant (ASC) for rail in the models with the ASC.

When the model does not include an alternative specific constant, the local interaction variable takes on this role at high densities. Since the alternative specific constant happened to be positive for this case study in a baseline model without any interaction, simple calculation can show that the coefficient on the local interaction variable will be negative for this case study in models without an alternative specific constant for high network density, since the "local" interaction variable itself will be negative (the sample mode share for rail is less than the sample mode share for car). This is the reason why we never saw the signature bimodal histogram in section 3 for the models without an alternative specific constant at high network density.

At low network densities in models with self loops, we have a problem in that the local interaction variable will be almost a perfect predictor for the dependent variable, particularly if we do not have time series data. At high network density in models without self loops and with an alternative specific constant, we are confronted with a double effect: the local interaction variable is almost perfectly correlated with unity whereby we have a violation in estimation due to the correlation between explanatory variables, and additionally a linear combination of the local interaction variable and unity is highly correlated with the choice variable itself, leading to the second violation in estimation. This linear combination is a perfect predictor for the model without self loops and with an alternative specific constant when network density $p = 1$.

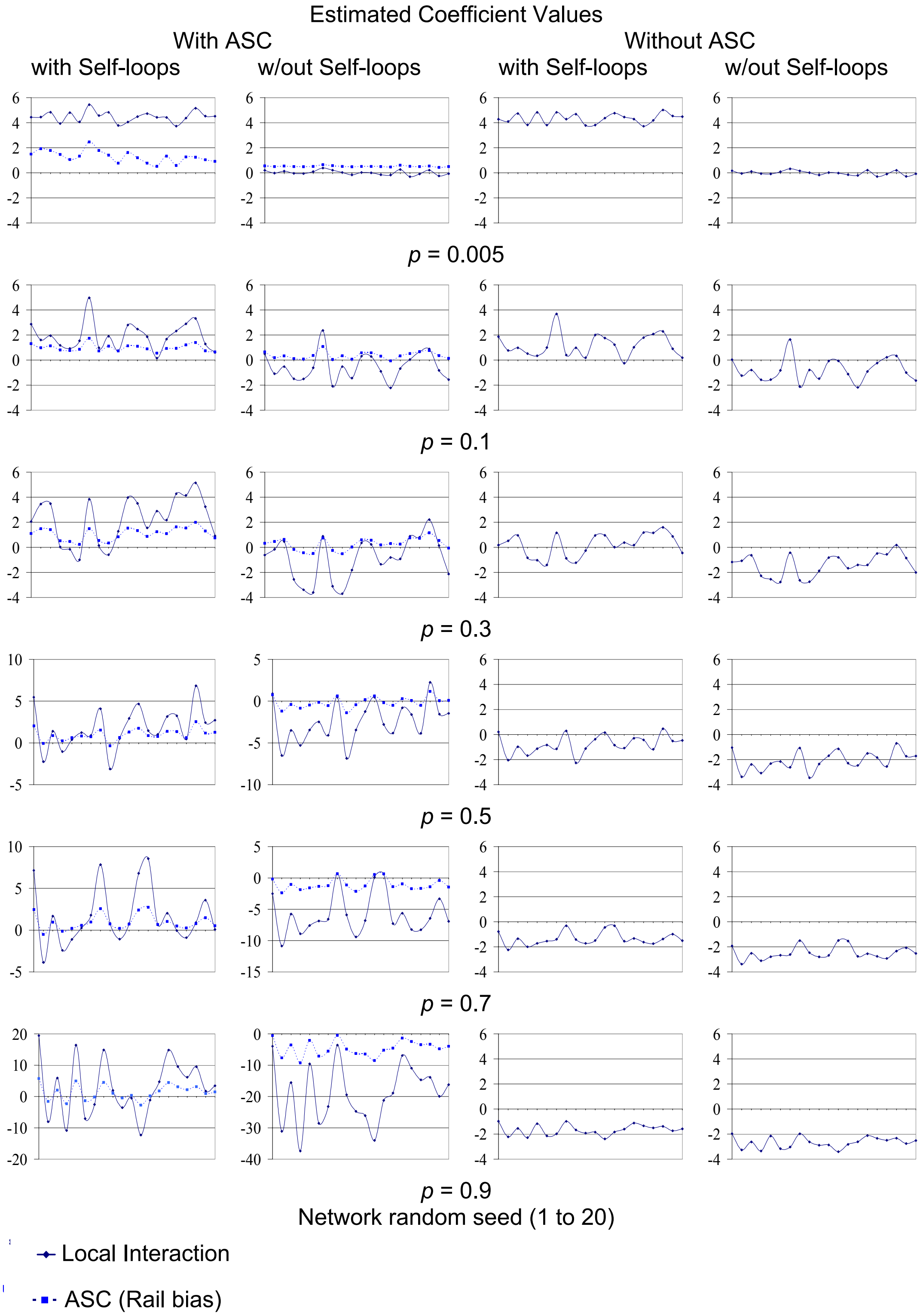

Figure 4: Estimated coefficient values for the four model specifications with and without an alternative specific constant, and with and without self loops; there are 20 network instantiations per density value for the sweep of network density from $p = 0.005$ to 0.9

5 CONCLUSIONS

In this paper, we have explored a multi-agent based model of discrete choices with interdependence of decision-makers' choices. By applying the model to an example of intercity travel demand using empirical data, we introduced individual agent heterogeneity beyond that induced by the local interaction structure. We found that the model's characteristic phase transition is dependent on network density in an example with Erdős-Rényi graphs, as well as on the importance of the estimated value of the coefficient for the local interaction variable relative to other coefficients in the binary model. Furthermore, we find that the estimation process to determine the set of coefficients can be highly sensitive to the small variations in the different instantiations, particularly in models including an alternative specific constant.

Special care must be taken in estimation of empirical models with networks with:

- very low network densities when the model includes self loops;
- very high network densities when the model includes an alternative specific constant (ASC),, especially in a model without self loops.

In general, preference goes to models with an ASC in order to ensure the error terms in the utility function have zero mean and the estimated coefficients are unbiased. Whether self-loops are implemented or not in an empirical model depends on the rationale of the system, and ideally on availability of panel data over multiple time periods.

In addition to this central contribution, we hope that our work also serves a secondary function to highlight good practice with multi-agent based social simulation. A key feature of agent-based modelling is internal verification, or otherwise said, how can the researcher be confident that the agent-based model is performing the actions that it is expected to do? What is the evidence that the programming implementation of the abstract or conceptual model is correct? To address this fact, we began our modelling endeavor with a very simplified model studied previously by others as a cornerstone, and built up our multi-agent based model step by step, and adding different layers of complexity one at a time. In our case, this meant adding different kinds of heterogeneity. In section 2.1, the dynamics of the model are driven by choices made by agents with global information. For this simple model, there is an analytical solution. The analytical benchmark gives us behavioral insights for corner solutions in parameter space, and also serve as a cross-check that the subtleties of scheduling, event simulation and sequences of random draws in our model behave as expected. In section 2.2, there is additional heterogeneity due to the network structure and the fact that agents have local information, rather than global information. We experiment with a well-known abstract class of networks to see the effect of density, and drawn on established results about connectivity in such graphs to guide behavioral hypotheses. Finally, in the empirical demonstration, we add heterogeneity due to individual characteristics of agents (gender, travel purpose) as well as agent-specific attributes of choice alternatives (travel time, travel cost).

6 RECOMMENDATIONS

In order to be able to apply the agent-based model for policy purposes, more extensive data would be desirable than what was available to us for this exploratory methodological study applying abstract classes of networks. Manski (1995) highlights three hypotheses in his classic monograph "to explain the common observation that individuals belonging to the same group tend to behave similarly... endogenous effects, wherein the propensity of an individual to behave in some way varies with the prevalence of that behavior in the group; contextual effects, wherein the propensity of an individual to behave in some way varies with the distribution of background characteristics in the group; and correlated effects, wherein individuals in the same group tend to behave similarly because they face similar institutional environments or have similar individual characteristics." The first two hypotheses express inter-agent causality in a model. The third hypothesis does not. The important distinction between the two inter-agent causal effects is that the first involves feedback that can be reinforcing over the course of time depending on the strength of the certainty parameter in relation to the rest of the utility function as we have seen in our agent-based model. The policy implications of the approaches are widely different, especially if there exists a case of an inherent dynamic with feedback. Access to temporal panel data is highly desirable in order to better empirically distinguish the effects during the estimation of the utility parameters.

In addition to the availability of empirical data on the change in the choice distribution over time, as well as changes in agent characteristics and agent-specific attributes of the choice alternatives over time, another consideration in applying the agent-based model for policy purposes is the availability of data on the possible change in the population itself, both its size and its network structure. In the agent-based model in this study, we have fixed the population in the initialization phase of the model and this population continues at each time step throughout all iterations of a simulation run until time T. In a policy application however, the links in the base population may change among existing agents, and furthermore perhaps some agents may leave and other new agents may enter.

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