Robust Control of Air-Flow in Air-Heating Set

Marek Dlapa
Department of Automation and Control Engineering
Faculty of Applied Informatics
Tomas Bata University in Zlin
nám. T. G. Masaryka 5555, 760 01 Zlín, Czech Republic
E-mail: dlapa@fai.utb.cz

KEYWORDS

Algebraic approach, controller design, PID control, structured singular value, Differential Migration, evolutionary algorithms.

ABSTRACT

The paper deals with the robust control of air-flow in air-heating set. The identification is perrfomred using step responses and approximation via second order system. The controller is designed using the structured singular value (SSV or μ) and algebraic approach. Since the cost function is nonconvex the optimization cannot be done using standard tools. Hence an evolutionary algorithm - Differential Migration (DM) is employed yielding good results in this case. The functionality of the comtroller is verified though experiments on the real plant.

INTRODUCTION

This contribution is another article documenting the power of the robust control in practical design. The robust control treats both parametric and dynamic structured uncertainties. The most advanced methods in the field are derived from Zames' small gain theorem (Zames 1981) and consequent structured singular value theory denoted SSV or μ yielding the tools for both the robust stability and performance evaluation taking into account the parametric and dynamic structured uncertainties (Doyle 1982). There are two well known methods for derivation of the controller: the *D-K* iteration (Doyle 1985) and μ-*K* iteration (Lin et al. 1993). In these two methods, the state-space formulae for the $\mathbf{H}_\infty$ suboptimal controller (Doyle et al. 1989, Gahinet and Apkarian 1994) imply the necessity of the open-loop to be stable. This requirement is a limiting factor for achieving the asymptotic tracking. The cure is the algebraic approach (Dlapa et al. 2009, Dlapa and Prokop 2010, Dlapa 2011) solving the issue and yielding simple controllers with no degradation consequent upon the simplification of the controller.

In this paper, the algebraic approach and *D-K* iteration are applied to control of the air flow in an air-heating set with nonlinear behaviour and the results are compared with other standard methods. The controllers are verified through experiments on the real plant.

Proceedings 27th European Conference on Modelling and Simulation ©ECMS Webjørn Rekdalsbakken, Robin T. Bye, Houxiang Zhang (Editors)
ISBN: 978-0-9564944-6-7 / ISBN: 978-0-9564944-7-4 (CD)

CONTROL OF AIR-HEATING TUNNEL

The laboratory plant has three control signals and seven measured quantities (see Table 1 and Figure 1 and 2). The control signals are the voltage on the bulb and the main and secondary fan. The control signals are generated by the CTRL unit connected to a standard IBM PC computer via serial port. The CTRL unit generates analogue signals to the transformation and unification unit giving the right voltage for the actuators. Similarly the analogue signals from sensors are transformed to the unified voltage 0 - 10 V.

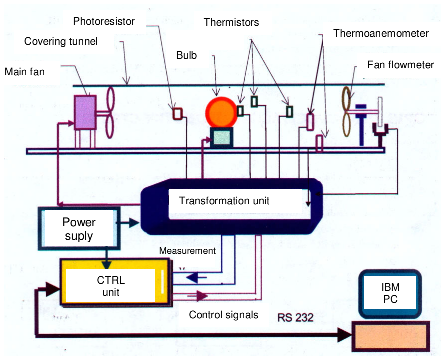

Figure 1. The scheme of air-heating tunnel

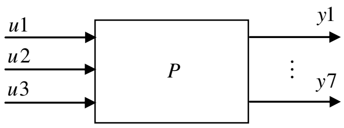

Figure 2. Inputs and outputs of the plant

Table 1: Inputs and outputs to the CTRL unit

Input channel	Sensor	Output channel	Actuator
$y1$	bulb brightness sensor	$u1$	voltage on the bulb
$y2$	temperature sensor close to the bulb T_2	$u2$	voltage on the main fan
$y3$	temperature of the bulb sensor T_3	$u3$	voltage on the adjacent fan
$y4$	temperature at the output of the tunnel sensor T_4		
$y6$	thermometer TA_6		
$y7$	fan airflow sensor		

IDENTIFICATION

The response to stepwise changes of the voltage on the main fan is in Figure 3 and 4. Via approximation of these characteristics second order transfer functions were obtained with a dead zone, which was omitted in the final model. These transfer functions make the set

$$\mathbf{P}_{27} \equiv \left\{ \frac{k}{(T_1 s+1)(T_2 s+1)} : \begin{matrix} k \in (0.58; 0.84), \\ T_1 \in (0.96; 1.86), \\ T_2 \in (1.80; 3.38) \end{matrix} \right\} \tag{1}$$

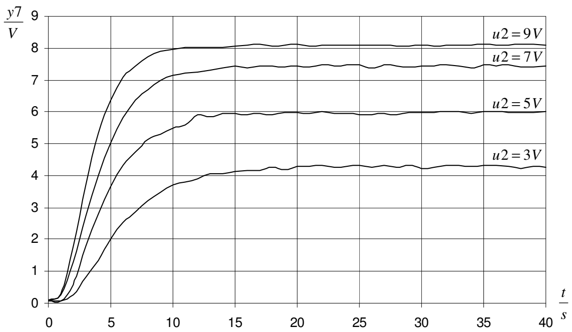

Figure 3. Responses of the measured quantity $y7$ to stepwise changes of $u2$ in the range 3-9V with other control signals at 0V

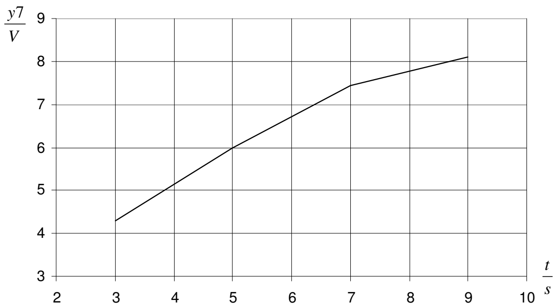

Figure 4. Static characteristics of the measured quantity $y7$ to stepwise changes of $u2$ in the range 3-9V with other control signals at 0V

μ-SYNTHESIS

The nominal plant has its parameters equal to mid values:

$$P_{27}(s) = \frac{0.71}{(1.41s+1)(2.59s+1)} \tag{2}$$

Parameter uncertainty is treated by additive and two quotient uncertainties connected in serial interconnection (see Figure 5).

The set $\mathbf{P}_{27}$ can be treated via LFT as follows

$$\tilde{P}_{27}(s,\Delta) = \frac{0.71+0.13\delta_2}{(1.41s+1)(2.59s+1)} \frac{1}{\left(1+\frac{0.45\delta_3 s}{1.41s+1}\right)\left(1+\frac{0.79\delta_4 s}{2.59s+1}\right)}, \tag{3}$$

$$\Delta = \begin{bmatrix} \delta_2 & 0 & 0 \\ 0 & \delta_3 & 0 \\ 0 & 0 & \delta_4 \end{bmatrix}, \ \delta_2, \delta_3, \delta_4 \in (-1; +1)$$

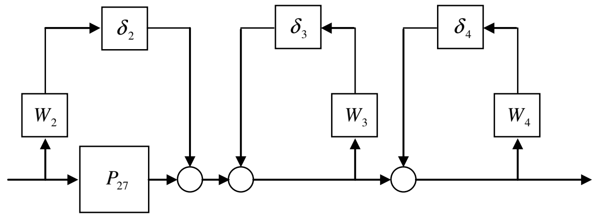

Figure 5. LFT interconnection for the family of plants $\mathbf{P}_{27}$

Denote

$$W_2(s) = \frac{0.13}{(1.41s+1)(2.59s+1)}, \ W_3(s) = \frac{0.45s}{1.41s+1},$$

$$W_4(s) = \frac{0.79s}{2.59s+1} \tag{4}$$

then

$$\tilde{P}_{27}(s,\Delta) = [P_{27}(s) + \delta_2 W_2(s)] \frac{1}{1+\delta_3 W_3(s)} \frac{1}{1+\delta_4 W_4(s)} \tag{5}$$

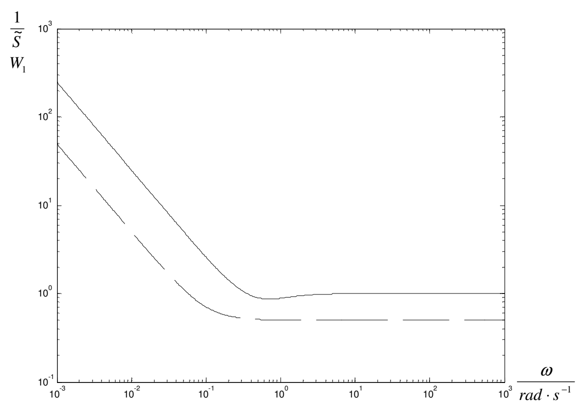

Figure 6. Bode plot of $1/\tilde{S}$ and W_1

The desired complementary sensitivity function has the form

$$\tilde{T}(s) = \frac{1}{(2s+1)^2} \tag{6}$$

which for performance condition $\|W_1 S\|_\infty < 1$ gives the weight (see Figure 6)

$$W_1(s) = \frac{10s+1}{20s+0.0001} \text{ and } \frac{10s+1}{20s} \tag{7}$$

for the *D-K* iteration and algebraic approach, respectively. The noise weight has a similar form as in previous cases

$$W_n(s) = \frac{s/0.001+1}{s/0.5+1} 0.0005 \quad (8)$$

The algebraic μ-synthesis is performed for the interconnection in Figure 7 and the perturbation matrix in the form:

$$\mathbf{\Delta} \equiv \{\text{diag}[\delta_1, \delta_2, \delta_3, \delta_4, \delta_n]: |\delta_{i,n}| < 1, \delta_{2,3,4} \in \mathbf{R}, \delta_{1,n} \in \mathbf{C}\} \quad (9)$$

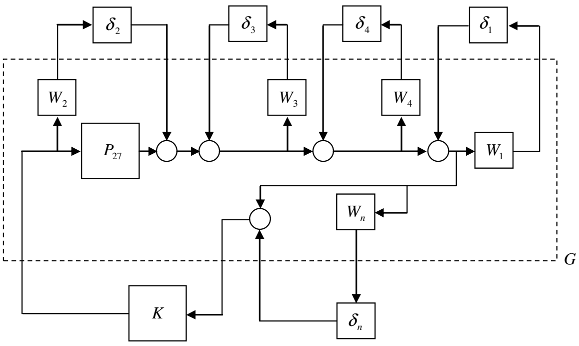

Figure 7. Interconnection for the μ-synthesis

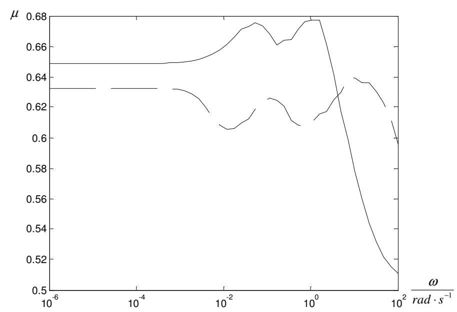

Figure 8. μ-plots for the algebraic approach (full) and the *D-K* iteration (dashed)

Stabilizing controller K satisfies the robust performance and stability condition if and only if for each frequency $\omega \in \mathbf{R}$ the structured singular value

$$\mu_\Delta[\mathbf{F}_l(\mathbf{G}, K)(j\omega)] < 1 \quad (10)$$

The aim of synthesis is to design a controller which satisfies the condition:

$$\sup_{\substack{\omega \\ K \text{ stabilizing } \mathbf{G}}} \mu_{\mathbf{\Delta}}\{\mathbf{F}_L[\mathbf{G}(\omega), K(\omega, \alpha_1, \ldots, \alpha_4)]\} \leq 1,$$
$$\omega \in (-\infty; +\infty) \quad (11)$$

where are the nominal closed loop poles in and $\mu_{\mathbf{\Delta}}$ denotes the structured singular value of LFT on generalized plant **G** and controller K with $\mathbf{\Delta}$ defined in (11).

The controller $K = N_K/D_K$ is obtained by solving the Diophantine equation

$$AD_K + BN_K = 1 \quad (12)$$

with $A, B, D_K, N_K \in \mathbf{R}_{\text{ps}}$. All feedback controllers N_K/D_K are given by

$$K = \frac{N_K}{D_K} = \frac{N_{K_0} - AT}{D_{K_0} + BT}, \; N_{K_0}, D_{K_0} \in \mathbf{R}_{\text{PS}} \quad (13)$$

where N_{K_0}, $D_{K_0} \in \mathbf{R}_{\text{PS}}$ are particular solutions of (12) and T is an arbitrary element of $\mathbf{R}_{\text{PS}}$.

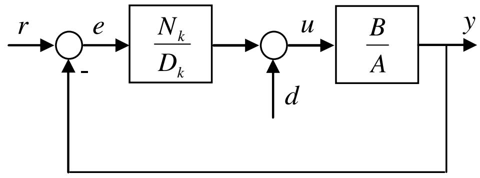

Figure 9. Nominal feedback loop

The controller K satisfying equation (12) guarantees the BIBO (bounded input bounded output) stability of the feedback loop in Figure 9. This is a crucial point for the theorems regarding the structured singular value. If the BIBO stability is held, then the nominal model is internally stable and theorems related to robust stability and performance can be used. The BIBO stability also guarantees stability of $\mathbf{F}_L(\mathbf{G}, K)$ making possible usage of performance weights with integration property implying nonexistence of state-space solutions using DGKF formulae (Doyle *et al.*, 1989) due to zero eigenvalues of appropriate Hamiltonian matrices. Such procedure results in zero steady-state error in the feedback loop with the controller obtained as a solution to equation (12). This technique is neither possible in the scope of the standard μ-synthesis using DGKF formulae, nor using LMI approach (Gahinet and Apkarian 1994) leading to numerical problems in most of real-world applications.

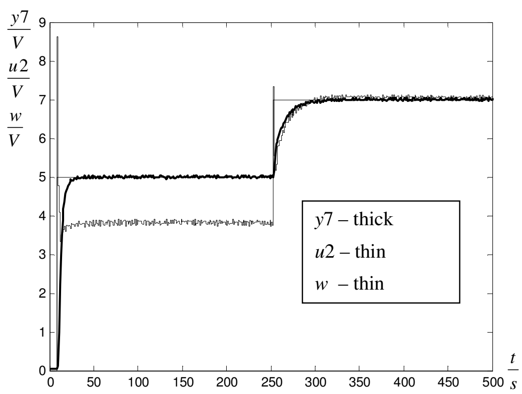

Figure 10. Control of the real plant for the algebraic μ-synthesis

In order to overcame the multimodality of the cost function (10) evolutionary algorithm Differential Migration is used (Dlapa 2009) giving the poles

$$\alpha_1 = 18.676;\ \alpha_2 = 0.787;\ \alpha_3 = 0.401;\ \alpha_4 = 0.094 \quad (14)$$

giving the proper PID controller

$$K_A(s) = \frac{17.69s^2 + 14.65s + 2.86}{s^2 + 18.86s} \quad (15)$$

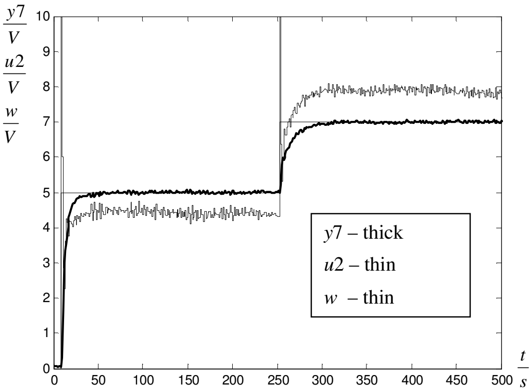

Figure 11. Control of the real plant for the *D-K* iteration

The controller obtained via the *D-K* iteration is approximated by a 5[th] order system

$$K_{D-K}(s) = \frac{0.017s^5 + 1.408s^4 + 3.896s^3 + 2.061s^2 + 0.311s + 0.001}{0.00001s^5 + 0.0015s^4 + 0.1443s^3 + 1.7942s^2 + 0.0071s} \quad (16)$$

The μ-plot and control of the real plant is in Figure 8 and 10. It is apparent that the frequency properties of the controller obtained via the algebraic approach are almost the same as for the *D-K* iteration. However, it must be considered that the *D-K* iteration uses the weight without the unstable pole on the imaginary axis. For the weight with unstable pole the $\mu \to \infty$ for $\omega \to 0$. The real plant control for both controllers are similar, however, the algebraic approach yields lower control signal at the beginning and faster tracking.

SYNTHESIS IN Rps WITH THE POLES IN ONE POINT

Synthesis in $\mathbf{R}_{ps}$ with the poles in one point is applied to the nominal plant (2) and the perturbed plant

$$P'_{27}(s) = \frac{0.84}{(1.86s + 1)(3.38s + 1)} \quad (17)$$

The minimum of $\eta(\alpha)$ occurs for $\alpha = 0.4$. The control law for this value has the transfer function

$$K_{RPS}(s) = \frac{0.686s^2 + 0.606s + 0.132}{s^2 + 0.505s} \quad (18)$$

The controller has a high overshoot for step from 0 to 5 V, for the step from 5 to 7 V there is no overshoot. No oscillation of the measured quantity is present (see Figure 12).

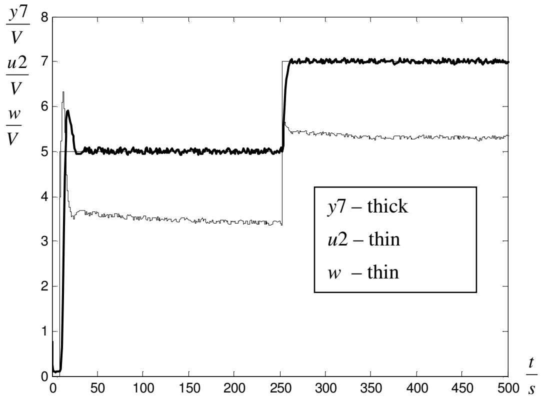

Figure 12. Control of the real plant for the design in $\mathbf{R}_{ps}$ with poles in one point

NASLIN METHOD

The Naslin method for the nominal plant P_{27} and the desired overshoot 1% ($\alpha = 2.4$) yields the PI controller

$$Q_N(s) = \frac{1.18s + 0.50}{s} \quad (19)$$

Control of the real plant exhibits a high overshoot for low temperatures and oscillations (see Figure 13).

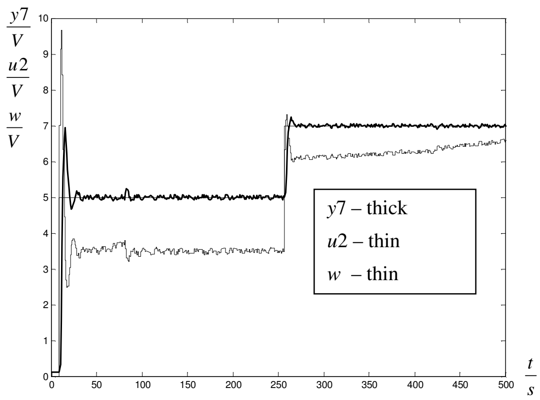

Figure 13. Control of the real plant for the Naslin method

The Naslin method has 40% overshoot for the step from 0 to 5 V (Figure 13). For the step from 5 to 7 V the overshoot decreases to 10%, which is due to different set point in the upper part of the static characteristic. Both the algebraic μ-synthesis and the *D-K* iteration yield no overshoot for both steps of the reference (Figure 10 and 11) due to considering parameter uncertainties of the controlled plant. Moreover, the measured quantity has only small oscillations from the set point. Compared to the *D-K* iteration algebraic approach has smaller control effort at the beginning and faster tracking of the reference signal.

ACKNOWLEDGEMENTS

This work was supported by the European Regional Development Fund under the Project CEBIA-Tech No. CZ.1.05/2.1.00/03.0089.

REFERENCES

Dlapa, M. 2009. "Differential Migration: Sensitivity Analysis and Comparison Study," *Proceedings of 2009 IEEE Congress on Evolutionary Computation (IEEE CEC 2009)*, pp. 1729-1736, ISBN 978-1-4244-2959-2.

Dlapa, M.; R. Prokop and M. Bakošová. 2009. "Robust Control of a Two Tank System Using Algebraic Approach," *Proceedings of EUROCAST 2009*, pp. 603-609, ISBN 978-84-691-8502-5.

Dlapa, M. and R., Prokop. 2010. Algebraic approach to controller design using structured singular value, *Control Engineering Practice*, Vol. 18, No. 4, pp. 358–382, ISSN 0967-0661.

Dlapa, M. 2011. Controller Design for a Two Tank System Using Structured Singular Value and Direct Search Methods, *18th World Congress of the International Federation of Automatic Control (IFAC)*, Milan, Italy, pp. 7511-7516, ISSN: 1474-6670.

Doyle J.C. 1982. "Analysis of feedback systems with structured uncertainties," *Proceedings of IEE, Part-D*, 129, pp. 242-250.

Doyle J.C. 1985. "Structure uncertainty in control system design," *Proceedings of 24th IEEE Conference on decision and control*, pp. 260-265.

Doyle J.C.; P.P. Khargonekar and B.A. Francis. 1989. State-space solutions to standard H_2 and H_∞ control problems, *IEEE Transactions on Automatic Control*, Vol. 34, No. 8, pp. 831-847.

Gahinet, P. and P., Apkarian 1994. A linear matrix inequality approach to H_∞ control, *International Journal of Robust and Nonlinear Control*, Vol. 4, pp. 421-449.

Lin J. L.; I. Postlethwaite and D. W. Gu. 1993. "μ-K iteration: a new algorithm for μ synthesis," *Automatica*, **29**, 219-224.

Zames G. 1981. "Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms, and approximate inverse," *IEEE Trans. Auto. Control*, **26**, pp. 301-320.

AUTHOR BIOGRAPHIES

MAREK DLAPA was born in Zlín, Czech Rep. and went to the Tomas Bata University in Zlín, where he studied chemical technology and obtained his degree in 2000. After a couple of years he was given Ph.D. degree at TBU in Zlín in 2004. Now he works as a research worker at TBU in Zlín. His web page can be found at http://dlapa.cz.