A COMPREHENSIVE FORMULATION FOR RAILROAD BLOCKING PROBLEM

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ABSTRACT

In rail transportation, there have been several attempts to determine the best routes for shipments (a number of cars with the same origin-destination) through the network. Shipments enter into several intermediate yards on their routes to separate and regroup shipments into new trains. Most of time, every entering into a yard includes excessive delay for shipments; therefore it is an ideal plan that each shipment has been transported by an exclusive train service. Mainly because of restriction on a number of running trains, this plan cannot be adapted in most cases. Finding the optimal route over the rail network with above and other common constraints would be handled by *railroad blocking problem*. From 1980 to the present, many mathematical programs have been proposed in this issue. In literature, however, there has not been any model with all realistic constraints. In this article, we give a brief literature review on railroad blocking problem. Afterwards, we present a comprehensive mixed integer programming formulation for this problem. Finally, we solve a test instance to show our model is in line with our expectation.

INTRODUCTION

A rapid, efficient, and reliable transportation service has been considered as a main factor in development of countries. In most countries, different modes of transport (including road, sea, water, and rail) have been used to provide a better customer service. Due to the intense competition between transportation providers, if customers understand any decline in the quality of delivery service in one mode, it cannot easily survive in such competition for long run. Among different modes of transportation, railway has carried a large amount of domestic freight services. Better environmental compatibility, more fuel efficiency, and more economy often lead to have an endorsement of the governments. Hence, the duty of rail mangers is not only keeping the level of rail delivery system acceptable, but also considering its further development as a part of the national transportation system.

With respect to the investment level, decision making level, and time horizon, railway transportation planning has been divided into three hierarchical levels of planning: *strategic*, *tactical*, and *operational*. In practice, considering the three levels in the form of only one problem is obviously intractable. Thus these levels have been performed individually and sequentially. In this case, we adapt the output of strategic planning as in input of tactical planning, and output of tactical planning as an input of operational planning.

At the strategic level, macro decisions (made by high-rank mangers) require a lot of investment resources, and perform at long-time horizons more than one year. The main issues which arise in the strategic planning include: developing and improving the rail network (Hooghiemstra et al. 1999); determining optimal location of yards and their capacities (Alumur and Kara 2008). Tactical planning focuses on what to do to use efficiently existing railway infrastructures and resources. At this level, most decisions have been taken for interval of one month to one season. In fact, tactical planning plays the role of a bridge between strategic and operational planning. Outline of some issues in this level includes: crew scheduling (Caprara et al. 1997), maintenance management (Shafahi and Hakhamaneshi 2009), and railroad blocking problem (Barnhart et al. 2000). Operational planning explains the short period ways of achieving the goals and detailed daily activities in the rail transportation planning. Timetabling (D'ariano et al. 2007) is discussed in this stage.

In this paper, we only consider the railroad blocking problem which is an important issue in the railroad planning process. This article is organized into six sections. We define the main terms of railroad blocking problem in section 2. Section 3 includes a brief review on relevant studies in blocking problem. In section 4 we present a mathematical formulation that aims at minimizing both user and provider costs over the network, and discuss the scale of the model in a real-size rail network. In section 5 we make use of the proposed model in a test network. We finish the article with some concluding comments and future researches in section 6.

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RAILROAD BLOCKING PROBLEM

Rail (physical) network includes a number of yards linked together by rail tracks to ship a quantity of cargo. In this paper, we only consider the classification yards which have the necessary equipment to separate, sort, and group the incoming cars and finally dispatch them by outgoing trains.

There are two extreme transportation policies for dispatching shipments over the rail network. The first policy is to dispatch the shipments with the first outgoing train from the yard. Reducing shipment lost times, and forming long-length trains are among its advantages. On the contrary, the cars should pass several intermediate yards on their routes and switch their trains to reach the final destination. Jin (1998) reported that using this policy would dramatically increase average delay per car. In the other policy, while one train (transporting only one shipment) is scheduled to depart at the origin, it will bypass any intermediate yard until it reaches to its destination. Although this policy leads to minimization of the time spent at intermediate yards, the train will haul a number of cars less than maximum car which a train can pull.

A common way to decrease the delay is to apply a combination of two above policies. This policy considers the shared arcs in the path of shipments as *blocking arcs*. In the other words, each blocking arc is defined as a virtual arc that represents several successive arcs and yards in the physical network. First and last yards of these consecutive arcs are the beginning and end node of the blocking arc, respectively. While shipments do not reach the end of a blocking arc, no classification operations would be allowed. Set of all blocking arcs a shipment meets from the origin to the destination named a *blocking path*. The *blocking network* is a collection of all blocking arcs and their corresponding start and end nodes. In the blocking network, the weight of each arc is equal to the travel time required to traverse it.

To clarify these definitions, suppose that origin and destination of one shipment are node 1 and node 4, respectively (see figure 1). Also node 2 and node 3 are intermediate classification yards. All candidate paths shipping cars over network have been shown in figure 1. For example, consider blocking path 3. A shipment starts moving from node 1, bypasses node 2, enters into node 3 for classification operations and finally proceeds its route to deliver the shipment at node 4. Obviously all candidate paths traverse the same physical arcs over the physical network, though there are clearly different blocking paths over the blocking network.

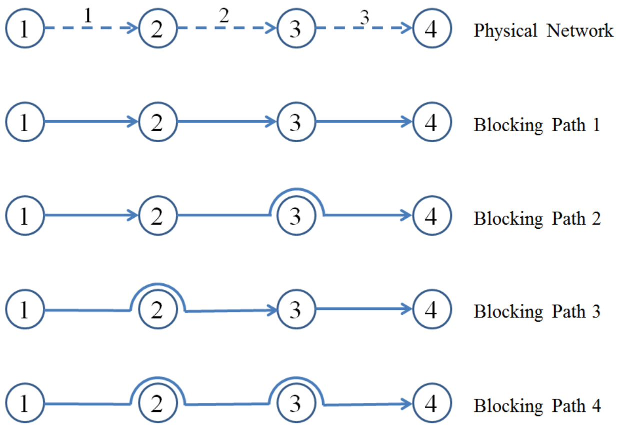


Figure 1: A physical rail network and all candidate blocking paths

LITERATURE REVIEW

Throughout the years, several researchers have proposed a variety of real-world blocking models using mathematics, and some papers also have developed solution techniques for these models. In the following, we briefly review the literature with an emphasis on modeling formulation in this field.

Assad (1980) was one of the first researchers who simplified the representation of blocking problem in the form of quantitative model. Its objective function consists of minimizing the sum of two costs: freight car and locomotive running costs; and classification costs inside yards as a linear function of incoming traffic. Assad (1980) also held the following constraints: conservation of flow which has been widely used in network optimization, and setting upper bound on the total flow moving through each blocking arc. The second constraint guaranteed that no shipment was assigned to the blocking arc not in optimal solution. This article mainly dealt with modeling formulation and just made an offer to adapt the Branch and Bound algorithm as an effective approach for solving the model exactly.

Bodin et al. (1980) proposed a mixed-integer nonlinear programming for railroad blocking problem. There are three differences between Assad (1980) and Bodin et al. (1980). At first, the former did not consider the limitation to total number of cars classifying at each yard, but latter did. Moreover, this model set the maximum on the number of blocking arcs with positive flow formed at every yard. The last difference is in the existence of delay term in objective function. The delay function, associated with incoming cars at each yard, can be presented as linear functions corresponding to the number of outgoing trains at the yard so that an increase in the number of trains dispatched from yards incurs less waiting time. Keaton (1989) proposed a mixed integer program for blocking problem. Unlike earlier efforts, he neglected the delay term and only considered the objective function as a summation of car and locomotive costs. The importance of this research is mainly due to application of Lagrangian relaxation (a technique for approximating large scale integer programming) to solve the blocking model.

Marín and Salmerón (1996) model was able to obtain the number of trains in blocking arcs in addition to the optimal blocking path of shipments. The objective function involves the investment cost for buying more running locomotives in addition to car and locomotive holding costs. This function is minimized subjected to constraints which are: conservation of car flow, restriction on a number of cars moving on all blocking arcs, and setting a number of blocking arcs formed at every yard. Imposing the number of train variables would significantly increase the computational efficiency in seeking the optimal solution. Therefore, the authors employed three most promising meta-heuristics: taboo search, simulated annealing, and descending methods.

After the Mississippi river flood in 1993, there was a report showing that if there had been a quick method to update planning the active yards on the rail system, the US government would quickly provide relief to whom affected by flood. Accordingly, Newton et al. (1998) represented a model which took into account the common railroad limitations and objective function which have been introduced in earlier efforts. The authors also proposed the algorithm base on Branch and Bound to maintain a relatively small running time of the algorithm in practice. Jin (1998) used Lagrangian relaxation technique to partition Netwon and etal (1998) model into smaller subproblems. For the real data of U.S. railroads, significant improvements have been obtained compare to Newton and etal (1998).

Fugenschuh et al. (2009) proposed the mixed integer programming problem. In addition to constraints commonly used in previous works, the authors obtained more constraints including: the upper bound on travel time for each shipment through network, limitation on the number of outcoming trains from every yard, and the allowable length and weight on each train assembled at yards. This work mainly focused on the modeling formulation and did not discuss on solution approaches.

MODEL FORMULATION

Recent formulations of blocking problem have been along with shortcomings. The main decision tools and realistic constraints that are important to rail mangers would be removed from the models mainly due to CPU-time and memory limitations. Thus primary focus of this paper is on a new formulation here for blocking problem and revising flaws in earlier models. For simplicity, we use the below notations to develop the model:

Parameters:

- φ : The value of travel time ($ per day-car),
- A : Set of all blocking arcs,
- K : Set of all shipments,
- $Q(k)$: Set of all candidate blocking paths for shipment k,
- m : Number of classification yards,
- t_a : Travel time on blocking arc *a* (day),
- PT : Planning time period (day),
- α : Given parameter (usually 2),
- ε : An arbitrarily small positive quantity,
- $\eta(a)$: The yard at starting-point of blocking arc *a*,
- $w_{\eta(a)}$: Time duration required for one car preparation to depart yard $\eta(a)$ (day),
- δ_a^q : 1 if blocking arc *a* is on path *q* and 0 otherwise,
- χ_a : Operating costs for one train running on arc *a* ($ per train),
- d^k : Number of cars belonging to shipment *k* (car),
- ξ_i^a : 1 if *i* is the origin of arc *a* and 0 otherwise,
- $W(i)$: Maximum number of blocking arcs with positive value beginning formed at yard *i*,
- $V(i)$: Maximum number of cars handled in yard *i* (car),
- $N(i)$: Maximum number of trains dispatched from yard *i* (train),
- M_a : Maximum number of trains running on blocking arc *a* (train),
- T^k : Upper bound on the transit time of shipment *k* permitted to spend in the network (day),
- u_a : Maximum number of cars can pull by one train on blocking arc *a* (car per train),
- l_k : Length of one car belonging to shipment *k* (meter per car),
- L_a : Maximum length of one train running on blocking arc *a* (meter),
- w_k : Weight of one car belonging to shipment *k* (ton per car), and
- W_a : Maximum tonnage transported on blocking arc *a* (ton).

Decision Variables:

- n_a : Number of trains running on blocking arc *a* (train),
- f_q^k : The portion of shipment *k* on blocking path *q*,

- Z_a: 1 if blocking arc a used and 0 otherwise, and
- X_q^k: 1 if shipment k used blocking path $q \in D(k)$ and 0 otherwise.

The proposed model would be formulated as the following mathematical program:

$$Min\ Z$$
$$= \varphi.\sum_{k \in K} \sum_{q \in Q(k)} \left[\sum_{a \in A} (t_a + w_{\eta(a)} + \frac{PT}{\alpha(n_a + \varepsilon)}).\delta_a^q \right].f_q^k.d^k$$
$$+ \sum_{a \in A} \chi_a.n_a \quad (1)$$

$$\sum_{q \in Q(k)} \delta_a^q.X_q^k \le |Q(k)|.Z_a \quad \forall a \in A; k \in K \quad (2)$$

$$\sum_{q \in Q(k)} f_q^k = 1 \quad \forall k \in K \quad (3)$$

$$\sum_{a \in A} \sum_{k \in K} \sum_{q \in Q(k)} \xi_i^a.\delta_a^q.d^k.f_q^k \le V(i) \quad \forall i = 1,...,m \quad (4)$$

$$\sum_{a \in A} \xi_i^a.Z_a \le W(i) \quad \forall i = 1,...,m \quad (5)$$

$$f_q^k \le X_q^k \quad \forall k \in K; q \in Q(k) \quad (6)$$

$$\left[\sum_{a \in A} (t_a + w_{\eta(a)} + \frac{PT}{\alpha(n_a + \varepsilon)}).\delta_a^q \right].X_q^k \le T^k$$
$$\forall k \in K; q \in Q(k) \quad (7)$$

$$\sum_{k \in K} \sum_{q \in Q(k)} \delta_a^q.d^k.f_q^k \le u_a.n_a \quad \forall a \in A \quad (8)$$

$$n_a \ge Z_a \quad \forall a \in A \quad (9)$$

$$\sum_{a \in A} \xi_i^a.n_a \le N(i) \quad \forall i = 1,...,m \quad (10)$$

$$\sum_{q \in Q(k)} \delta_a^q.f_q^k.l_k \le L_a.n_a \quad \forall k \in K \quad (11)$$

$$\sum_{q \in Q(k)} \delta_a^q.f_q^k.w_k \le W_a.n_a \quad \forall k \in K \quad (12)$$

$$f_q^k \ge 0; n_a \ge 0 \ and\ Integer;$$
$$Z_a, X_q^k\ binary; \forall k \in K; a \in A; q \in Q(k),$$

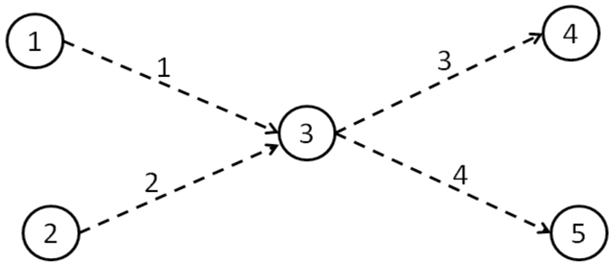
Objective function of this program, (1), consists of two parts: *user cost* and *provider cost*. The user cost is equal to monetary value of total travel time which is defined by the travel time in all blocking paths with positive car flow connecting all origins to destinations. In this component, travel time on one blocking path, q, includes three components: time period moving in track rail; time period to handle into classification yards, and waiting time to dispatch trains from yards. The provider cost is equal to the product of unit of train operating cost and number of running trains for all blocking arcs.

Constraint (2) ensures that if one blocking arc, a, does not belong to any path for all shipments, then the model will not allow to pass even one car through arc a. Constraint (3) makes all cars belonging to shipment k moving from origin to destination and not lost in network. Due to geometric design consistency and proper operations into yards, number of cars handled at every yard would be limited. Inequality (4) captures this point. The number of blocking arcs with positive flow from each yard, i, is limited to $W(i)$ by constraint (5). Constraint (6) represents the logical relationship meaning that if one car is moving on path q for shipment k, then X_q^k must be one. In other words, while path q is an active path for shipment k, at least one car must flow on it. Constraint (7) ensures that travel time for shipment k does not exceed allowable travel time. Constraint (8) makes number of cars moving in the blocking arc a depending on the number of trains running on this arc and also maximum number of cars pulled by one train at that blocking arc. Constraint (9) states that if the flow of a blocking arc is positive, then at least one train should move through the arc. Constraint (10) guarantees the number of trains dispatched from a yard, i, must be less than yard capacity, $N(i)$. Due to geometric design restriction or prohibition of railroads and yard-length requirement, standards have been made by managers on the train-length shown in constraint (11). Obviously, passing the heavy weight trains can cause sustaining damages on the railroads in a long-run. Thus providers have stated the restrictions on the weight of train sent from every yard in some countries in the form of constraint (12).

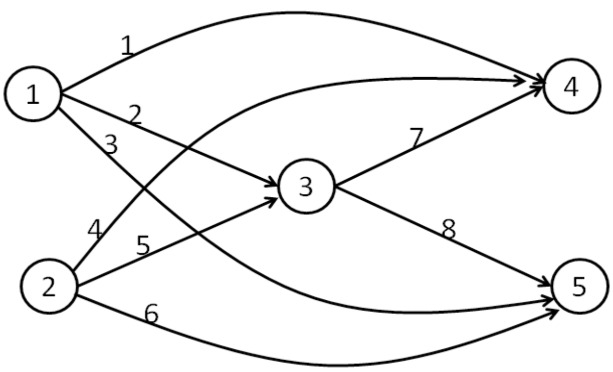
At the end of this section, we estimate the size of proposed model for a real-size network. let the network (Iran railways is in this size) which has about 100 yards (where the trains can be classified), 50 origins, 50 destinations, 5000 shipments moving over network, and consider only ten active paths between each origin-destination. Assume that we can make one blocking arc from any origin to any destination, any origin to any yard, from any yard to any other yard, any yard to any destination. In this case, the total number of candidate blocking arcs is equal to 50×50 + 50×100 + 100×100 + 100×500 = 22500. Thus the proposed model has 22500 non-negative integer variables to determine the number of train for each blocking arc; 22500 binary variables to select or not blocking arcs; 5000×10=50000 binary variables to select or not paths for shipments; 50000 non-negative flow variables; and approximately one hundred million constraints. For this case, optimization problem with this large-scale, exceed the capacity of existing solvers and software tools, especially because of nonlinearity.

AN EXAMPLE OF A SMALL SAMPLE NETWORK

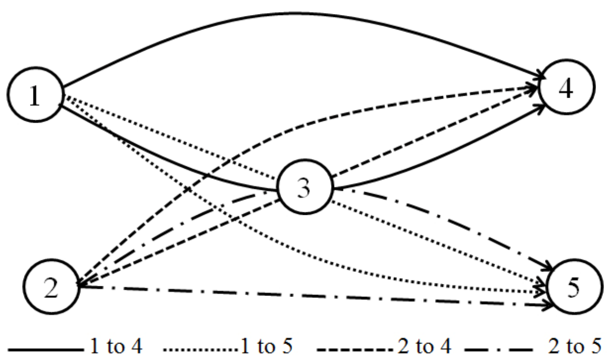
In this section, we use the proposal model on a small instance to test the model. The small physical network consists of origins (node 1 and 2), classification yards (node 1-3), destinations (node 4 and 5), and four physical arcs, shown in figure 2.a. Figure 2.b shows all blocking arcs of the sample network. For example, blocking arc 1 means the car first flows on physical arc 1, bypass the node 3, and terminate at node 4. Our model is a path-based formulation, thus we should list all feasible blocking paths for modeling shown in figure 2.c. In real-scale network, an enumeration of these paths would be nearly impossible. To overcome this problem, Barnhart et al. (2000) suggested using those paths for each OD pair which have the travel time within 150% of length of shortest path.



(a)



(b)



(c)

Figure 2: (a) Physical network, (b) blocking arc network, and (c) blocking path network

We now define the example data as follows: the number of cars is 200 cars from node 1 to node 4, 300 cars from node 1 to node 5, 400 cars from node 2 to node 4, and 500 cars from node 2 to node 5. Travel times along physical arcs 1, 2, 3, and 4 are 10, 15, 15 and 10 days, respectively. Maximum numbers of cars classified at all nodes are equal to 2000 cars. Maximum number of blocking arcs with positive flow for all nodes is assumed to be two. Train pulling capacities along arcs 1, 2, 3, and 4 are 34, 40, 40 and 20 cars, respectively. Operating costs on arcs 1, 2, 3, and 4 are 1020, 1680, 2940, and 2010 in unit of cost per car. Length and weight of one car are respectively 20 meters and 40 tons. Maximum of length and weight allowed moving along all arcs are 800 meters and 1600 tons respectively. Duration of planning is assumed to be 100 days.

The example can be solved within a 0.1 second to give the optimal solution (with objective function $904854) using *GAMS23.4* and optimal value of a number of operating trains is shown in figure 3. Results are obtained on a 2.67 GHz Dual Core computer with 4 GB free RAM.

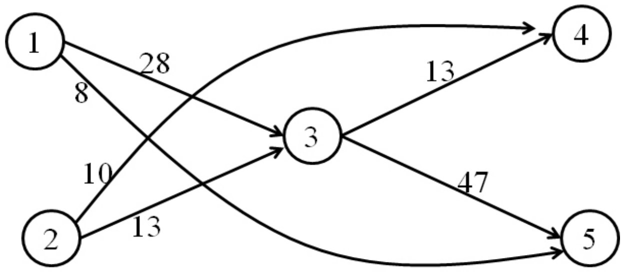


Figure 3: optimal value for n's variables in optimal solution

CONCLUSION

The blocking problem is an important problem in railway planning. By 2013, a variety of studies has been done in Operations Research for modeling this particular issue with realistic constraints. Despite several attempts, additional researches would be necessary to achieve a practical model. This research is an attempt to present the comprehensive optimization model, as a mixed-integer nonlinear program, with the realistic constraints. However the authors have shown that the application of the real networks makes the problem too large to be solved by the exiting commercial softwares. Thus the obvious future research emerging from this article is to introduce an efficient algorithm to solve the model. Effective techniques obtaining quick solutions would help the rail mangers to make their decisions. The authors are currently working on these techniques.

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