MODELLING OF BROADBAND ELECTRIC FIELD PROPAGATION IN NONLINEAR DIELECTRIC MEDIA

Matteo Conforti
PhLAM/IRCICA UMR 8523/USR 3380
CNRS-Université Lille 1
F-59655 Villeneuve d'Ascq, France
Email: conforti.matteo@gmail.com

Fabio Baronio
Dipartimento di Ingegneria dell'Informazione
Universitá di Brescia
Via Branze 38, 25123 Brescia, Italy
Email: fabio.baronio@unibs.it

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ABSTRACT

We derive unidirectional pulse propagation equations to describe extreme high-intensity and ultra-broadband optical interactions in uniaxial crystals, showing both second- and third-order nonlinear optical susceptivities. Two nonlinearly coupled first order (in the propagation coordinate) equations describe the dynamics and interactions of the ordinary and extraordinary field polarizations, and are valid for arbitrarily wide pulse bandwidth. We exploit this model to predict harmonic and supercontinuum generation in BBO crystals under strong and competing influence of quadratic and cubic susceptivities.

INTRODUCTION

In recent years there has been a great deal of interest in research on second-harmonic (SHG) (Mironov et al. 2009), high-order harmonic (HHG) (Krauzs and Ivanov 2009), and supercontinuum (SC) generation (Dudley et al. 2006) in nonlinear optical media for such diverse applications as frequency metrology, few-cycle pulse generation, spectroscopy, biological and medical analyses.

The SHG of super-strong ultrashort (tens of femtoseconds) laser pulses, using the $\chi^{(2)}$ nonlinearities in optical crystals, is a very important task, because the process can be used not only for wavelength conversion, but for significant improvement of temporal intensity contrast ratio and pulse shortening. SHG is especially important for Ti:sapphire laser facilities operating at 800 nm (Aoyama et al. 2003) and optical parametric amplifiers at 910 nm (Lozhkarev et al. 2007).

SC generation has been performed conventionally using the $\chi^{(3)}$ nonlinearities in optical fibers. Due to the high nonlinearity and engineerable dispersion available in fibers, spectra spanning multiple octaves can be achieved (Farrell et al. 2012; Fang et al. 2012). However, reaching the mid-infrared spectral region with $\chi^{(3)}$-based SC sources is challenging (Price et al. 2007). A promising alternative approach consists on the exploitation of the $\chi^{(2)}$ nonlinearities of optical crystals for SC generation (Conforti et al. 2010a; Phillips et al. 2011a). SC interactions can readily be achieved in birefringent or quasi-phase matched (QPM) crystals (Conforti et al. 2010b; Phillips et al. 2011b), with high-intensity light pulse excitation. Quadratic SC generation, difference frequency generation, optical parametric generation and QPM engineering are currently active areas of research (Conforti et al. 2007a; Zhou et al. 2012; Levenius et al. 2012).

Nowadays, technological advances in ultrafast optics have permitted the generation of ultraintense light pulses comprising merely a few field oscillation cycles. Peak intensities approaches $10^{15} W/cm^2$ (Sung et al. 2010), opening the study of an entirely new realm of nonlinear interactions in solid materials.

Beta-Barium-Borate (β-BaB_2O_4, BBO) is a very popular crystal, among all solid-state optical materials: BBO has a high damage threshold, low dispersion and $\chi^{(2)}$ nonlinearities of few pm/V allowing for efficient quadratic frequency conversion interactions (Nikogosyan 2005).

In this work, we explore the use of BBO crystals in extreme optical regimes, where dispersion effects and cubic nonlinearities play an essential role. In particular, we derive a comprehensive model to describe the propagation of extreme high-intensity and ultra-broadband optical pulses in BBO crystals. This model provides a powerful tool due to its generality and simplicity, and can be easily solved with a modest computational effort.

The paper is organized as follows. In Section 2, we recall the derivation of the master equations in uniaxial media, discussing the validity of the model. We consider both the second- and third-order nonlinear contributions, and their angular dependences. We take into account all possible second- and third-order interactions, including ones typically non-phase-matchable. In Section 3, we present some numerical examples of second harmonic generation and supercontinuum generation in BBO crystals, showing the key role of cubic susceptivity. Eventually we draw our conclusions in Section 4.

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DERIVATION OF THE MODEL

In this section we review and extend the derivation of the unidirectional nonlinear vector field equations reported in (Conforti et al. 2011) (also called Forward Maxwell Equations, FME (Housakou and Herrmann 2001), or Unidirectional Pulse Propagation Equation, UPPE (Kolesik et al. 2002)), describing the propagation of the ordinary and extraordinary polarizations of the electric field in uniaxial crystals with both $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities.

We start from Maxwell equations written in MKS units, in the reference frame $x'y'z'$

$$\nabla' \times \mathbf{E}' = -\frac{\partial \mathbf{B}'}{\partial t} \tag{1}$$

$$\nabla' \times \mathbf{H}' = \frac{\partial \mathbf{D}'}{\partial t} \tag{2}$$

$$\mathbf{B}' = \mu_0 \mathbf{H}' \tag{3}$$

$$\mathbf{D}' = \mathbf{D}'_L + \mathbf{P}'_{NL} \tag{4}$$

where $\mathbf{D}'_L$ and $\mathbf{P}'_{NL}$ account for the linear and nonlinear response of the medium, respectively. The components of the linear displacement vector for a dispersive anisotropic medium reads (assuming summation over repeated indexes)

$$D'_{L,j} = \varepsilon_0 \int_{-\infty}^{\infty} \varepsilon'_{jk}(t-t')E'_k(t')dt' \,. \tag{5}$$

In the reference frame of the principal axes of a uniaxial crystal, the dielectric permittivity tensor is the diagonal matrix $\varepsilon = diag(\varepsilon_o, \varepsilon_o, \varepsilon_e)$, where $\varepsilon_o, \varepsilon_e$ are the ordinary and extraordinary relative dielectric permittivity, respectively. The reference frame of the principal axes of the crystal $(x'y'z')$ is not convenient for the derivation of the propagation equations. We introduce a reference frame xyz that is rotated by (θ, ϕ) with respect to crystal axes. Namely, θ is the angle between the propagation vector (parallel to z) and the crystalline z' axis (the crystal optical axis), and ϕ is the azimuthal angle between the propagation vector and the $x'z'$ crystalline plane. The two reference frames are linked by the orthogonal rotation matrix A:

$$A = \begin{bmatrix} \cos\phi\cos\theta & \sin\phi\cos\theta & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\phi\sin\theta & \cos\theta \end{bmatrix}. \tag{6}$$

The dielectric permittivity tensor in the xyz frame is no longer diagonal, and it can be written as

$$\begin{aligned} \varepsilon &= A\varepsilon' A^T \\ &= \begin{bmatrix} \varepsilon_o\cos^2\theta + \varepsilon_e\sin^2\theta & 0 & (\varepsilon_o-\varepsilon_e)\cos\theta\sin\theta \\ 0 & \varepsilon_o & 0 \\ (\varepsilon_o-\varepsilon_e)\cos\theta\sin\theta & 0 & \varepsilon_o\sin^2\theta + \varepsilon_e\cos^2\theta \end{bmatrix}. \end{aligned} \tag{7}$$

In the reference frame xyz, it is possible to decompose the electromagnetic field into two linear and orthogonal polarizations of $\mathbf{D}$, both transverse to the propagation direction z (Landau and Lifshitz 1984): $\mathbf{D} = (0, D_y, 0)^T + (D_x, 0, 0)^T$. We assume the propagation of plane waves, so the electric field and displacement vectors depend upon the z coordinate (and time) only. It is worth noting that this decomposition is rigorous for linear propagation only, since the nonlinearity can rotate locally the polarization. However it is reasonable to consider the nonlinearity as a perturbative term whose effect is to couple the orthogonal polarized field vector components during propagation. If we neglect dispersion and nonlinearity, just for the moment, the electric field vector can be straightforwardly computed as:

$$\mathbf{E} = \varepsilon_0^{-1}\varepsilon^{-1}\mathbf{D} = \varepsilon_0^{-1} \begin{bmatrix} \left(\frac{\cos^2\theta}{\varepsilon_o} + \frac{\sin^2\theta}{\varepsilon_e}\right) D_x \\ \varepsilon_o^{-1} D_y \\ \frac{\varepsilon_e-\varepsilon_o}{\varepsilon_e\varepsilon_o}\cos\theta\sin\theta D_x \end{bmatrix} \tag{8}$$

By eliminating the magnetic field from Maxwell equations we obtain the vector wave equation:

$$\nabla \times \nabla \times \mathbf{E} - \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \mathbf{D}_L}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 \mathbf{P}_{NL}}{\partial t^2} \tag{9}$$

Note that obviously $\nabla \cdot \mathbf{D} = 0$, but $\nabla \cdot \mathbf{E} \neq 0$. By writing (9) in components we obtain

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 D_{L,x}}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 P_{NL,x}}{\partial t^2} \tag{10}$$

$$\frac{\partial^2 E_y}{\partial z^2} - \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 D_{L,y}}{\partial t^2} = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 P_{NL,y}}{\partial t^2} \tag{11}$$

$$0 = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2 P_{NL,z}}{\partial t^2} \tag{12}$$

The last equation witnesses the fact that the decomposition into two independent orthogonal polarizations is rigorous only in the linear case. We neglect $P_{NL,z}$, in the reasonable hypothesis of small nonlinearity.

Exploiting the relation (5) we obtain:

$$\begin{aligned} \frac{\partial^2 E_m(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\int_{-\infty}^{+\infty} E_m(z,t')\varepsilon_m(t-t')dt' \\ = \frac{1}{\varepsilon_0 c^2}\frac{\partial^2}{\partial t^2}P_{NL,m}(z,t)\,, \quad m = x, y \end{aligned} \tag{13}$$

where we have defined

$$\varepsilon_x = \left(\frac{\cos^2\theta}{\varepsilon_o} + \frac{\sin^2\theta}{\varepsilon_e}\right)^{-1} \tag{14}$$

$$\varepsilon_y = \varepsilon_o \tag{15}$$

We thus have obtained the propagation equations for an ordinary polarized wave E_y and an extraordinary polarized wave E_x.

By defining the Fourier transform $\mathcal{F}[E](\omega) = \hat{E}(\omega) = \int_{-\infty}^{+\infty} E(t)e^{-i\omega t}dt$, we can write (13) in the frequency domain:

$$\frac{\partial^2 \hat{E}_m(z,\omega)}{\partial z^2} + \frac{\omega^2}{c^2}\hat{\varepsilon}_m(\omega)\hat{E}_m(z,\omega) = -\frac{\omega^2}{\varepsilon_0 c^2}\hat{P}_{NL,m}(z,\omega), \tag{16}$$

where c is the velocity of light in vacuum, ε_0 is the vacuum dielectric permittivity, $\hat{\varepsilon}_m(\omega) = 1 + \hat{\chi}_m(\omega)$, $\hat{\chi}_m(\omega)$ is the linear electric susceptibility and $k_m(\omega) = (\omega/c)\sqrt{\hat{\varepsilon}_m(\omega)}$ is the propagation wavenumber.

We now proceed to obtain, from the second order vector wave equation (16), an equation, first order in the propagation coordinate z, describing electromagnetic fields propagating in the forward direction only. Several techniques have been proposed in literature in order to achieve a pulse propagation equation with minimal assumptions (Brabec and Krausz 2000; Housakou and Herrmann 2001; Kolesik et al. 2002; Kolesik and Moloney 2004; Genty et al. 2007; Kinsler et al. 2005; Kinsler 2010; Kumar 2010). The interested reader can find in (Kinsler 2010; Kolesik et al. 2012) an exhaustive discussion on the different derivation styles. Here we decided to follow the approach outlined in the review paper (Kolesik et al. 2012), that combines minimal assumptions and straightforward derivation.

We write the electric field components in spectral domain as the sum of a forward (F) and a backward (B) propagating part, that with our definition of the Fourier transform reads:

$$\hat{E}_m(z,\omega) = \hat{F}_m(z,\omega)e^{-ik_m(\omega)z} + \hat{B}_m(z,\omega)e^{ik_m(\omega)z}. \quad (17)$$

By plugging Ansatz (17) into (16), we get:

$$\left(\frac{\partial^2 \hat{F}_m}{\partial z^2} - 2ik_m(\omega)\frac{\partial \hat{F}_m}{\partial z}\right) e^{-ik_m(\omega)z} + \left(\frac{\partial^2 \hat{B}_m}{\partial z^2} + 2ik_m(\omega)\frac{\partial \hat{B}_m}{\partial z}\right) e^{ik_m(\omega)z} = -\frac{\omega^2}{\varepsilon_0 c^2}\hat{P}_{NL,m},$$

that can be rewritten as:

$$\frac{\partial}{\partial z}\left(\frac{\partial \hat{F}_m}{\partial z}e^{-ik_m(\omega)z} + \frac{\partial \hat{B}_m}{\partial z}e^{ik_m(\omega)z}\right) - -ik_m(\omega)\left(\frac{\partial \hat{F}_m}{\partial z}e^{-ik_m(\omega)z} - \frac{\partial \hat{B}_m}{\partial z}e^{ik_m(\omega)z}\right) = -\frac{\omega^2}{\varepsilon_0 c^2}\hat{P}_{NL,m}, \quad (18)$$

from where it is trivial to see that vector wave equation (16) is satisfied exactly, if the forward and backward components satisfy the following first order equations:

$$\begin{aligned} \frac{\partial \hat{F}_m(z,\omega)}{\partial z} &= -\frac{i}{2k_m(\omega)}\frac{\omega^2}{\varepsilon_0 c^2}\hat{P}_{NL,m}(z,\omega)e^{+ik_m(\omega)z} \\ \frac{\partial \hat{B}_m(z,\omega)}{\partial z} &= +\frac{i}{2k_m(\omega)}\frac{\omega^2}{\varepsilon_0 c^2}\hat{P}_{NL,m}(z,\omega)e^{-ik_m(\omega)z}. \end{aligned} \quad (19)$$

It is worth noting that up to this point we did not make any assumptions, so the model is equivalent to the starting equations. Equations (19) represent a nonlinear boundary value problem that cannot be solved with direct methods, but must be solved iteratively. However in the great majority of cases of interest, we can assume that (i) there are no reflections and (ii) that nonlinear polarization does not couple forward and backward waves (perturbative regime). In this case we can assume $\hat{B}_m(z,\omega) \approx 0$ and Eqs. (19), through (17), reduce to the Forward Maxwell Equations:

TABLE I

EFFECTIVE QUADRATIC NONLINEAR COEFFICIENTS. $d_{22} = 2.2$PM/V, $d_{31} = 0.04$PM/V.

Coefficient	Expression	Interaction
d_0	$-3d_{31}\cos^2\theta\sin\theta - d_{22}\cos^3\theta\sin 3\phi$	eee
d_1	$-d_{22}\cos 3\phi\cos^2\theta$	eeo, oeo, oee
d_2	$-d_{31}\sin\theta + d_{22}\cos\theta\sin 3\phi$	ooe, eoo, oeo
d_3	$d_{22}\cos 3\phi$	ooo

$$\frac{\partial \hat{E}_m(z,\omega)}{\partial z} + ik_m(\omega)\hat{E}_m(z,\omega) = -i\frac{\omega}{2\varepsilon_0 c n_m(\omega)}\hat{P}_{NL,m}(z,\omega). \quad (20)$$

We consider an instantaneous nonlinear polarization composed of a quadratic and cubic parts (summation over repeated indexes is assumed)

$$P'_{NL,j} = \varepsilon_0\left(\chi^{(2)}_{jkl}E'_kE'_l + \chi^{(3)}_{jklm}E'_kE'_lE'_m\right), \quad (21)$$

where $\chi^{(2)}_{jkl}$ and $\chi^{(3)}_{jklm}$ are the second and third order nonlinear susceptivity tensors, that are usually given in the crystal axes reference frame. In order to obtain the effective nonlinearity (Midwinter and Warner 1965a; Midwinter and Warner 1965b), we have to rotate the polarization vector with matrix A, following the prescription

$$\mathbf{P}_{NL}(\mathbf{E}) = A\mathbf{P}'_{NL}(A^T\mathbf{E}). \quad (22)$$

After some calculations, we can write:

$$\begin{aligned} \frac{\partial \hat{E}_x}{\partial z} + ik_x(\omega)\hat{E}_x &= \frac{-i\omega}{cn_x(\omega)}\hat{P}_x \\ \frac{\partial \hat{E}_y}{\partial z} + ik_y(\omega)\hat{E}_y &= \frac{-i\omega}{cn_y(\omega)}\hat{P}_y \end{aligned} \quad (23)$$

where the nonlinear terms P_x, P_y read as follows:

$$\begin{aligned} P_x &= d_0E_x^2 + 2d_1E_xE_y + d_2E_y^2 \\ &+ \frac{1}{2}\left(c_0E_x^3 + 3c_1E_x^2E_y + 3c_2E_y^2E_x + c_3E_y^3\right), \quad (24) \\ P_y &= d_1E_x^2 + 2d_2E_xE_y + d_3E_y^2 \\ &+ \frac{1}{2}\left(c_1E_x^3 + 3c_2E_x^2E_y + 3c_3E_y^2E_x + c_4E_y^3\right). \quad (25) \end{aligned}$$

where d_m, $m = 0, \ldots, 3$, are the effective nonlinearity for quadratic interactions, whereas c_m, $m = 0, \ldots, 4$ are the effective cubic nonlinearities. The values of the effective nonlinearity depend upon the crystal and their values can be found in literature (Nikogosyan 2005; Midwinter and Warner 1965a; Midwinter and Warner 1965b; Banks et al. 2002). In Tables I, II we report the effective nonlinearity for the crystals of class $3m$, to which BBO belongs, and specify the kind of interaction. For example, eeo ($e + e \rightarrow o$) indicates the sum frequency generation of two extraordinarily polarized electric fields (E_x) that generate an ordinarily polarized field (E_y).

Equations (23) are first order in the propagation coordinate, conserve the total field energy and retain their validity for

TABLE II

EFFECTIVE CUBIC NONLINEAR COEFFICIENTS. $c_{11} = 5.6 \cdot 10^{-22}\mathrm{M}^2/\mathrm{V}^2$, $c_{10} = -0.24 \cdot 10^{-22}\mathrm{M}^2/\mathrm{V}^2$, $c_{16} = -1.4 \cdot 10^{-22}\mathrm{M}^2/\mathrm{V}^2$ (BACHE ET AL. 2013).

Coefficient	Expression	Interaction
c_0	$c_{11}\cos^4\theta + c_{33}\sin^4\theta + \frac{3}{2}c_{16}\sin^2 2\theta$ $-4c_{10}\sin 3\phi \sin\theta \cos^3\theta$	eeee
c_1	$\frac{3}{2}c_{10}\cos 3\phi \sin 2\theta \cos\theta$	eeoe, eeeo
c_2	$-\frac{1}{3}c_{11}\cos^2\theta + c_{16}\sin^2\theta$ $+c_{10}\sin 2\theta \sin 3\phi$	ooee,eeoo
c_3	$c_{10}\cos 3\phi \sin\theta$	oooe, ooeo
c_4	c_{11}	oooo

arbitrary wide pulse bandwidth. The computational effort needed to solve these equations, by a standard split step Fourier method exploiting Runge-Kutta for the nonlinear step, is of the order of magnitude of that needed for solving the standard three-wave equations universally exploited to describe light propagation in quadratic crystals (Conforti et al. 2007b; Baronio et al. 2008; Baronio et al. 2010). However Eqs. (23) are far more general, and are equivalent to Maxwell equations when dealing with unidirectional propagation (Kolesik et al. 2002; Kolesik et al. 2012).

EXAMPLES

In this section, we first show a representative example of the modeling of supercontinuum generation by means of competing quadratic and cubic nonlinearities. Then, we present simulations of soliton compression and blue-shifted dispersive waves generation in BBO.

Supercontinuum generation

We fix the orientation angles of the BBO crystal to $\theta = 19^o$ and $\phi = 90^o$.

We consider the propagation of an ordinarily polarized pulse of duration $T = 20$ fs, peak intensity of 120GW/cm^2, central wavelength $\lambda_0 = 1200$ nm, where BBO shows normal dispersion ($\beta'' = 0.27\mathrm{ps}^2/\mathrm{m}$). Under such assumptions, considering a type I ($o + o \rightarrow e$) quadratic interaction, the mismatch is $\Delta k = k_e(2\omega) - 2k_o(\omega) = 3.3 \cdot 10^4 m^{-1}$, that give rise to an effective cascaded negative (defocusing) Kerr nonlinearity. The combination of normal dispersion and defocusing nonlinearity allows for solitary wave propagation. However, intrinsic cubic nonlinearities in the material are self-focusing and can compete with the induced quadratic self-defocusing effects (Bache et al. 2008).

The cascaded quadratic and cubic Kerr nonlinearities are expressed as $\gamma_2 = -(\frac{\omega d_{eff}}{nc})^2 \frac{1}{\Delta k}[m/V^2]$ and $\gamma_3 = \frac{3}{8}\frac{\omega\ c_{eff}}{nc}[m/V^2]$, with d_{eff} and c_{eff} effective nonlinear coefficients of Tables I, II. In the present case we find that the strongest interactions are $o + o \rightarrow e$ (quadratic), and $o+o+o+o \rightarrow o$ (cubic), so we can approximate $d_{eff} \approx d_2$ and $c_{eff} \approx c_4$.

Figure 1a shows the time domain evolution of the ordinarily polarized (o) electric field envelope during the propagation in BBO crystal. With envelope we mean the inverse Fourier transform of the positive frequency components of the spectrum. This visualization permits to have an envelope-like appearance, without fast oscillations of the carrier, but accounts of all frequency components. The input pulse undergoes a strong compression up to $z = 0.6$ mm, where the minimum pulse duration and maximum of spectral extension is achieved. Figure 1b shows the evolution of the ordinarily polarized field spectrum. The compression is due to the cascaded quadratic effects ($\gamma_2 = -14 \cdot 10^{-16} m/V^2, \gamma_3 = 6.65 \cdot 10^{-16} m/V^2$). At the compression point the ordinary polarized pulse shows trailing oscillations, and subsequently radiation is emitted at a slower group velocity: a linear dispersive wave located in the red part of the spectrum at 2400 nm (Bache et al. 2010a; Bache et al. 2010b).

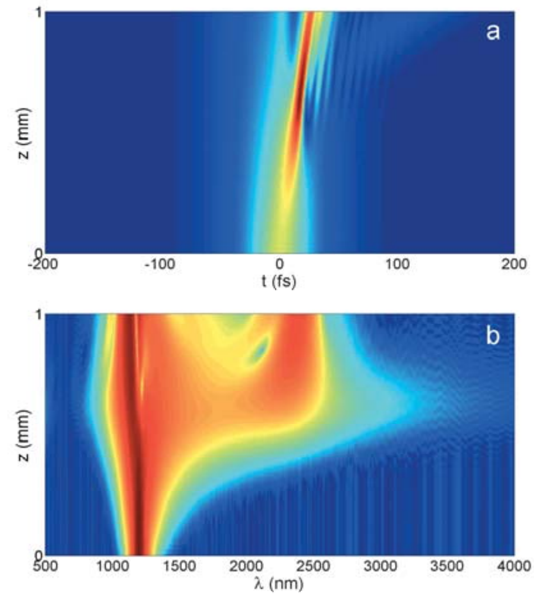

Fig. 1. (Color online) a) temporal propagation and b) field spectrum evolution (decibels) of the ordinarily polarized electric field envelope in BBO crystal. The initial pulse has duration $T = 20$ fs, wavelength $\lambda_0 = 1200$ nm, peak intensity of 120 GW/cm^2. Crystal's orientation: $\theta = 19^o$ and $\phi = 90^o$.

Then, we decrease the θ orientation angle of the BBO crystal to $\theta = 16.2^o$ ($\phi = 90^o$), keeping fixed the input pulse characteristics. In this case the dispersion is unaltered ($\beta'' = 0.27\mathrm{ps}^2/\mathrm{m}$), but the mismatch is $\Delta k = 7.1 \cdot 10^4 m^{-1}$.

Figure 2a shows the time domain evolution of the ordinarily polarized electric field during the propagation in BBO crystal, whereas figure 2b shows the evolution of the field spectrum. The scenario has been dramatically changed with respect to the previous case. In fact, the effective quadratic negative Kerr nonlinearity ($\gamma_2 = -6.6 \cdot 10^{-16} m/V^2$), induced by mismatched type I ($o + o \rightarrow e$) interaction, is perfectly balanced by the cubic nonlinearity of the medium ($o + o + o \rightarrow o$ interaction). The ordinarily polarized pulse propagates in the BBO crystal in the same way as the nonlinearities were vanishing, independently from input intensity.

Soliton compression and emission of resonant radiation

We fix the orientation angles of the BBO crystal to $\theta = 80^o$ and $\phi = 90^o$.

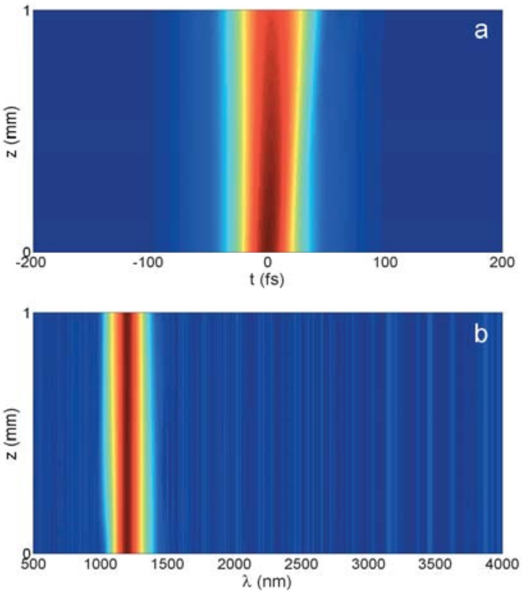

Fig. 2. (Color online) a) temporal propagation and b) field spectrum evolution (decibels) of the ordinarily polarized electric field envelope in BBO crystal. The initial pulse has duration $T = 20$ fs, wavelength $\lambda_0 = 1200$ nm, peak intensity of 120 GW/cm^2. Crystal's orientation $\theta = 16.2^o$ and $\phi = 90^o$.

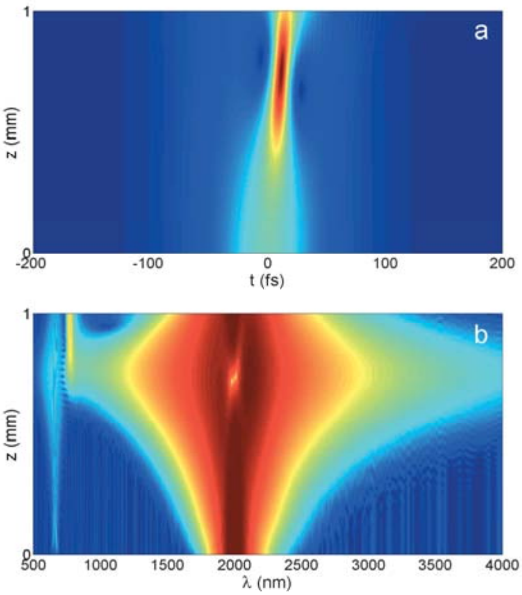

Fig. 3. (Color online) a) temporal propagation and b) field spectrum evolution of the ordinarily polarized electric field envelope in BBO crystal. The initial pulse has duration $T = 30$ fs, wavelength $\lambda_0 = 2000$ nm, peak intensity of 130 GW/cm^2. Crystal's orientation: $\theta = 80^o$ and $\phi = 90^o$.

We consider the propagation of an ordinarily polarized pulse of duration $T = 30$ fs, with intensity of 130 GW/cm^2, central wavelength $\lambda_0 = 2000$ nm, where BBO shows anomalous dispersion ($\beta'' = -0.09\text{ps}^2/\text{m}$). Under such assumptions, considering a quadratic type I ($o + o \rightarrow e$) interaction, the mismatch is $\Delta k = -6 \cdot 10^5 m^{-1}$, that give rise to an effective cascaded positive (focusing) Kerr nonlinearity. The combination of anomalous dispersion and focusing nonlinearity can allow for solitary wave dynamics. The cubic nonlinearities in the material are self-focusing too, and are stronger with respect to the induced cascaded quadratic self-focusing effects: in fact we have $\gamma_2 = 1.8 \cdot 10^{-18} m/V^2, \gamma_3 = 4 \cdot 10^{-16} m/V^2$.

Figure 3a shows the time domain propagation of the ordinarily polarized electric field during the propagation in BBO crystal, figure 3b shows the evolution of the field spectrum. The input pulse undergoes a strong compression up to $z = 0.8$ mm, where the minimum pulse duration and maximum of spectral extension is achieved. The compression is due to high-order cubic soliton excitation. A linear dispersive wave (Wai et al. 1987), located in the blue part of the spectrum at 900 nm, has been generated.

CONCLUSIONS

We have derived unidirectional pulse propagation equations to describe extreme high-intensity and ultra-broadband optical interactions in anisotropic crystals showing both quadratic and cubic nonlinear optical susceptibilities, taking BBO as the most relevant example. This model can be used to model harmonic and ultrabroadband generation in BBO crystals under strong and competing influence of quadratic and cubic susceptivities.

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Matteo Conforti was born in Brescia, Italy, in 1978. He received the Laurea degree and the Ph.D. degree in electronic engineering from University of Brescia, Brescia, in 2003 and 2007, respectively. He has been Research Fellow at the University of Brescia from 2004 to 2012. Currently he is Research Officier at PhLAM laboratory (CNRS-University of Lille). His main research interests include nonlinear optics and numerical methods for electromagnetism. He is the author and coauthor of over 100 refereed papers and conference presentations.

Fabio Baronio was born in Brescia (Italy). In March 2001, he received the Laurea degree (BS, MS) in Electronic Engineering (summa cum laude) from the University of Brescia, Italy. In March 2005 he received the PhD degree in Electronic and Telecommunication Engineering from the University of Padua, Italy. From March 2005 he is an Assistant Professor in Electromagnetic Fields at the University of Brescia. The scientific research activity of Fabio Baronio deals with Electromagnetism, Photonics and Telecommunications. He has authored or co-authored of over 100 refereed papers and conference presentations. He is a member of the Optical Society of America and of the IEEE Laser and Electro-Optics Society.