A SOCIAL INTERACTION MODEL FOR CRIME HOT SPOTS

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ABSTRACT

A common feature of mapped crime patterns is a strong spatial and temporal clustering into crime "hot spots". In this paper we explore a social interaction model for the evolution of the attractiveness of the crime environment for criminal activity. We see how hot spots may arise when the idiosyncratic attractiveness of the environment is not encouraging for criminal activity. The stability of these hot spots is determined to depend on both the size of the hot spot and the social interaction function itself.

INTRODUCTION

Criminal activities ranging from homicides to burglaries are unevenly distributed within an environment and amongst victims and offenders (Johnson et al., 2007; Johnson, 2010). The strong clustering of elevated levels of crime in space and time is often referred to as a crime "hot spot". The environment in which the crime occurs may play a role in the generation and accessibility of crime opportunities and may even provoke criminal activity. Studies of the influence of the role of these opportunities in crime occurrence date back to the nineteenth century (Weisburd et al., 2009; Johnson, 2010); yet, systematic studies of the interaction between the offender and the environment is a relatively recent pursuit in what has been coined environmental criminology (Brantingham and Brantingham, 1981).

While much focus has been placed on the mapping of crime hot spots, and sociological theories developed to account for potential causes of the formation of crime hot spots; mathematical modeling to aid in understanding the mechanisms of the genesis, spread, and dissipation of crime hot spots is in the nascent phase. Mechanistic models have explored agent based simulation and reaction diffusion approaches to the risk of victimization(Short et al., 2008, 2010), social interaction models of the propensity of the criminal agent to act(Berestycki and Nadal, 2010), and incorporating Levy flights to describe non-local movement of offenders (Chaturapruek et al., 2013).

We develop and explore a model for the nonlocal aggregation of environmental attractiveness for criminal activity. The model brings together the ideas of routine activity theory, crime pattern theory, and rational choice theory to explore how social interactions amongst recent and potential victims and offenders influence the aggregation of environmental attractiveness for criminal activities and the formation of crime hot spots. Routine activity theory asserts that societal organization from the routine activities of victims to placement of 'guardians' impacts the attractiveness of the victim for victimization (Felson, 2008). Crime pattern theory asserts that the spatial organization of crime concentration reflects the collective awareness of offenders to suitable crime opportunities (Brantingham and Brantingham, 2008). Rational choice theory asserts that the decision to continue or desist from criminal activity by the offender is based on an assessment of the relative risks and potential rewards of the criminal act as perceived by the offender (Cornish and Clarke, 2008). The model uses these theories to present a realization of the 'broken windows theory' that signs of disorder attract more disorder and diminishing those signs will diminish the attraction of disorder (Keizer et al., 2008).

The model presented is based on considerations taken in the Berestycki and Nadal model (Berestycki and Nadal, 2010); however, we perform an analysis of the equilibrium solutions when an opinion dynamic for criminal activity is formed through a social interaction of environmental factors that influence the continuance or desistance of crime at a location. The resultant model that we analyze is a reformulation of the seminal model of Amari for the formation of localized activity states in lateral inhibition type neural fields (Amari, 1977). We follow the analysis of Amari to present the necessary and sufficient conditions for the potential equilibrium solutions including hot spots and provide the corresponding taxonomy of equilibrium solutions based upon an idiosyncratic attractiveness of the environment to crime. In the Amari analysis the stability of equilibrium solutions is exhibited through considerations of the stability of the width of the equilibrium solution. Departing from Amari's analysis, we will present the stability analysis by a standard linearization technique of the

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model system. The mechanisms we incorporate in this model are generic mechanisms that could apply to many different criminal activities and we do not specialize to any one crime type. However, the model should be considered as describing the attractiveness for one crime type as the description of social interaction given here is representative of a 'communication of risk' about areas where crime has occurred.

MODEL

The object of interest in developing maps of crime hot spots is the level of criminal activity, $u(x,t)$, at some position x in the domain Ω and time t. We assume the level of criminal activity to depend on the typical perceived attractiveness of a given location, x, in the environment for criminal activity, $A(x,t)$. That is, $A(x,t)$ represents a coarse grained view of the environment that can be thought as describing a typical reward ($A > 0$) or risk ($A < 0$) for committing a criminal act at location x on time t. In line with routine activity theory, the dynamics of this attractiveness will depend on the presence of potential offenders, targets, and deterrent forces (Cohen and Felson, 1979; Felson, 2008; Berestycki and Nadal, 2010) This attractiveness variable is analogous to the risk of victimization modeled by Short *et. al.* (Short et al., 2008, 2010) and is an alternative interpretation of the propensity to act modeled by Berestycki and Nadal(Berestycki and Nadal, 2010). The spatio-temporal field $A(x,t)$ may represent, for example, general environmental cues or specific offender knowledge about the vulnerability of the area for criminal activity(Short et al., 2010).

The crime level is considered to be a non-linear function of the attractiveness of the environment

$$u(x,t) = \Lambda[A(x,t)]] \tag{1}$$

where the 'acting out' function $\Lambda[A]$ is a monotonically non-decreasing, saturating function satisfying $\Lambda[A] = 0$ for $A \leq 0$. That is, for relatively unattractive environments there is no crime and as attractiveness increases so will the crime level, approaching some maximal crime level normalized to a value of 1. For simplicity in the proceeding analysis we will consider the acting out function to be a step function

$$\Lambda[A] = \begin{cases} 0 & \text{if } A \leq 0 \\ 1 & \text{if } A > 0 \end{cases} \tag{2}$$

This choice reflects the binary nature of the decision to act out, or, perform the criminal activity.

We model the time evolution of the attractiveness of the environment incorporating an opinion dynamic represented as a social interaction term similar to that presented by Berestycki and Nadal(Berestycki and Nadal, 2010):

$$\tau \frac{\partial A}{\partial t} = -A(x,t) + W(x,t) + \int_{\Omega} j(x,x')\, u(x',t)\, dx'. \tag{3}$$

$W(x,t)$ describes the inherent level of attractiveness for criminal activity at a location x and t in the absence of criminal activity. While the inherent attractiveness of a location could be modified over time, we consider this timescale to be long relative to the dynamics of hot spot formation and consider $W(x,t)$ to be time independent. Furthermore, we will make the simplifying assumption that the environment we are considering is uniform in the inherent attractiveness; that is, $W(x,t) = w$ where w is the average value of $W(x,t)$ over the spatial domain. We consider the field to be homogeneous; that is, the weighting function depends only on the distance between locations x and $x^{'}$ and not the specific locations in the environment; that is, $j(x,x') = j(|x - x'|)$. Furthermore, the crime hot spots that we consider are stable persistent elevations of the crime level which is tantamount to an equilibrium solution of (3). Therefor in this description of the social interaction we neglect any time lag between crime events reflected in the crime level and the corresponding impact on the attractiveness of the environment. Both the temporal and spatial scale for the dynamics of attractiveness represented by τ and description of $j(|x - x'|)$ respectively, are not clear from available data. As such we rescale our time units to be in terms of the attractiveness time scale, $t \to \frac{t}{\tau}$. With this normalization τ is set to unity in equation (3). Additionally, spatial location x is given in terms of the attractiveness spatial scale.

Knox analysis of data for various types of crimes in numerous locations demonstrate an ubiquitous feature of co-occurrences of criminal events that are proximal in time and space that are significantly more common than would be expected if the occurrence of criminal activities were random events (Johnson et al., 2007; Johnson, 2010). This near repeat victimization may reflect a foraging strategy where offenders utilize knowledge from previous activities to assist in future targeting decisions (Johnson, 2010). Thus there is a communication of risk that is reflected by the weighing function for social interaction $j\left(|x - x^{'}|\right)$. The Knox analysis suggest that the strengthening of the attractiveness for future crime events is strongest at the same location as where an event has occurred and decreases as the proximity decreases. Just as knowledge of routine activities of victims in an area may increase the attractiveness of the area, so would knowledge about deterrents against criminal activities in an area decrease the attractiveness of the area. Given the information from the Knox analysis it is reasonable to assert that in areas where crime has occurred these deterrents are not as strong as the factors that would increase the attractiveness; however, at a sufficient distance from an area where crime has occurred elevations in co-occurrence of criminal activities diminish suggesting that the extent of deterrent forces to the attractiveness of an area is broader than those that would enhance the attractiveness for a criminal activity. In fact, the presence of a guardian or deterrent force in response to elevated crime levels does not necessarily displace crime to an adjacent setting (Keizer et al., 2008; Short et al., 2010) indicating a potential greater distal impact of the deterrent force relative to an attractive force. Alternatively, the broader extent could represent an optimal foraging behavior whereby offenders concentrate towards areas of high attractiveness. The opportunity for crime to occur requires the presence of both victims and offenders; hence, an absence of offenders creates a decreased attractiveness for crime to occur.

A social interaction function, $j(|x|)$, that encompasses near-repeat victimization via foraging behavior of criminals and the same mechanisms for the spread of deterrent information through the environment should have a positive local maxima at $|x| = 0$, one root preceding a local minima which is negative, and $\lim_{|x|\to\infty} j(|x|) = 0$. An example of such a function is given by the difference of Gaussians function

$$j(x) = \frac{j_1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{x^2}{2\sigma_1^2}\right) - \frac{j_2}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{x^2}{2\sigma_2^2}\right)$$

where $j_1 > j_2 > 0$ and $\sigma_2 > \sigma_1 > 0$.

Previous work (Berestycki and Nadal, 2010) considers a social interaction weighting function of the form

$$j(x, x') = j_0 + j_1(x, x') \tag{4}$$

where $j_1(x, x')$ is positive for "close" locations x and x' and zero otherwise. Under such assumptions the integral term in (3) is well approximated by a diffusive form:

$$\int_\Omega j(x, x^{'})u(x^{'}, t)dx' \approx j_0\overline{u}(t) + D\nabla^2 u(x, t)$$

where $\overline{u}$ is the average crime level over the domain. Additionally in that work social deterrence is incorporated as a separate field that modulates an effective cost to the propensity to commit a crime at a location where there is a non-zero crime level. Separate analysis are provided in that work for the case where there is no social interaction term and deterrence is purely local (i.e. no influence on deterrence level at position x from any other location in the environment) and the case where there is no deterrence and social interaction is global providing an equal influence at all locations (i.e. $j_1(x, x^{'}) \equiv 0$ in (4)).

In the subsequent analysis we will consider the domain $\Omega = \mathbb{R}^1$. With this domain, the model presented here is a reinterpretation of Amari's seminal model for describing neural fields with lateral inhibition (Amari, 1977) to describing crime dynamics. We use techniques developed for analyzing this neural field model and reformulate the results for the given application in crime dynamics.

EQUILIBRIUM SOLUTIONS

Hot spots are characterized as relatively stable localaized areas where persistent criminal activity is concentrated. As such to determine necessary and sufficient conditions for the existence and stability of hot spots we analyze equilibrium solutions of (3). At equilibrium $\frac{\partial A}{\partial t} = 0$ and corresponding equilibrium solutions satisfy

$$A(x) = w + \int_\Omega j\left(|x - x^{'}|\right) u(x')\, dx'. \tag{5}$$

From the description of the 'acting out' function (1), $u(x) = 0$ for $A(x) \leq 0$ so we define the opportunity sets for criminal activity from the set function:

$$R[A] = \{x | A(x) > 0\}$$

which is the region of the field that is considered attractive for a criminal act and hence has a non-negative crime level. With this notation we rewrite (5) as

$$A(x) = w + \int_{R[A]} j\left(|x - x^{'}|\right) dx'. \tag{6}$$

Any equilibrium solution with $R[A] = \emptyset$ (i.e. $A(x) \leq 0$ for any x) will be termed a 'quiet' or $\emptyset$-spot. Any equilibrium solution with $R[A] = (-\infty, \infty)$ (i.e. entire field is attractive to criminal activity) will be termed a 'rampant' or ∞-spot. A hot-spot is a localized region of elevated (non-zero) crime level which is represented here as a finite interval for which $A(x) > 0$. that is, $R[A] = (h_1, h_2)$. Given the homogeneity of the field, without loss of generality we can consider a hot spot of length h to satisfy $R[A] = (0, h)$ and will refer to such equilibrium solutions as h-spots.

Before identifying the necessary and sufficient conditions for the existence of $\emptyset$-, ∞-, and h-spot solutions we define some pertinent features of the integral term in equation (6),

$$J(x) = \int_0^x j(x')dx'.$$

From this definition, $J(0) = 0$ and $J(-x) = -J(x)$. We further define the quantities:

$$J_\infty = \lim_{x\to\infty} J(x)$$
$$J_m = \max_{x>0} J(x).$$

Theorem 1: Necessary and Sufficient conditions for equilibrium solutions.
a) There exists a $\emptyset$-spot if and only if $w < 0$.
b) There exists a ∞-spot if and only if $2J_\infty > -w$.
c) There exists a h-spot if and only if $w < 0$ and $h > 0$ satisfies $w + J(h) = 0$.
Proof:
a) If there exists a $\emptyset$-spot then $A(x) = w$ and $R[A] =$

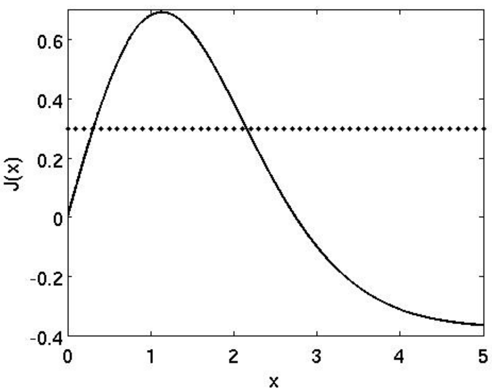


Fig. 1. Finding h-spot widths from J(x): h-spots are found from the intersection of the graphs for $J(x)$ (solid line) and $-w$ (dotted line). In this example we find two h-spots with widths $h_1 < h_2$. Parameters: $j_1 = 7.5, \sigma_1 = 1, j_2 = 8.3, \sigma_2 = 1.65$

$\emptyset$ requires $w < 0$. Conversely, if $w < 0$ then $A(x) = w$ is a $\emptyset$-spot solution.
b) If there exists a ∞-spot then $A(x) = w + \int_{-\infty}^{\infty} j(x - x')dx' = w + 2J_\infty > 0$. Conversely, if $2J_\infty > -w$ then $A(x) = w + 2J_\infty$ is a ∞-spot solution
c) If there exists a h-spot then

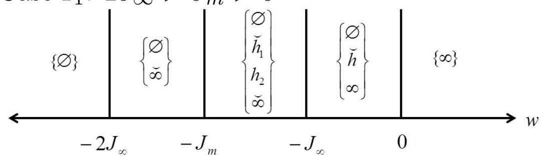
$$\begin{aligned} A(x) &= w + \int_0^h j\left(x - x'\right) dx' \\ &= w + J(x) - J(x-h). \end{aligned} \qquad (7)$$

From the continuity of $A(x)$, $A(0) = A(h) = 0$ implying $w + J(h) = 0$. Conversely if $w + J(h) = 0$ then a solution defined by (7) satisfies $A(0) = A(h) = 0$. Furthermore $\frac{dA}{dx} = j(x) - j(x-h)$ which is positive at $x = 0$ and negative at $x = h$ so $A(x) > 0$ for $0 < x < h$ and provided w is sufficiently negative, $A(x) < 0$ outside this interval. □

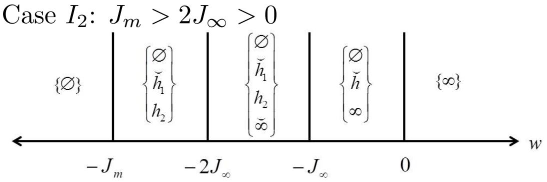
From the theorem 1 we note that the existence of the various types of equilibrium solutions depend on the interaction of the idiosyncratic attractiveness of the environment and the properties of the social interaction function. For example, to have a zero crime level requires the idiosyncratic attractiveness of the environment to reflect an expected risk to engaging in criminal activity. However this alone does not guarantee a zero crime level environment, if the net social interaction is large enough relative to the idiosyncratic attractiveness, then there is potential for rampant crime where there is a non-zero crime level throughout the environment. Theorem 1 allows us for a given social interaction function, $j(|x|)$, to develop a taxonomy of hot spot equilibrium based on the idiosyncratic level of attractiveness of the environment for criminal activity. Figure 1 shows an example of finding the width h of an h-spot from $J(x)$ and a given value for w.

To obtain a taxonomy of equilibrium solutions for varying idiosyncratic attractiveness levels of the environment we note that there are three cases to consider. In the following diagrams for each of the cases we show the sets of equilibrium solutions for various levels of idiosyncratic attractiveness w. Those equilibrium values which are not stable are indicated with a ˘ symbol above the spot width. Stability is of these solutions is shown in the next section.

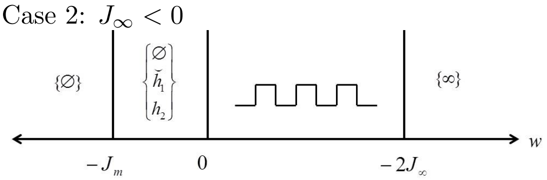
Case I_1: $2J_\infty > J_m > 0$



Case I_2: $J_m > 2J_\infty > 0$



Case 2: $J_\infty < 0$



We note for certain choices of w in all cases the field admits a bi-stability where both the quiet spot and hot spot solutions are stable. When $J_\infty < 0$ and $0 < w < -2J_\infty$ there is a multi-peak solution and no localized solution.

As noted in theorem 1, a quiet spot required $w < 0$. We can see from the taxonomy that if the idiosyncratic attractiveness is sufficiently negative a quiet spot is the only equilibrium solution. This would correspond to an environment where there is a high risk for committing a criminal activity. Achieving such an environment would be potentially resource intensive and likely too expensive to implement. As w is increased from these high risk values stable hot spot solutions become possible. Once the idiosyncratic attractiveness promotes crime, we find cases where the only equilibrium is one where crime persists everywhere in the environment. For stable hot spot solutions, it is required that the idiosyncratic attractiveness of the environment represent a risk for committing a criminal act. A risk that is enough to discourage crime without creating an environment where no crime exists may be accomplished through many guardianship mechanisms ranging from neighborhood watch programs to the criminal Justice system.

STABILITY

Hot spots of criminal activity are persistent in time implying that the h-spot solutions we have found should be stable equilibrium solutions. In this section we establish the stability of equilibrium solutions of (3) against perturbations of these equilibrium. The stability analysis performed by Amari developed a differential equation to described the evolution of the width of an equilibrium solution. Then the stability of the width of an equilibrium solution is used as a proxy for the stability of the equilibrium solution of the original system. Here, we establish the stability of the equilibrium solution by a linearization of (3) around an equilibrium solution adapting the analysis of Blomquist *et al* (Blomquist et al., 2005) to a single state variable. Before engaging this analysis we note that for quiet spots $A(x) = w$ are considered stable. Additionally, solutions can only grow to ∞-spots if $J_\infty + w > 0$; while such solutions can not be properly perturbed, $infty$-spots with $J_\infty < -w$ are not considered stable.

Let $A_e(x)$ denote an equilibrium solution of (3)

$$A_e(x) = \int_{-\infty}^{\infty} j\left(x - x'\right) \Lambda\left(A_e(x')\right) dx'.$$

We consider a perturbed state of $A_e(x)$

$$A(x,t) = A_e(x) + \chi(x,t). \tag{8}$$

For the form of the acting out function given in (2) a Taylor expansion about the equilibrium solution yields

$$\Lambda\left(A_e + \chi\right) = \Lambda\left(A_e\right) + \delta\left(A_e\right)\chi + ... \tag{9}$$

where $\delta(x)$ denotes a Dirac delta function and it is assumed $|\chi| \ll |A_e|$ so we may keep only the first two terms of the Taylor expansion. Plugging the perturbed state (8) into (3) and using the Taylor series approximation (9) we deduce a non-local evolution equation for the perturbation $\chi(x,t)$

$$\frac{\partial \chi}{\partial t} = -\chi(x,t) + \int_{-\infty}^{\infty} j(x - x')\delta\left(A_e(x')\right)\chi(x')dx' \tag{10}$$

We seek solutions of (10) of the form

$$\chi(x,t) = e^{\lambda t}\chi_1(x) \tag{11}$$

where the eigenvalues λ determine the the disturbance is growing ($\lambda > 0$) or decaying ($\lambda < 0$) indicating an unstable or stable equilibrium respectively. Plugging (11) into (10) we find

$$(1+\lambda)\chi_1(x) = \int_{-\infty}^{\infty} \delta\left(A_e(x')\right)\chi_1(x')dx'.$$

Noting that for a h-spot $|\frac{d}{dx}A_e(0)| = |\frac{d}{dx}A_e(h)| = |j(0) - j(h)|$,

$$(1+\lambda)\chi_1(x) = \frac{\left[j(x)\chi_1(0) - j(x-h)\chi_1(h)\right]}{|j(0) - j(h)|}. \tag{12}$$

We refer the reader to appendix B of Blomquist *et al* (Blomquist et al., 2005) for details of this computation. Evaluating (12) at the end points of the h-spot $x = 0$ and $x = h$, achieves the system of homogeneous linear equations, $M\vec{\chi_1} = \vec{0}$

$$\begin{bmatrix} (1+\lambda) - M_1 & M_2 \\ -M_2 & (1+\lambda) + M_1 \end{bmatrix} \begin{bmatrix} \chi_1(0) \\ \chi_1(h) \end{bmatrix} = \vec{0}$$

where,

$$M_1 = \frac{j(0)}{|j(0) - j(h)|},\ M_2 = \frac{j(h)}{|j(0) - j(h)|}.$$

As we consider non-trivial perturbations $\chi_1(x)$, we find the λ by solving $\det(M) = 0$

$$\lambda_{+,-} = \frac{\pm\sqrt{j^2(0) - j^2(h)}}{|j(0) - j(h)|} - 1.$$

The root λ_- is always negative while the root λ_+ is negative when $j(h) < 0$ and positive when $j(h) > 0$. This means that h-spot solutions with $j(h) > 0$ exhibit a saddle point instability and those with $j(h) < 0$ are stable. Unstable solutions are noted in the equilibrium solution taxonomy tables by a $\breve{}$ symbol over the corresponding spot width.

The stability of an h-spot solution depends on both the width h, and the social interaction function $j(x)$. For example we see from figure 1 that in cases where we find two h-spot equilibrium, the narrower h-spot corresponds to $j(h) > 0$ and is unstable while the broader h-spot corresponds to $j(h) < 0$ and is stable. The lack of stability of a narrow h-spot means that these spots can be dissipated by a small deterrence; whereas, dissipation of a broad h-spot would require a significant deterrence effort.

DISCUSSION

The routine activity theory makes micro and macro level assertions. On the micro-level the theory states that the convergence of potential victims and offenders in the absence of capable guardianship against the crime may lead to the emergence of crime. At a macro-level the theory asserts that features of the larger society and community or environment, may make these convergences more likely (Felson, 2008). The model presented here quantifies the capability of the guardian against the crime by an idiosyncratic attractiveness level w. In this case a lack of capable guardianship would be quantified be a positive idiosyncratic attractiveness that could potentially lead to rampant crime equilibrium solution consistent with the routine activity theory. The presence of a guardian is important to deter the occurrence of crime events; however, the presence of a guardian may not be sufficient to ensure that there is no crime. The level of capability of the guardian to deter the crime is quantified by negative idiosyncratic attractiveness. The presence of the hot-spot solutions that emerge from our model require the presence of a level of guardianship that can deter rampant crime; yet, is not so strong a deterrent as to prohibit all crime. Guardianship against crime acts can take many forms ranging from policing to alarm systems to the implementation of the criminal justice system for example. The ubiquity of hot-spots of criminal activity would imply that currently implemented levels of such guardianship are in this range of deterring rampant crime solutions while not quieting all crime. A dissipation of the hot spot would require a temporary increase in the guardianship to decrease the width of the hot-spot to below the narrow width or unstable width hot-spot solution for the idiosyncratic background level. However, to protect against the resurgence of hot-spots of criminal activity would require maintaining a high level of guardianship. Such high levels of guardianship may be cost prohibitive to implement and require levels of guardianship that are socially unacceptable. While the success of the implementation of a surge and maintain strategy to dissipate and protect against the re-emergence of crime hot-spots remains to be seen in practice, Brazil appears to be implementing such a strategy in its preparation for the 2014 World Cup (Associated Press, 2014).

In this paper we have presented a model for the non-local aggregation of environmental attractiveness for criminal activity via a social interaction mechanism. This social interaction takes the form of an opinion dynamic mimicking a voter model. In a voter model a binary decision or vote is made by one agent and neighboring agents may be influenced to change their vote based on this vote (Xia et al., 2011). In this case the vote is for the occurrence of crime based on local attractiveness that is influenced by non-local votes for crime. The influence to vote for or against crime in this manner is distance dependent via a social interaction weighting function.

We build on earlier work considering a social interaction model (Berestycki and Nadal, 2010) for the communication of risk in influencing the propensity of the offender to commit a crime activity. Rather than considering local costs for repeat victimization for a particular location, decreases in the attractiveness of the environment or deterrence for criminal activity at a location are communicated through the social interaction function. The decrease in attractiveness could be a consequence of, for example, the presence of guardians owing to the crime level or an attraction of offenders to a more attractive location for criminal activity. We analyze the existence and stability of hot spots of criminal activity as equilibrium solutions of the model when deterrent forces are non-local and attractive forces are non-constant across the whole domain. We find that hot spots exist so long as the there is an overall risk in the environment for committing a criminal activity and that a bi-stability can occur between quiet and hot spot environments.

The model studied is a reformulation of Amari's seminal model for studying lateral inhibition type neural fields (Amari, 1977). We followed Amari's analysis to give necessary and sufficient conditions for the existence of different types of equilibrium solutions including hot spot solutions and provided a taxonomy of equilibrium solutions such as provided by Amari for neural fields. However, we departed from Amari's approach in consideration of the stability of the equilibrium solutions. Our approach of a direct analysis of perturbations of equilibrium solutions for (3) provides more information about the nature of the instability that arises than Amari's approach of considering the stability of the spot width itself.

In this model we considered an environment where the inherent attractiveness of the environment is homogeneous $W(x,t) = w$. Amari considered for neural fields a stationary input stimulus, $W(x,t) = W(x) > 0$, and found that the localized activity states would be centered at local maxima of the input stimulus. In this context we could think of $W(x)$ as a function that has negative minima at areas of strong guardianship and potentially positive at areas where the environmental cues suggest a low risk for criminal activity such as in the broken windows theory (Keizer et al., 2008). We would expect in following the Amari analysis that hot spots would form around a positive local maxima of this stationary inherent attractiveness.

Additionally we have only considered the properties of one hot spot. Typically crime maps show a patterning of distinct separated crime hot spots. Amari considered the interaction of localized excitation patterns in neural fields. Following Amari's analysis we would expect in this model that for two crime hot spots that are sufficiently close the two crime hot spots will attract each other to form one crime hot spot. At a more intermediate distance the two hot spots will repel one another until they are sufficiently separated to have no influence on each other. The exact distance that hot spots separate will depend on the social interaction function.

A common policing strategy known as "cops on the dots" is to send police to the crime hot spots to provide a strong deterrence. The result of this strategy may be to either dissipate crime or locate it to another location (Braga, 2001). Recently using this strategy to dissipate a hot spot in a reaction diffusion model (Zipkin et al., 2013) and existence for traveling wave solutions in a reaction diffusion model for criminal propensity to act (Berestycki et al., 2013). Amari introduced a "two layer" version of his neural field model that mimics the cops on the dots strategy. In this extension the deterrent force would be considered a separate field variable that is excited only at the location of the crime and provides a spread of deterrent to the attractiveness field. Amari exhibited that such a two layer network can exhibit traveling wave solutions. The social interaction function presented here can be derived from such a two layer network when the deterrent layer is considered to be in a quasi-steady state with the description of the level of attractiveness.

An additional feature of crime hot spots for future consideration in this modeling framework is the influence of the topology of the environment on the formation of crime hot spots. If an offender is unaware of a target location then the offender can not commit a crime at that location. The topology of crime concentration reflects a communication of potential targets and risks (Brantingham and Brantingham, 2008; Johnson, 2010) that may be reflected in the social interaction function. Explorations of neural field equations like that of Amari that incorporate the topology of the neural network have shown new non-trivial equilibrium and traveling wave solutions (Haskell and Bressloff, 2003; Haskell and Paksoy, 2011; Salomon and Haskell, 2012, 2013). It would be interesting to further study these field equations integrating a topology that arises from environmental criminology for continued comparison and advancement of the understanding of the influence of network topology on crime hot spots.

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