PATH DEPENDENCY IN INVESTMENT STRATEGIES – A SIMULATION BASED ILLUSTRATION

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KEYWORDS
Path dependency, leverage, financial simulation, risk

ABSTRACT

In finance, the term path dependency is typically used when valuing derivative assets like American or Asian type options. Our simulation based example illustrates that the final payoff of an investment strategy could also depend on the previous historical price movements of the asset in our portfolio even if the final selling price of the asset itself is independent of it.

As illustration, we use the real life monthly return data of the shares of the Hungarian oil company (MOL), and we show that it does matter what path the stock price follows from the purchase to the date of selling if we finance our portfolio from a debt requiring regular payments throughout the holding period. In our model the investor covers the required cash outflows by selling some of the shares originally bought. Over a ten year period one may achieve a total return between -100.0 and 1,026.0 per cent depending on the path of the share quotation generated randomly by mixing real life monthly returns. In 7.95 per cent of the cases we would even go bankrupt before the 10 years are over.

INTRODUCTION

In the financial literature the term path dependency is used in different contexts. Sometimes, just as in the social sciences, we refer with this expression to the fact that our current or future payoffs, decisions or strategic options are limited or determined by our previous choices or our history (e.g. Graves (2011) on entrepreneurial finance in Southern states of US or Bianco et al. (1997) on financial systems). Pierson (2000) offers a conceptualisation of "path dependence" in this relation, while Dobusch and Kapeller (2013) contrast a number of recent articles showing the different approaches when using the term in this meaning.

In other cases we use a more narrow meaning and consider something path dependent, once the value of the given asset depends not only on the price of another item *at a set point in time*, but also on the price dynamics of the other asset *during a period of time*. (E.g. Thompson (1995) on contingent claims, Baule and Tallau (2011) on bonus certificates or Jazaerli and Saporito (2013) on the path dependence of the greeks of options.) In this article we use the expression in its later meaning, where it is not our historical decisions but a set of past events out of our control that influence our final payoff. This kind of definition in relation to investment in a project is very well presented and contrasted to real options by Adner and Levinthal (2004). The basic idea of this paper first appeared in the article of Száz (2013).

We examine a very stylized company with E=1 equity and D amount of debt. Taxes, dividends and transaction costs are ignored. Furthermore, our company does not trade, produce or offer services, but its activity is limited to investing in and holding of one given financial asset called *stock*. The only decision the company may and has to take is whether to buy the stock only using equity (*Strategy A*) or creating a leveraged portfolio (*Strategy B*). Next we describe the two strategies in detail.

Strategy A: This is our benchmark strategy. The firm takes no debt (D=0), and spends the total of its shareholder capital (the equity, E) on buying stocks. After T years we close the position and sell the stocks.

Strategy B: Taking some debt (D) with maturity of T years the company spends at the start the total of its own and borrowed capital (E+D=V) on buying stocks. Interest and principal payments of the debt have to be covered from selling the appropriate quantity of the stocks owned. After T years we close the position and sell the remaining stocks. Payments on the debt are made monthly, the interest rate is r per annum, and principal payments are made monthly in equal sums across the whole lifetime of the debt. For this article for the sake of simplicity we assume D=2.

Proceedings 28th European Conference on Modelling and Simulation ©ECMS Flaminio Squazzoni, Fabio Baronio, Claudia Archetti, Marco Castellani (Editors)
ISBN: 978-0-9564944-8-1 / ISBN: 978-0-9564944-9-8 (CD)

The relative performance of the two strategies is obviously dependent on three factors:

- the price dynamics of the stock;
- the amount of debt (D) (level of leverage: $L=V/E$);
- the interest rate of the debt (r).

A REAL LIFE EXAMPLE

For comparing the two strategies, we have chosen the actual price dynamics of an existing stock, namely MOL (Hungarian oil and gas company). The period examined is 1998-2008, hence T=10 years. Figure 1 illustrates the price dynamics of MOL shares during this period. For easier overview we rescaled the data by choosing the initial stock price on the first day to be the basis (100%).

As one may observe in the first half of the period MOL was travelling horizontally without any relevant up- or downtrend. After the first half of 2003 a strong uphill began, but some high drops also occurred. In the 10-year period, the price of MOL shares multiplied by 4.5 so the average growth rate was 16.3% p.a. This means that investing in this stock would have resulted in an annual yield of 16.3% in case of *Strategy A*.

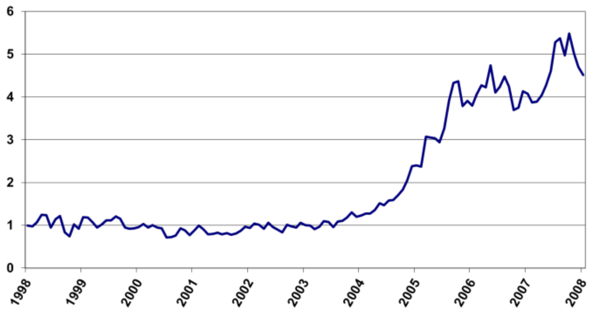

Figure 1: Price dynamics of MOL shares at BSE (monthly closing price data between 1998-2008, beginning of 1998=100%)
Source of data: Budapest Stock Exchange (www.bse.hu)

Now let us assume that *Strategy B* is available at an interest rate r=12.0%. Should we take a loan at 12.0 per cent if you can invest the money in a stock yielding 16.3%? The answer, at least at first glance, seems to be simple but it is not. Do not forget that you have to repay the loan by selling the stocks. This redemption profile makes the strategy path dependent: it is not only the final stock price, but also the interim dynamics that count.

Let us check how *Strategy B* performs if D=2. Since the stock price starts from 1, *Strategy A* buys 1, while *Strategy B* purchases 3 units of stocks. After 120 months, *Strategy A* still has 1 stock and the value of the position is 4.5. However, the balance of *Strategy B* is shocking: we have only 0.35 unit of stock, which means that we have lost almost 90 percent of our start-up portfolio and underperformed *Strategy A* significantly.

Figure 2 shows the net position dynamics of the two strategies. (Net position is the value of the stocks owned minus the outstanding debt.) *Strategy B* is above *Strategy A* in a few months at the beginning, but after the second year it remains below it all the way along. Hence, it would not have been a good strategy to finance the stock investment with an annual growth rate of 16.3% from a loan at an interest rate of 12%. The reason is the path of the stock price: loan payments during the long horizontal travelling of the stock price consume too much of our portfolio. By the time price will start to increase sharply, we have already sold approximately 75 percent of our original stock reserve.
It is also notable that the net equity value of *Strategy B* goes negative several times. In these periods outstanding debt is higher than the value of assets. Although we allowed for this in our model, in the real life typically there is a minimum (positive amount) requirement for the shareholder's capital. (E.g. if that is not met for a period of time the firm has to be liquidated under the Hungarian law.) However, as long as the number of stocks owned is above zero, we still have some chance to repay the debt, if the stock price increases significantly (as in our case). The really serious problem occurs in case we would run out of stocks before repaying the total of the debt. The number of stocks cannot increase in *Strategy B*, hence once we have sold all of them, the firm goes bankrupt for sure.

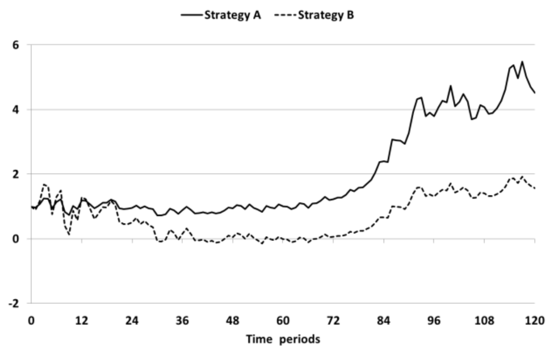

Figure 2: Net equity value of the two strategies (D=2 and r=12% in *Strategy B*)

In the rest of the paper, we seek the answer to the following questions:

- What is the *break-even interest rate* of the loan that makes the final net equity positions equal in the two strategies?
- How does the break-even interest rate depend on the size of leverage?
- What happens if we keep the initial and final stock prices, but change the path between them? How will this affect the performance and the break-even interest rate of *Strategy B*?

Break-even interest rate

Since the 12 per cent interest rate in the first numerical example was too high to make *Strategy B* profitable, the break-even interest rate must be under this level for sure. However, the exact answer was a bit surprising even for the authors: only interest rates under 5.11% could have been accepted so that we still have one stock at the end (that is, the two strategies end up with the same net position).

Table 1 collects the ultimate number of stocks in *Strategy B* for different leverage and interest rate values. The break-even interest rates are those with exactly 1 remaining stock. Some basic features of the data in Table 1 are as anticipated. For example, the higher the interest rate at a given leverage level, the less stock remains under *Strategy B*. Visually, the values are descending from the top to the bottom
An interesting result is the shaded row which shows that in case of 5.11% interest rate *Strategy B* will always have exactly 1 stock at the end. With other words, *the break-even interest rate is independent of the leverage*. In what follows, we show that this relationship is general and not the result of our specific data. Formalising the problem we have the following equations.

Periodic principal payment of the loan:

$$\mathrm{P_t} = \mathrm{P} = \frac{\mathrm{D_0}}{\mathrm{nT}} \tag{1}$$

Outstanding principal of the loan:

$$D_t = D_{t-1} - P_t = D_{t-1} - \frac{D_0}{nT} = \frac{nT-t}{nT} D_0 \tag{2}$$

Periodic interest payment on the loan:

$$I_t = \frac{r}{n} D_{t-1} = \frac{r}{n}\frac{nT-t+1}{nT} D_0 \tag{3}$$

Number of shares in *Strategy A*:

$$N_{t,A} = N_A = \frac{E_0}{S_0} \tag{4}$$

Number of shares in *Strategy B*:

$$N_{t,B} = \frac{E_0+D_0}{S_0} - \sum_{i=1}^{nt} \frac{P_i+I_i}{S_i} \tag{5}$$

where r is the annual interest rate, T is the number of years examined, n is the number of payment periods within a year, $t=0, 1, \ldots, nT$ is the index of time periods, and S is the price of the stock.

Our goal is to find the interest rate that makes the two strategies indifferent, hence where $N_{nT,A} = N_{nT,B}$. Substituting the previous equations will yield the following formula for r:

$$\frac{E_0}{S_0} = \frac{E_0 + D_0}{S_0} - \sum_{i=1}^{nT} \frac{P_i + I_i}{S_i}$$

$$\frac{D_0}{S_0} = \sum_{i=1}^{nT} \frac{P_i + I_i}{S_i}$$

$$\frac{D_0}{S_0} = \sum_{i=1}^{nT} \frac{D_0 + (\frac{r}{n})(nT - i + 1)D_0}{nTS_i}$$

$$\frac{nT}{S_0} = \sum_{i=1}^{nT} \frac{1 + (\frac{r}{n})(nT - i + 1)}{S_i}$$

$$r = n\frac{\frac{nT}{S_0} - \sum_{i=1}^{nT}\frac{1}{S_i}}{(nT+1)\sum_{i=1}^{nT}\frac{1}{S_i} - \sum_{i=1}^{nT}\frac{i}{S_i}} \tag{6}$$

Table 1: Number of remaining stocks in *Strategy B*

Interest rate	Leverage (V/E) 1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00
0%	1.060	1.121	1.181	1.241	1.302	1.362	1.422	1.483	1.543	1.603	1.664	1.724
1%	1.049	1.097	1.146	1.194	1.243	1.291	1.340	1.388	1.437	1.485	1.534	1.582
2%	1.037	1.073	1.110	1.147	1.184	1.220	1.257	1.294	1.330	1.367	1.404	1.440
3%	1.025	1.050	1.075	1.100	1.124	1.149	1.174	1.199	1.224	1.249	1.274	1.299
4%	1.013	1.026	1.039	1.052	1.065	1.078	1.091	1.105	1.118	1.131	1.144	1.157
5%	1.001	1.003	1.004	1.005	1.006	1.008	1.009	1.010	1.011	1.013	1.014	1.015
5.11%	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6%	0.989	0.979	0.968	0.958	0.947	0.937	0.926	0.915	0.905	0.894	0.884	0.873
7%	0.978	0.955	0.933	0.910	0.888	0.866	0.843	0.821	0.799	0.776	0.754	0.731
8%	0.966	0.932	0.897	0.863	0.829	0.795	0.761	0.726	0.692	0.658	0.624	0.590
9%	0.954	0.908	0.862	0.816	0.770	0.724	0.678	0.632	0.586	0.540	0.494	0.448
10%	0.942	0.884	0.826	0.769	0.711	0.653	0.595	0.537	0.479	0.422	0.364	0.306
11%	0.930	0.861	0.791	0.721	0.652	0.582	0.512	0.443	0.373	0.303	0.234	0.164
12%	0.919	0.837	0.756	0.674	0.593	0.511	0.430	0.348	0.267	0.185	0.104	0.022

Since D does not appear in the last formula, the break-even interest rate is indeed independent from the leverage. Rather it depends on the length of the total investment T, the frequency of cash payments needed n and the path of the share prices S_i.
However, it is important to emphasize that the leverage is not irrelevant for the final net position of Strategy B. Higher leverage increases our profit in case r is lower that the break-even level, and decreases it if r is above the determined level.

The intuition behind the irrelevance of D to the break-even interest rate is the following. The break-even interest rate is the interest rate where D is neutral (the two strategies result the same final position), hence its level does not count in this problem. An in-depth understanding might be achieved if we imagine a firm financed only from one source (D) for which the cost of capital needs to be paid monthly. With a given price development path we can determine whether it is worth investing in the share. Our final decision would be independent of the actual quantity of capital as both return and cost of capital would be proportionate to the investment. Whether it is worth taking this capital or not is also independent of whether we have some extra capital (E) without regular cost of capital to pay. The leverage itself would only determine the extent of the effect, the direction of the effect is derived from the comparison of the path (series of returns) and the cost of capital.

PATH DEPENDENCY

We have already observed that the performance of *Strategy B* is path dependent: it is not enough to know that the price of the stock increases by 16.3% annually; it is also important how this average growth rate is achieved. Path dependency is a well-known term in connection with derivative products. American, Asian or other exotic options are path dependent since their payoff (and consequently their price) depends on the dynamics of the underlying product's price.
A great description of path dependent derivatives might be found in Wilmott (2006). Some aspects of the link between price dynamics and optimal hedging is described by Dömötör (2012, pp. 73-81.) for currency futures. Ruttiens (2013) describes the same path dependency problem when arguing that volatility is not always a good measure of risk.

Now we will illustrate how path dependency influences a simple leveraged stock investment, namely *Strategy B*. We will not deal with all the possible or probable ways that the stock price could have taken between the initial 1 and the final 4.5 values. (The number of such paths is of course infinite.) We are only examining paths with the same monthly logreturns as those actually occurred.

We determine the monthly logreturns (y_t) of the stock in the common way:

$$y_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{7}$$

Basic descriptive statistics of the monthly logreturns in our sample are summarized in Table 2.

Table 2: Descriptive statistics of the logreturns

Minimum	-38.01%
Maximum	32.01%
Mean	1.26%
Standard deviation	10.57%

Extreme paths

We will show the significance of path dependency by changing the order of occurrence of the logreturns. First, we discuss two extreme trajectories, and sort the actual logreturns in ascending and descending order. Figure 3 shows the actual and the two extreme paths of the stock price. Since the initial value and the set of monthly logreturns are the same in all the three cases, the stock price will always be 4.5 at the end and the average annual growth rate is also unchanged (16.3%). However, rearranging the logreturns causes massive variations in the paths, so much so that we had to apply different axis-scaling for the descending scenario.

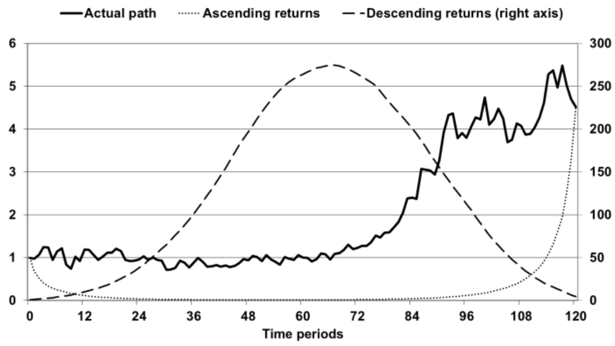

Figure 3: Paths of the stock price with actual and ascending (left axis), and descending (right axis) order of given logreturns

It is trivial that with given logreturns the descending scenario is the best, while the ascending one is the worst case for *Strategy B*. In these extreme scenarios, with D=2 and r=12%, the leveraged strategy ends up with 2.82 and -88.12 stocks respectively. The negative sign of the second data shows that in the worst case scenario we run out of stocks and cannot repay the loan. In other words we go bankrupt before the end of the investment period.

Actually, we go bankrupt very quickly: we start with 3 stocks, but we are not able to finance the loan payments through more than 14 periods. That is, in almost one year we have to sell all of our stocks and we still have approximately 1.75 debt to repay. Since the returns are in ascending order is this case, big negative returns come first in the early periods, and by the time better (positive) returns would occur we will have already no stocks and cannot benefit from the favourable price movements.

The significance of the path is even more obvious if we determine the break-event interest rates in the two extreme scenarios: it is 270% in case of descending returns and -17% with ascending returns. The later means there is no realistic macroeconomic situation (r) where leverage would pay off unless there would be a regular cash inflow from our investment, for example we could collect significant dividend payments.

Random paths

After the two extreme scenarios we examine how random occurrence of the logreturns would perform. We simulated 2,000 scenarios by combining the given logreturns randomly. Figure 4 shows the actual and three simulated paths. The initial and the final stock prices are still 1 and 4.5 respectively; since logreturns are additive and changing the order does not affect the sum.

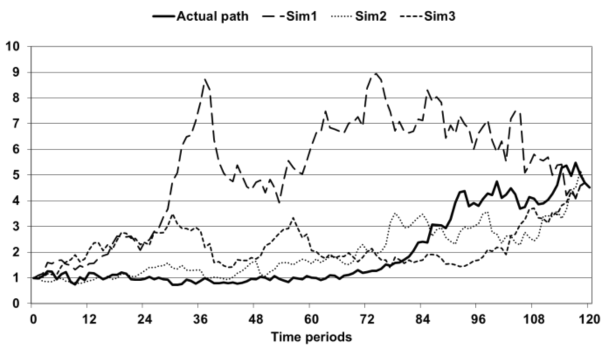

Figure 4: Actual and simulated paths with given set of logreturns

We have also calculated the break-even interest rates and the final number of stocks in *Strategy B* with the previous parameter settings (D=2 and r=12%). We also quantified the amount of cash inflow (e.g. dividend, capital increase by shareholders, fresh debt) needed to save a path generating bankruptcy (negative number of shares). Table 3 summarizes the descriptive statistics of these calculations, while Figure 5 and Figure 6 show the relative frequencies of the simulated data.

Out of the 2,000 scenarios simulated, the maximum number of shares is 2.28 which means that the maximum final net position is as high as 10.26 (2.28*4.5). Since we achieved it with 1 unit of equity, this growth equals to 1,026% total return over the 10-year investment horizon. Similar calculations show that the average total return is 464%, while the median is 527%.

Table 3: Descriptive statistics of the break-even interest rate, the number of stocks and the cash needed to save in *Strategy B* (D=2, r=12%)

	Break-even interest rate	Number of remaining stocks	Cash needed to save
Minimum	-9.59%	-5.32	-2.08
Maximum	64.55%	2.28	0.00
Mean	15.85%	1.03	-0.08
Std. dev.	10.57%	0.72	0.29
Median	14.61%	1.17	0.00
1st quartile	8.06%	0.67	0.00
1st decile	3.59%	0.13	0.00

These figures are equivalent to 26.2% (maximum), 16.6% (mean) and 18.1% (median) annual returns. We may observe that the average annual return is very close to that of Strategy A (16.3%).

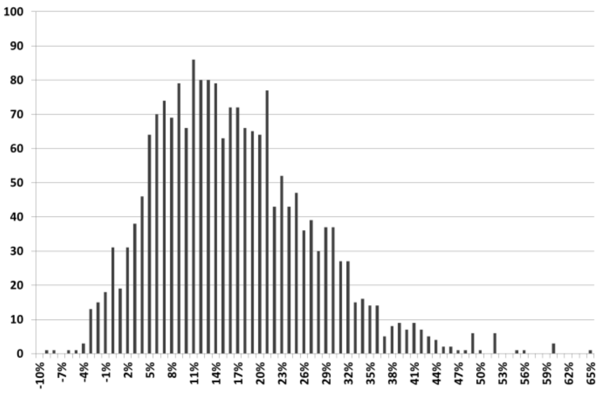

Figure 5: Frequencies of break-even interest rates from 2,000 simulated paths

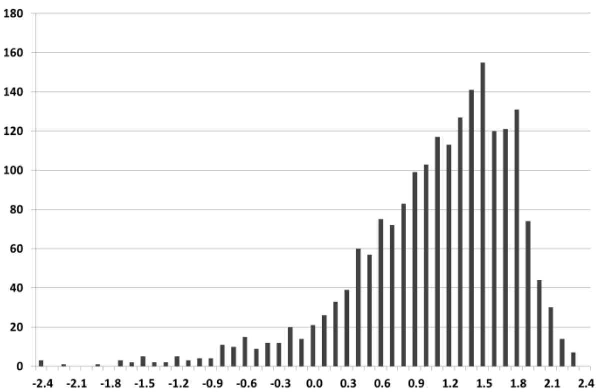

Figure 6: Frequencies of the final number of stocks in *Strategy B* from 2,000 simulated paths

Although our average annual return on the stock investment (*Strategy A*) is 16.3%, in 56.15% of the cases a similar cost for financing would have decreased our returns. In half of the paths an interest rate below 14.61% would be required, and we need to push down interest until 8.06% to have 75% chance to boost our profit by using the given leverage. For a 90% chance to win on the debt one needs a rate below 3.59%. Even if debt is for free we have just 95.8% chance that we will be able to cover all principal payments on time.

Using our initial interest rate of 12%, only 59.7% of all paths make leverage more profitable than *Strategy A* with no debt. Out of 2,000 only 1,841 (92.05%) scenarios will end up with positive number of shares, this means that the probability of bankruptcy is 7.95%. Given the possibility to survive bankruptcy by using fresh capital the total amount of money needed to save our company in these 159 cases is shown in Figure 7. (No cost for capital assumed.) To save our firm in all cases we would need 2.08 unit of money that is more than the double of the initial equity capital (*E*) available. To push the probability of bankruptcy down to 5% (that is to save 2.95% of the paths) we need 70% more start-up equity kept in form of cash, while an extra cash reserve equal to original *E* would reduce chance of failure to 3.9%.

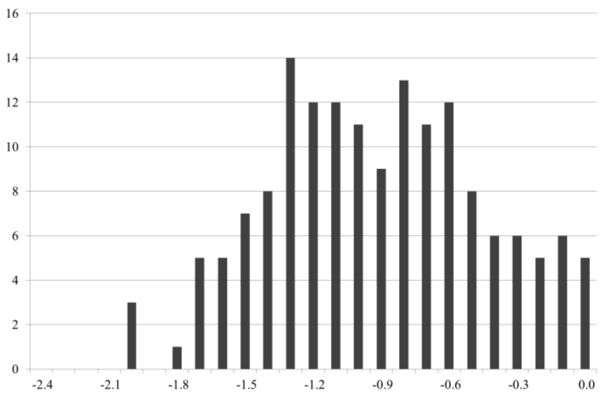

Figure 7: Frequencies of fresh capital needed to save 159 bankruptcy generating simulated paths

CONCLUSION

In this paper we investigated path dependency in case of an investment strategy using no path dependent investment assets. We illustrated how the use of a path dependent "negative" asset (debt) would make our strategy path dependent. This kind of risk is very often overviewed by investors making them unable to correctly judge total risk taken.

We have shown that the break-even interest rate for using debt in financing any portfolio is independent of the leverage itself. Although, this well-known general rule in financing is dramatically different in case of path dependency: the critical rate is not necessarily equal to the return that can be achieved by investing the capital, rather than it may be considerably lower or higher.
The risk path dependency generates is not negligible. In case of our random simulation in 7.95% of the cases the firm went bankrupt while earning an average asset return higher than the interest rate of debt over the total investment period. These failed firms would need serious amount of fresh capital to survive but even then most of those would produce loss.

Our research has also raised several further questions. For the debt different repayment schedules could be considered, fresh capital may have some alternative cost, and also length of surviving could be measured at different leverage levels. Further, our model could be developed by requiring a certain level of equity for allowing a company to move on to the next period of time.

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