UNIFIED REPRESENTATION OF DECOUPLED DYNAMIC MODELS FOR PENDULUM-DRIVEN BALL-SHAPED ROBOTS

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KEYWORDS

Modelling and simulation in robotic applications, ball-shaped robots, robot dynamics, Euler-Lagrange equation, multi-body simulation

ABSTRACT

Dynamic models describing the ball-robot motion form the basis for developments in ball-robot mechanics and motion control systems. For this paper, we have conducted a literature review of decoupled forward-motion models for pendulum-driven ball-shaped robots. The existing models in the literature apply several different conventions in system definition and parameter notation. Even if describing the same mechanical system, the diversity in conventions leads into dynamic models with different forms. As a result, it is difficult to compare, reproduce and apply the models available in the literature. Based on the literature review, we reformulate all common variations of decoupled dynamic forward-motion models using a unified notation and formulation. We have verified all reformulated models through simulations, and present the simulation results for a selected model. In addition, we demonstrate the different system behavior resulting from different ways to apply the pendulum reaction torque, a variation that can be found in the literature. For anyone working with the ball-robots, the unified compilation of the reformulated dynamic models provides an easy access to the models, as well as to the related work.

INTRODUCTION

Ball-shaped vehicles have been under development already over the last 120 years. The first patents on self-propelled spherical toys were filed in the end of 19th century. Studies on dynamic modelling and steering of motor-driven balls started in 1990's leading into emergence of computer controlled spherical mobile robots. (Ylikorpi and Suomela 2007) Recent studies on ball-shaped robots have described a variety of applications in different environments, including marine, indoors, outdoors and planetary exploration. Lately, commercial spherical robots have been introduced to the markets. The practical applications include surveillance, rehabilitation and gaming.

Ball-shaped robots offer interesting and challenging modelling and control problems due to their extraordinary dynamic nature. In development of robot mechanics and control, simulation tools play an utterly important role. Simulators regularly represent the robotic system and its behavior, and they are used to verify the performance of the control system. The core of the simulator is the dynamic model describing the ball-robot motion, which also forms the base for the control system development. Thus, the properly formulated dynamic model is of a great importance for development of simulators and control algorithms.

We have conducted a literature review of decoupled forward-motion models for pendulum-driven ball-shaped robots. The survey covered 12 different robots and their models presented in 22 published papers. For describing the robot dynamic model, these publications present several different conventions in system parameters definition and notation, including various model simplifications. This divergence makes it difficult to compare, reproduce and apply the models available in the literature. In this paper, we reformulate in a unified notation all commonly found decoupled forward-motion models of pendulum-driven ball-shaped robots. Our reformulated models, without any simplifications, provide a detailed description of the used assumptions as well as the selected coordinate systems. The unified compilation of the reformulated dynamic models provides an easy access to the existing models.

As is the common practice in the literature, we have verified the performance of each dynamic model through simulations in Matlab-software of MathWorks Inc. (Version 7.5.0.342, R2007b). Additionally, a comparative simulation was performed for each model in Adams multi-body simulation software of MSC.Software Corporation (Version MD Adams R3, Build 2008.1.0). In this context, such model validation

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with two parallel and independent simulation tools has been rarely presented before. In addition, we demonstrate the different system behavior resulting from different ways to apply the pendulum reaction torque, a variation that can be found also in the literature. For anyone working with the ball-shaped robots, we present in Appendix 1 the models in a hand-book style providing a clear and easy access to the models, as well as to the related work behind them.

RELATED WORK

Ball-shaped robots represent a family of mobile robots that can be realized with several different mechanisms for actuation, some of which were briefly reviewed by Ylikorpi and Suomela (2007). Plenty of prior work has been conducted on kinematic and dynamic modelling of these robots. Li and Canny (1990), Jurdjevic (1993), and Bicchi et al. (1995) discuss the classical ball-plate – problem. Halme et al. (1996) introduce a ball-robot equipped with a single driving wheel inside the hollow sphere. Bicchi et al. (1997) present another ball robot with a unicycle driving inside the sphere. Camicia et al. (2000) continue the work developing a more advanced dynamic model. Zhan et al. (2011) present another ball-robot based on a unicycle. Mukherjee et al. (1999, 2002) discuss the application of the ball-plate problem, path planning, and steering of a ball robot, while Das and Mukherjee (2004, 2006) develop more complex rolling paths.

Svinin and Hosoe (2008), Svinin et al. (2012a, 2012b), and Morinaga et al. (2012) discuss kinematics, dynamics and control of a ball-robot carrying six flywheels. Karimpour et al. (2012), Joshi et al. (2007, 2010), and Joshi and Banavar (2009) conduct an extensive discussion on a spherical robot driven by three and four momentum wheels.

A motor-actuated hanging pendulum creates one possible driving mechanism, applied for several different ball robots (Koshiyama and Yamafuji 1992, 1993; Michaud and Caron 2002; Bruhn et al. 2005; Kaznov and Seeman 2010; Yoon et al. 2011). Jia et al. (2009), Sang et al. (2011), and Zheng Y.L. (2011) add a momentum wheel on the pendulum.

There are two popular methods to present the equations of motion of a pendulum-driven robot; A coupled model presents the full motion of the complete system. Various mathematical methods, such as Kane's method, Euler-Lagrange equation, and Maggi's equations are often applied to create the coupled model (Jia et al. 2008, 2009; Liu et al. 2008; Zhuang et al. 2008; Liu and Sun 2010; Sang et al. 2011; Yu et al. 2011; Zheng, M. et al. 2011; Zheng, Y.L. 2011; Gajbhiye and Banavar 2012; Balandin et al. 2013).

Different from the coupled model, a decoupled model discusses steering and forward-driving motions separately. To mention some methods, decoupled models have been created with application of Newton-Euler-equations, Euler-Lagrange equation, and Boltzmann-Hamel-equations. We have chosen to apply the Euler-Lagrange equation. Along with our new reformulated models, Appendix 1 presents the reference information for the original works. This survey concentrated on those 22 published models presenting the forward motion of pendulum-driven ball-robots.

COMMON VARIATIONS IN MODEL PRESENTATION

The decoupled forward-motion state of a pendulum-driven ball-robot can be conveniently presented with the ball rotation angle, the pendulum rotation angle, and their time derivatives. The two rotation angles are commonly nominated as the *generalized coordinates* chosen to describe the system state. Ball position along the rolling plane couples directly to the ball rotation through a kinematic rolling constraint.

Figure 1 shows one definition of the generalized coordinates θ_1 and θ_2, also used by Kim et al. (2009). In this convention, the ball rotation angle θ_1 measures from the ground vertical, and the pendulum elevation angle θ_2 is measured with respect to a reference fixed on the ball. Alternatively, the pendulum angle can be chosen to be the absolute one, while presenting the ball rotation with respect to the pendulum (Koshiyama and Yamafuji 1993). Yet, as one more alternative, both rotation angles can be presented as absolute with respect to the ground (Cai et al. 2011). In addition and opposite to the case shown in Figure 1, the positive rotation direction of the ball can be selected to be clockwise (Yue et al. 2006; Kamaldar et al. 2011; Kayacan et al. 2012a). Table 1 shows the possible permutations available for definition of the two generalized coordinates. All six variations can be found in the literature.

In addition to the different definitions of the generalized coordinates, also the presentation of the ball inertia has different forms. In most of the cases the ball inertia is calculated around the ball center, but some models present the inertia around the contact point (Koshiyama

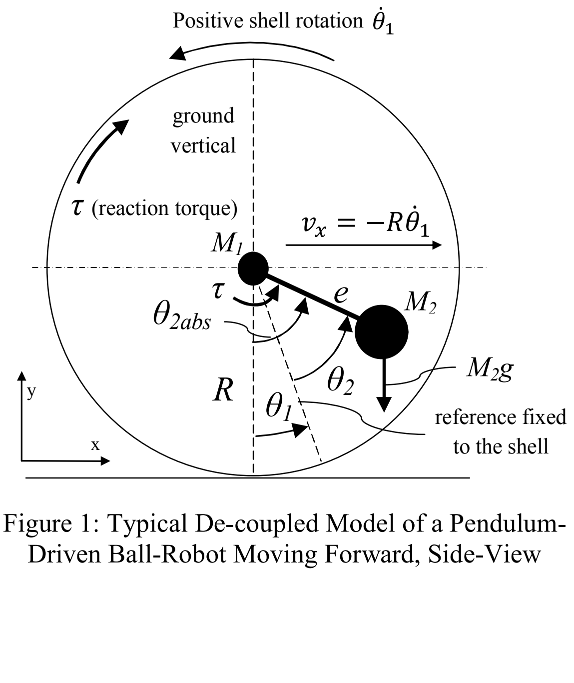

Figure 1: Typical De-coupled Model of a Pendulum-Driven Ball-Robot Moving Forward, Side-View

Table 1: Variants in Definition of the Generalized Angular Coordinates

	Options						Number of options
Selected absolute coordinate	Absolute Ball angle, relative Pendulum angle		Absolute Pendulum angle, relative Ball angle		Both absolute		3
Positive rotation direction of the ball with respect to the pendulum	Same	Opposite	Same	Opposite	Same	Opposite	2 x 3 = 6
Illustration in Appendix 1	C)	D)	A)	F)	E)	B)	
Occurrences in the selected literature	2	3	2	1	1	13	

and Yamafuji 1993; Cai et al. 2011; Kamaldar et al. 2011). Our representation calculates the inertia around the ball center.

Different from the other references, Koshiyama and Yamafuji (1992, 1993) present the absolute pendulum rotation angle with respect to the ground horizontal. Our unified representation in Appendix 1 measures the absolute rotation angle from the ground vertical for all models.

The literature presents cases where the pendulum absolute angular velocity is assumed small and the products of the angular velocities can then be neglected (Kim et al. 2009). In addition, sometimes small angles have been assumed thus changing the appearance of trigonometric functions (Kim et al. 2009; Liu et al. 2009). In contrast, we present the complete dynamic equations without simplifications.

Finally, the dynamic model is often presented in a matrix form. Literature shows a couple of different arrangements where the matrix elements and the coordinate vectors are shown in a different order (Koshiyama and Yamafuji 1993; Yue et al. 2006). Appendix 1 presents all models in a uniform arrangement.

THE UNIFIED MODEL

Selection of the Generalized Coordinates

Figure 1 illustrates one possible selection of the generalized coordinates. Ball rotation angle θ_1 is measured counterclockwise from the ground vertical to a reference fixed on the ball. Pendulum rotation angle θ_2 is measured counterclockwise from the reference on the ball towards the pendulum arm. Table 2 explains the parameters and variables used in Figure 1 and in our unified notation.

In the convention shown in Figure 1, the ball angle θ_1 is expressed as an *absolute coordinate*. The absolute coordinate presents directly the ball rotation angle with respect to the inertial world-coordinate system. In contrast, the pendulum rotation angle θ_2 is expressed as a *relative coordinate*. The relative coordinate tells only the pendulum position with respect to the ball. As an alternative presentation for the pendulum orientation, Figure 1 presents also the absolute pendulum angle θ_{2abs} that measures the pendulum position directly from the ground vertical towards the pendulum arm.

Derivation of the Equations of Motion

The Euler-Lagrange equation can be used to create the equations of motion (Symon 1960; Goldstein et al. 2002), and it has been often applied also with ball-shaped robots (Liu et al. 2008; Jia et al. 2009; Zhang et al. 2009; Kayacan et al. 2012a). Lagrangian function L is defined as the difference between the kinetic energy T and the potential energy V, as shown in (1). Generalized forces Q_i affecting the system can then be solved through derivation of the Lagrangian with respect to time and the generalized coordinates q_i, as presented in (2), known as the Euler-Lagrange equation.

$$L = T\text{-}V \tag{1}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \tag{2}$$

Referring to the convention in Figure 1, eqs. (3) and (4) present the kinetic and potential energy of the spherical shell and the pendulum. The Lagrangian L in (1) and the differentials on the left side of (2) can then be solved.

Table 2: Parameters for the Dynamic Models in Figure 1 and Appendix 1

M_1	ball mass	M_2	pendulum mass
R	ball radius	e	pendulum length
J_1	ball inertia	J_2	pendulum inertia
θ_1	ball angle	θ_2 θ_{2abs}	pendulum angle absolute pendulum angle
$\dot{\theta}_1$	ball angular velocity	$\dot{\theta}_2$	pendulum angular velocity
c_1	ball rolling friction coefficient	c_2	pendulum joint friction coefficient
T_1	ball kinetic energy	T_2	pendulum kinetic energy
V_1	ball potential energy	V_2	pendulum potential energy
v_x	horizontal ball velocity	τ	pendulum motor torque

$$T_1 = \frac{1}{2} M_1 R^2 \dot{\theta}_1{}^2 + \frac{1}{2} J_1 \dot{\theta}_1{}^2$$

$$T_2 = \frac{1}{2} M_2 \left(\left(-R\dot{\theta}_1 + e(\dot{\theta}_1 + \dot{\theta}_2)\cos(\theta_1 + \theta_2) \right)^2 \right) \quad (3)$$

$$+ \frac{1}{2} M_2 \left(e(\dot{\theta}_1 + \dot{\theta}_2)\sin(\theta_1 + \theta_2) \right)^2 + \frac{1}{2} J_2 (\dot{\theta}_1 + \dot{\theta}_2)^2$$

$$V_1 = 0$$

$$V_2 = -M_2 g e \cos(\theta_1 + \theta_2) \quad (4)$$

As presented by Koshiyama and Yamafuji (1993), the result from (2) can be expressed in a *configuration space* according to (5), where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ and $\boldsymbol{G}$ denote the matrices including mass and inertia terms, centrifugal terms, coriolis terms, viscous friction, and gravitational forces respectively. Torque vector $\boldsymbol{Q}$ includes the generalized forces, i.e. the torques affecting the system.

$$\begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} B11 & B12 \\ B21 & B22 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix} +$$

$$\begin{bmatrix} C11 \\ C21 \end{bmatrix} (\dot{\theta}_1 \dot{\theta}_2) + \begin{bmatrix} D11 & D12 \\ D21 & D22 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} G11 \\ G21 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \quad (5)$$

The left side of (5) can be acquired through the derivations shown in (2). However, the contents of the torque vector $\boldsymbol{Q}$ on the right side deserve some discussion, which has not been conducted in the related literature before: The two generalized torques Q_1 and Q_2 relate to the two angular coordinates θ_1 and θ_2 shown in Figure 1. The ball-robot carries a motor that drives the pendulum with respect to shell. The choice of the generalized coordinates defines how the motor torque and its reaction torque project to the generalized torques. Symon (1960, p. 354) notes: *'...the mutual forces which the particles exert on each other ordinarily depend on the relative coordinate.'* Upon application of the Euler-Lagrange equation, the relative motion between the bodies takes into consideration also the reaction forces between the bodies. Contradictorily, the use of absolute coordinates neglects the reaction forces that then need to be separately taken into account.

To give an example, Figure 1 presents the pendulum angle θ_2 relative to the ball angle. Because of the relative expression of the pendulum angle, the reaction torque from the pendulum motor becomes automatically into consideration through the Euler-Lagrange equation. The pendulum driving torque τ is then included in the system input Q_2 in (5), but the reaction torque is not added explicitly in the ball torque Q_1. However, if the absolute pendulum angle θ_{2abs} was used instead, the reaction torque must be added also as an input on the ball torque Q_1.

A proof for the above made statement can be found by calculating symbolically eqs. (1) - (5) using both absolute and relative pendulum angles and notifying the appearance of torques Q_1 and Q_2 in the result. The straightforward calculation is omitted here.

The literature presents both approaches, applying either relative or absolute pendulum coordinate. However, the convention in application of the reaction torque varies. We apply the reaction torque consistently upon need, as is described above, confirmed by the parallel simulations in Adams, and reported in Appendix 1. For comparison, our simulation results present also the different system behavior resulting from the different application of the reaction torque.

Modelling the Viscous Friction

We supplement all dynamic models with viscous friction, which has been previously presented for some formulations. For the given velocity vector $\dot{\boldsymbol{\theta}}$, the manually calculated friction matrix $\boldsymbol{D}$ provides proper resistance torque for the ball and the pendulum. In the definition of the friction matrix, it is important to note that the frame of reference must be similar to that used in derivation of the Lagrangian in (1). A similar friction model was presented by Koshiyama and Yamafuji (1993), as shown in Appendix 1 A).

Numerical Simulation

All six dynamic models, created with application of (1) and (2), were implemented in Matlab for verification. In the simulation, the dynamic model formulated in configuration space (5) was applied to solve the accelerations for the given input torque $\boldsymbol{Q}$. The joint velocities and angles were then integrated with ode45 – solver function. In addition, a parallel model of the system was built in Adams multi-body simulation software and the results were compared for validation.

The ball-robot model is defined for the Adams-software by describing the mechanical structure and the physical properties of the robot. Adams then autonomously creates the dynamic model needed for simulation. Thus, Adams provides a model that is independent from the one created for Matlab, and can be used as a reference in validation of the derived models.

SIMULATION RESULTS

All models collected in Appendix 1 were simulated both in Matlab and in Adams. Comparison of the simulation results verified the correctness of the models. For the sake of brevity, we present the simulation results only for the model according to Figure 1 and applying the formulation C) in Appendix 1.

Figure 2 A) shows the open-loop response for a given pendulum torque impulse. The input torque has a form of a cosine function with a 5-s period and 1-Nm peak value. The integration result in Matlab agrees well with the simulation result in Adams. No difference is visible between the two models in Figure 2 A). Regarding the earlier discussion on the observed variation in application of the reaction torque, the third simulation result in Figure 2 A) reveals the effect from the excess reaction torque in Q_1. Figure 2 B) demonstrates the identical behavior of the two independent simulation models; one in Matlab, another in Adams.

Figure 3 repeats the simulations in a closed-loop with a PI-controlled ball velocity. The target ball velocity is -5 rad/s and the controller gains are P = 0.1 and I = 0.3. In the third simulation, the effect from the excessive reaction torque is clear leading into different conclusion about system dynamics and highly different prediction of the needed pendulum motor torque. The result underlines the importance of the correct dynamic model, being the subject of this paper. Further development and discussion on the control algorithms remain as future work.

Simulations of all model formulations in Appendix 1 produce the same results. In the simulations, the robot model represents the *GimBall*-robot developed at Aalto University having the properties: M_1 = 3.294 kg, M_2 = 1.795 kg, R = 0.226 m, e = 0.065 m, J_1 = 0.0633 kgm^2, J_2 = 0.0074 kgm^2, c_1 = 0.02 Nms/rad, c_2 = 0.2 Nms/rad and g = 9.81 m/s^2.

The models were integrated in Matlab using ode45-solver with the following settings: RelTol = 10^{-6}, AbsTol = 10^{-10}, MaxStep = 10^{-3} and InitialStep = 10^{-6}. The simulator settings in Adams were the corresponding.

CONCLUSIONS AND FURTHER WORK

The dynamic models describing ball-robot motion form the basis for the developments in ball-robot mechanics and motion control algorithms. Thus, the dynamic model holds an extremely important position in the research on the ball-shaped robots.

Because of the existing diversity in notation and model contents, it is difficult to compare, reproduce and apply the models available in the literature. To facilitate model comparison and re-use, we have in this paper reformulated all common decoupled forward-motion models of pendulum-driven ball-shaped robots. The reformulated models, created with the application of Euler-Lagrange equation while applying a unified notation and harmonized formulation, are collected in Appendix 1.

All reformulated models have been validated with parallel simulations in Matlab and Adams multi-body simulation software. The two independent simulation results show an excellent agreement thus validating the models. Additional simulation results demonstrate the effect from the different conventions in application of reaction torque.

Clear, correct and harmonized description of the dynamic models in a hand-book style is useful for their application in further developments. These models, being the topic of this paper, set the basis for further work of the control algorithms. Our work continues with extension of the dynamic model to consider also other dynamic cases than the pure rolling along a level surface, as well as to consider a robot structure different from an ideal rigid sphere. Practical experiments and addition of the Coulomb friction model are foreseen.

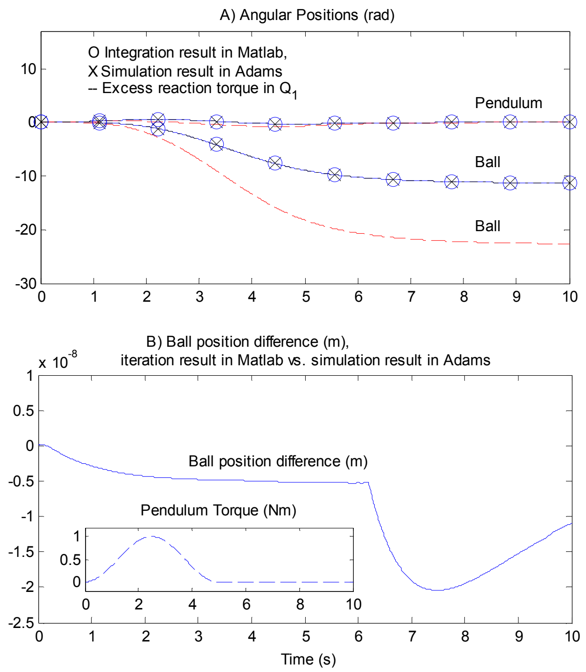

Figure 2: A) In an open-loop simulation, an excess reaction torque in Q_1 causes erroneous system response. B) Ball location difference between the simulation results is < $2 \cdot 10^{-8}$ m. Inset: the applied pendulum torque

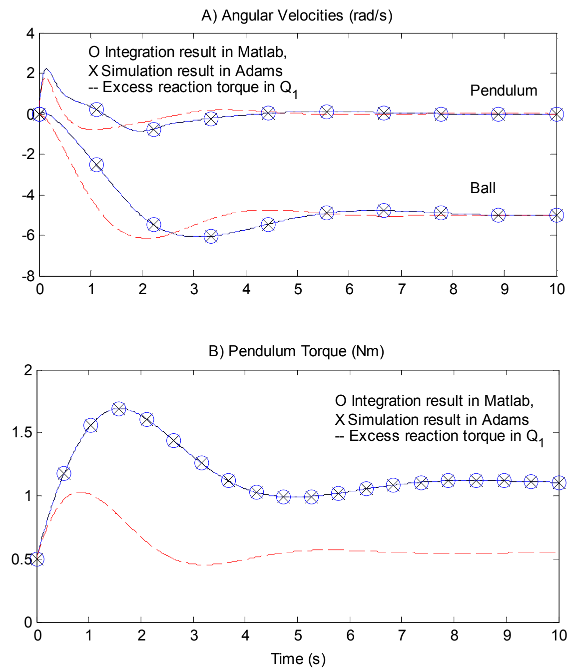

Figure 3: A) In a closed-loop, a PI-controller stabilizes the ball velocity. Excess reaction torque in Q_1 causes a different system response. B) Excess reaction torque causes error in the simulated pendulum driving torque

APPENDIX 1

THE COMPLETE DYNAMIC MODELS IN UNIFIED REPRESENTATION

Unless otherwise stated, the ball kinetic energy is: $T_1 = \frac{1}{2}M_1R^2\dot{\theta}_1{}^2 + \frac{1}{2}J_1\dot{\theta}_1{}^2$.

'τ' presents the pendulum motor torque.

The given elements for ***A***, ***B***, ***C***, ***D***, ***G***, and ***Q*** complete the configuration space presentation:

$$\begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} B11 & B12 \\ B21 & B22 \end{bmatrix}\begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix} + \begin{bmatrix} C11 \\ C21 \end{bmatrix}\left(\dot{\theta}_1\dot{\theta}_2\right) + \begin{bmatrix} D11 & D12 \\ D21 & D22 \end{bmatrix}\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} G11 \\ G21 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

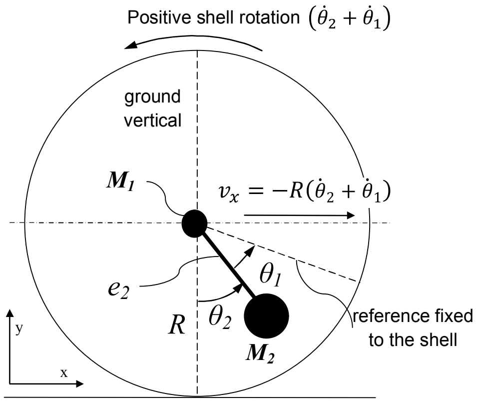

Figure A1: Absolute Pendulum Angle, Relative Ball Angle in Same Direction

A) Absolute Pendulum Angle, Relative Ball Angle in Same Direction

$A11 = J_1 + (M_1 + M_2)R^2$
$A12, A21 = J_1 + (M_1 + M_2)R^2 - M_2Re_2\cos(\theta_2)$
$A22 = J_1 + J_2 + (M_1 + M_2)R^2 + M_2e_2{}^2 - 2M_2Re_2\cos(\theta_2)$
$B12, B22 = M_2Re_2\sin(\theta_2)$
$D11 = (c_1 + c_2)$
$D12, D21, D22 = c_1$
$G2 = M_2ge_2\sin(\theta_2)$
$Q1 = -\tau_1$
$Q2 = 0$

Applicable References: (Koshiyama and Yamafuji 1992, 1993)

$V_2 = -M_2ge_2\cos(\theta_2)$
$T_1 = \frac{1}{2}M_1R^2(\dot{\theta}_2 + \dot{\theta}_1)^2 + \frac{1}{2}J_1(\dot{\theta}_2 + \dot{\theta}_1)^2$
$T_2 = \frac{1}{2}M_2\left(R(\dot{\theta}_2 + \dot{\theta}_1) - e_2\dot{\theta}_2\cos(\theta_2)\right)^2 + \frac{1}{2}M_2\left(e_2\dot{\theta}_2\sin(\theta_2)\right)^2 + \frac{1}{2}J_2\dot{\theta}_2{}^2$

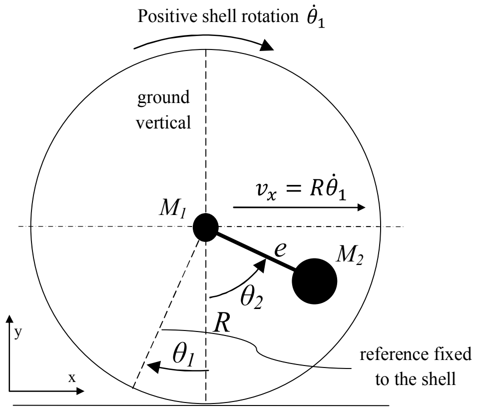

Figure A2: Absolute Ball Angle, Absolute Pendulum Angle in Opposite Direction

B) Absolute Ball Angle, Absolute Pendulum Angle in Opposite Direction

$A11 = J_1 + (M_1 + M_2)R^2$
$A12, A21 = M_2Re\cos(\theta_2)$
$A22 = J_2 + M_2e^2$
$B12 = -M_2Re\sin(\theta_2)$
$D11 = (c_1 + c_2)$
$D12, D21, D22 = c_2$
$G2 = M_2e\sin(\theta_2)g$
$Q1 = \tau$
$Q2 = \tau$

Applicable References: (Yue et al. 2006; Liu et al. 2009; Zhang et al. 2009; Ghanbari et al. 2010; Liu et al. 2012; Yu et al. 2012a; Yu et al. 2012b Yue and Liu 2012a; Yue and Liu 2012b; Mahboubi et al. 2013; Yue and Liu 2013;)

$V_2 = -M_2ge\cos(\theta_2)$
$T_2 = \frac{1}{2}M_2\left(\left(R\dot{\theta}_1 + e\dot{\theta}_2\cos(\theta_2)\right)^2 + \left(e\dot{\theta}_2\sin(\theta_2)\right)^2\right) + \frac{1}{2}J_2\dot{\theta}_2{}^2$

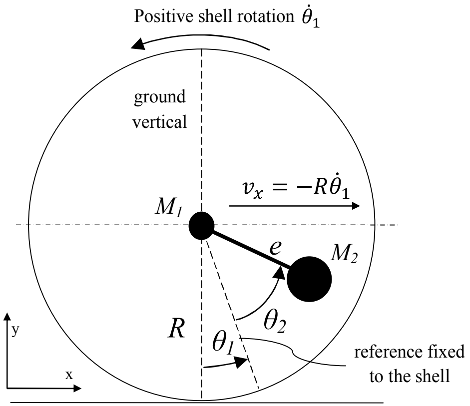

Figure A3: Absolute Ball Angle,
Relative Pendulum Angle in Same Direction

$$V_2 = -M_2 g e \cos(\theta_1 + \theta_2)$$

$$T_2 = \frac{1}{2}M_2\left(-R\dot{\theta}_1 + e(\dot{\theta}_1 + \dot{\theta}_2)\cos(\theta_1 + \theta_2)\right)^2 + \frac{1}{2}M_2\left(e(\dot{\theta}_1 + \dot{\theta}_2)\sin(\theta_1 + \theta_2)\right)^2 + \frac{1}{2}J_2(\dot{\theta}_1 + \dot{\theta}_2)^2$$

C) Absolute Ball Angle, Relative Pendulum Angle in Same Direction

$A11 = J_1 + J_2 + M_2e^2 + (M_1 + M_2)R^2 - 2M_2 R e \cos(\theta_1 + \theta_2)$
$A12, A21 = J_2 + M_2e^2 - M_2 R e \cos(\theta_1 + \theta_2)$
$A22 = J_2 + M_2e^2$
$B11, B12 = M_2 R e \sin(\theta_1 + \theta_2)$
$C11 = 2M_2 R e \sin(\theta_1 + \theta_2)$
$D11 = c_1$
$D22 = c_2$
$G1 = M_2 e \sin(\theta_1 + \theta_2) g$
$G2 = M_2 e \sin(\theta_1 + \theta_2) g$
$Q1 = 0$
$Q2 = \tau$

Applicable References: (Nagai 2008, Kim et al. 2009)

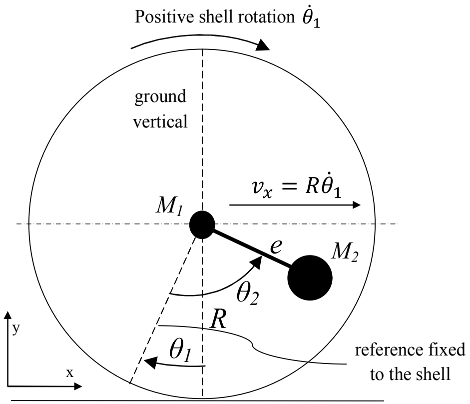

Figure A4: Absolute Ball Angle,
Relative Pendulum Angle in Opposite Direction

$$V_2 = -M_2 g e \cos(\theta_2 - \theta_1)$$

$$T_2 = \frac{1}{2}M_2\left(\left(R\dot{\theta}_1 + e(\dot{\theta}_2 - \dot{\theta}_1)\cos(\theta_2 - \theta_1)\right)^2\right) + \frac{1}{2}M_2\left(e(\dot{\theta}_2 - \dot{\theta}_1)\sin(\theta_2 - \theta_1)\right)^2 + \frac{1}{2}J_2(\dot{\theta}_2 - \dot{\theta}_1)^2$$

D) Absolute Ball Angle, Relative Pendulum Angle in Opposite Direction

$A11 = J_1 + J_2 + M_2e^2 + (M_1 + M_2)R^2 - 2M_2 R e \cos(\theta_1 - \theta_2)$
$A12, A21 = -J_2 - M_2e^2 + M_2 R e \cos(\theta_1 - \theta_2)$
$A22 = J_2 + M_2e^2$
$B11, B12 = M_2 R e \sin(\theta_1 - \theta_2)$
$C11 = -2M_2 R e \sin(\theta_1 - \theta_2)$
$D11 = c_1$
$D22 = c_2$
$G1 = M_2 e \sin(\theta_1 - \theta_2) g$
$G2 = -M_2 e \sin(\theta_1 - \theta_2) g$
$Q1 = 0$
$Q2 = \tau$

Applicable References: (Kayacan et al. 2012a, 2012b, 2012c)

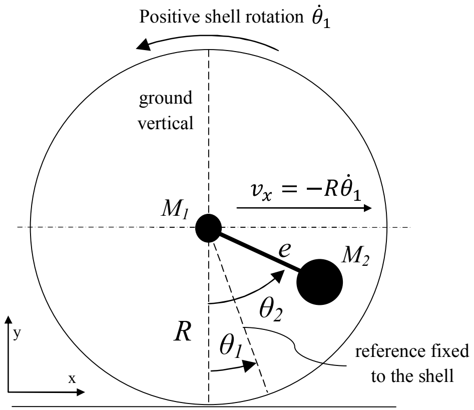

Figure A5: Absolute Ball Angle,
Absolute Pendulum Angle in Same Direction

$$V_2 = -M_2 g e \cos(\theta_2)$$

$$T_2 = \frac{1}{2}M_2\left(\left(-R\dot{\theta}_1 + e\dot{\theta}_2\cos(\theta_2)\right)^2 + \left(e\dot{\theta}_2\sin(\theta_2)\right)^2\right) + \frac{1}{2}J_2\dot{\theta}_2^{\,2}$$

E) Absolute Ball Angle, Absolute Pendulum Angle in Same Direction

$A11 = J_1 + (M_1 + M_2)R^2$
$A12, A21 = -M_2 R e \cos(\theta_2)$
$A22 = J_2 + M_2e^2$
$B12 = M_2 R e \sin(\theta_2)$
$D11 = (c_1 + c_2)$
$D12, D21 = -c_2$
$D22 = c_2$
$G2 = M_2 e \sin(\theta_2) g$
$Q1 = -\tau$
$Q2 = \tau$

Applicable Reference: (Cai et al. 2011)

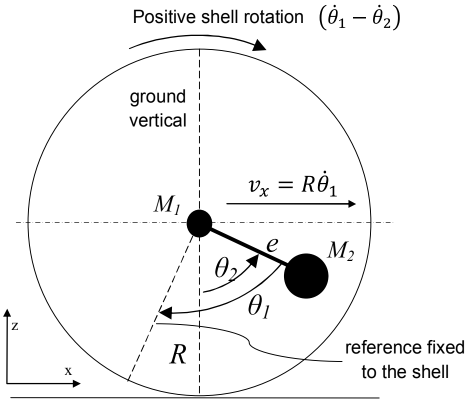

Figure A6: Absolute Pendulum Angle, Relative Ball Angle in Opposite Direction

$$V_2 = -M_2 g e\cos(\theta_2)$$
$$T_1 = \frac{1}{2}M_1\left(R(\dot{\theta}_1 - \dot{\theta}_2)\right)^2 + \frac{1}{2}J_1(\dot{\theta}_1 - \dot{\theta}_2)^2$$
$$T_2 = \frac{1}{2}M_2\left(\left(R(\dot{\theta}_1 - \dot{\theta}_2) + e\dot{\theta}_2\cos(\theta_2)\right)^2\right) + \frac{M_2}{2}\left(e\dot{\theta}_2\sin(\theta_2)\right)^2 + \frac{J_2}{2}(\dot{\theta}_2)^2$$

F) Absolute Pendulum Angle, Relative Ball Angle in Opposite Direction

$A11 = J_1 + (M_1 + M_2)R^2$
$A12, A21 = -J_1 - (M_1 + M_2)R^2 + M_2Re\cos(\theta_2)$
$A22 = J_1 + J_2 + M_2e^2 + (M_1 + M_2)R^2 - 2M_2Re\cos(\theta_2)$
$B12 = -M_2Re\sin(\theta_2)$
$B22 = M_2Re\sin(\theta_2)$
$D11 = (c_1 + c_2)$
$D12, D21 = -c_1$
$D22 = c_1$
$G2 = M_2e\sin(\theta_2)g$
$Q1 = \tau$
$Q2 = 0$

Applicable Reference: (Kamaldar et al. 2011)

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