INTERTEMPORAL CHOICE AND DYNAMICS OF RISK AVERSION

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KEYWORDS

Asymmetric volatility; Risk seeking; Prospect theory; TGARCH; EGARCH; Volatility dynamics; Market microstructure; Heuristic-driven trader

ABSTRACT

This paper provides a theoretical explanation for the heteroscedasticity of asset returns. In line with existing empirical results, our model yields an asymmetric relationship between stock return and volatility. Based on the simple assumptions that investors behave according to Prospect Theory and are subject to mental accounting in a dynamic setting, we analytically derive the unit-root versions of two of the best fitting heteroscedasticity models (EGARCH and TGARCH).

INTRODUCTION

Time-varying volatility (heteroscedasticity) of asset returns has attracted much research in the recent decades. Since the milestone papers of Engle (1982) and Bollerslev (1986) a great number of scholarly paper has been devoted to the topic. Their findings indicate that the phenomenon can be modeled by GARCH type models. Nevertheless, as the evidence shows below, no robust theoretical foundation has been proposed yet.

Another important aspect of the autoregression puzzle is the asymmetry in the volatility process. In particular, the phenomenon known as "asymmetric volatility" implies that changes in the price of the underlying asset are negatively correlated with the volatility of the subsequent period. Despite the wide amount of literature devoted to asymmetric volatility (Black, 1976; Christie, 1982; and Schwert, 1989), it still misses a robust explanation.

Until now three main explanations for this latter puzzle have been proposed. The first is the leverage effect noted by Black (1976), Christie (1982) and Schwert (1989). The authors assume that if the value of an equity drops, the firm becomes more leveraged, therefore, the volatility of equity returns rises according to the increased risk, hence, causing the negative relationship between return and subsequent volatility. They conclude, however, that volatility is indeed an increasing function of financial leverage, the effect by itself is not sufficient to account for the observed negative correlation.

The second explanation labeled as the volatility feedback hypothesis states that in cases of unexpected increase in volatility (e.g. exogenous shocks), expected volatility rises accordingly, and thus increasing the required return of the given asset in line with equilibrium asset pricing models. This latter has an immediate negative impact on current stock price; therefore, it strengthens or weakens the magnitude of a previous shock subsequent to losses or gains respectively hence causing the asymmetry. Numerous papers on the topic provided evidence in support of both explanations (Pindyck, 1984; or Kim et al., 2004), yet recent studies still yield controversial results: on the one hand, Bollerslev et al. (2006) find that the analysis of high-frequency data indicates no significant volatility feedback, while on the other hand, Bekaert and Wu (2000) conclude that the leverage effect is insignificant.

The third main explanation is given by McQueen and Vorkink (2004) – their initial setting is the closest to ours – proposing that volatility autocorrelation is due to investors' inclusion of the fluctuation of prices in their perceived utility (i.e. loss-averse behavior). The authors assume that volatility increases both following gains and losses as in volatility feedback hypothesis. This assumption comes from the paper of Barberis et al. (2001), which latter provides an asset pricing interpretation of Thaler and Johnson's (1990) experiment of prospect theory in a dynamic setting. The Barberis-Huang-Santos (2001) (henceforth BHS) paper assumes that perception of losses (gains) is more (less) painful (delightful) when they are subsequent to prior losses (gains). In other words, this latter means that previous losses increase and previous gains decrease risk-aversion. However, BHS do not take into account the entire analysis of Thaler and Johnson; the authors do not focus on the finding that investors become risk-seeking following losses and risk-averse subsequent to gains if the opportunity of breaking-even is included in the choice set, which, in fact, almost always applies to asset returns. As we show in the following section, the individual dataset we use in this paper provides support for the latter hypothesis instead of the assumption of BHS. In other words, the pattern obtained in empirical tests suggest that, in contrast to BHS and McQueen and Vorkink, volatility indeed increases following losses and decreases subsequent to gains in order to allow or prevent breaking-even respectively.

It is also worth mentioning that the original setting in which autoregressive conditional heteroscedastic models were defined was the expected utility theory, that is, the dynamics of volatility (i.e. the standard deviation of asset returns in a given period) have been analyzed mostly in the setting of the mean-variance optimization of standard asset pricing models (e.g. the CAPM). However, contradictory results of this approach to utility perception have been well-documented, which would lead to biased

Proceedings 30th European Conference on Modelling and Simulation ©ECMS Thorsten Claus, Frank Herrmann, Michael Manitz, Oliver Rose (Editors)
ISBN: 978-0-9932440-2-5 / ISBN: 978-0-9932440-3-2 (CD)

interpretations. Therefore, we build our theoretical model on an alternative approach: the prospect theory of Kahneman and Tversky (1979).

Based on Ormos and Timotity (2015), in our paper we apply this latter approach in a dynamic interpretation along with mental accounting and derive the microfoundations of heteroscedasticy. Our main findings are that (i) previous, unexpected shocks have negative, linear effect on the investors' required return; (ii) this pattern yields a negative effect of previous market shocks on market volatility; (iii) our setting provides the microfoundations of a unit-root, asymmetric, autoregressive volatility process similar to Threshold Generalized Autoregressive Conditional Heteroscedasticity (TGARCH) and Exponential Generalized Autoregressive Conditional Heteroskedastic (EGARCH) models in discrete and continuous time respectively.

The paper is structured as follows: in the subsequent section the theoretical setting is derived along with the main empirical findings of behavioral patterns necessary to interpret the theory. The last section summarizes the main conclusions of the paper and provides potential ways of further research.

THE MODEL

Previous market return has been shown to play a dominant role in the volatility dynamics of assets and is mainly responsible for the asymmetric response to shocks. We argue that this phenomenon can be explained by applying prospect theory in a dynamic setting.

Dynamics of the required return

Assuming that investors hold portfolios similar to the market portfolio, or at least that they diversify and hence invest into multiple assets, their portfolio is highly correlated with the market. In other words, a negative or positive market shock leads to losses and gains on investors' portfolios. Thaler and Johnson (1990) show in their experimental study that in such cases (if breaking even is in the choice set) investors become risk-seeking following losses and more risk-averse subsequent to gains; they aim to avoid realizing losses (exactly as in disposition effect (Shefrin and Statman, 1985 and Odean, 1998)) and are afraid of losing previous paper gains. This behavior comes from the S-shaped value function of loss-aversion: if we include the previous outcome as a reference point, the convexity of utility perception in the domain of losses results in risk-seeking behavior as the expected utility reaches its maximum at positive risk. In this specific case, considering the previous outcome as a fixed loss would cause greater pain than aggregating in time and hoping to break even; however, realizing the previous gain yields higher expected utility than taking the risk of losing the accumulated wealth.

Therefore, mental accounting (the mental aggregation or separation of pieces of information) in a dynamic setting leads investors to aggregate in time. Hence, they aim to obtain a given reference return at each period or, at least, earn this return on average. That is, if we assume the rational expectations of outcomes as the reference point (Koszegi and Rabin, 2006), the subsequent required return is decreased by the previous abnormal return to be able to obtain the pre-defined reference return on average. Analytically the aforementioned is described with the following equation

$$\mu_t = \mu_{t-1} + \alpha(\mathrm{r}_{t-1} - \mu_{t-1}) + r_{f,t} - r_{f,t-1}: \alpha \in [-1,0]. \quad (1)$$

where $r_{f,t}$, r_t and μ_t stand for the risk-free rate, the portfolio return and the required portfolio return of a given investor respectively. In particular, if we assume that investors form their expectations rationally and allocate their portfolio accordingly (i.e. they choose from the efficient portfolios), μ_t represents both the required and the expected return of their portfolio as the latter two become equal. Therefore, $\mathrm{r}_{t-1} - \mu_{t-1}$ stands for the abnormal portfolio return in the previous period, which modifies the current period required (expected) return through α. The economic interpretation of this latter variable is defined as the sensitivity of an investor to mental accounting (the aggregation of previous outcomes). It would make no sense to assume that market participants adjust the required return by more than the previous shock itself; hence, we set its lower boundary at -1. Its negative value is due to the definition: aggregating in time increases or decreases the required return subsequent to losses or gains respectively. The $r_{f,t} - r_{f,t-1}$ term is added as the correction for the change in the risk-free rate or inflation.

It is worth mentioning here that autoregressive conditional heteroscedasticity (henceforth ARCH) models (Engle, 1982) were created in the setting of standard asset pricing models that are based on the expected utility theory (EUT); however, the latter would not yield the behavior described above. In contrast to prospect theory, EUT assumes concave utility in all domains of wealth, and therefore, would never induce risk-seeking behavior following losses. Therefore, the aforementioned behavior cannot be analyzed in a standard asset pricing structure, hence in order to give a coherent setting, the following section provides the definition of the risk-return relationship in prospect theory.

The mean-volatility relationship in prospect theory

The application of prospect theory in asset pricing attracted close attention in behavioral finance. Out of these, we discuss the most relevant findings that are related to our model. Levy and Levy (2004) argues that the mean-variance optimization of standard asset pricing models applies to prospect theory as well. In particular, they find that the prospect theory efficient set is a subset of the mean-variance efficient frontier and even by including probability distortion, the two sets almost coincide. Their results are confirmed and extended to asset pricing models in the paper of De Giorgi et al.

(2003) and Barberis and Huang (2008). The latter papers show that if the financial market equilibrium exists then the security market line theorem of CAPM holds under cumulative prospect theory as well. This finding also means that diversifying investors hold portfolios from the capital market line, and therefore, the relationship between volatility and expected return is linear for efficient portfolios.

Adding this linearity to the theory of the inclusion of previous gains and losses (as in Eq. (1)) leads to linearly decreased and increased portfolio volatility subsequent to gains and losses respectively.

The dynamics of portfolio volatility

In the followings, we present an analytical derivation of the dynamics of volatility. We define the intertemporal change of volatility in Eq. (2) and (3). Here we assume that in an equilibrium setting the price of risk does not change over time; nonetheless, the required return is not constant but follows the dynamics of

$$\mu_t = r_{f,t} + \beta\sigma_t = \mu_{t-1} + \alpha(\mathrm{r}_{t-1} - \mu_{t-1}) + r_{f,t} - \\ -r_{f,t-1} = r_{f,t} + \beta\sigma_{t-1} + \alpha\left(\mathrm{r}_{t-1} - r_{f,t-1} - \beta\sigma_{t-1}\right). \quad (2)$$

Here we applied the aforementioned linearity between risk and expected return of the CAPM setting. As long as we assume that investors hold well-diversified portfolios, only systematic risk is priced; therefore, σ_t stands for the market-related portfolio risk (henceforth volatility). β represents the slope of capital market line or the price of risk. The economic interpretation of Eq. (2) is that subsequent to losses investors aim to obtain higher expected return; however, according to equilibrium pricing, they can only achieve their goal by investing in riskier assets or increasing leverage. Solving the latter equation for the dynamics of volatility yields

$$\sigma_t = \sigma_{t-1} + \frac{\alpha}{\beta}\left(\mathrm{r}_{t-1} - r_{f,t-1} - \beta\sigma_{t-1}\right) = \sigma_{t-1} + \frac{\alpha}{\beta}e_{t-1} = \\ \sigma_{t-1} + \frac{\alpha}{\beta}\sigma_{t-1}\Delta W_{t-1} = \sigma_{t-1}\left(1 + \frac{\alpha}{\beta}\Delta W_{t-1}\right), \quad (3)$$

where e_{t-1} and ΔW_{t-1} represent a normally distributed error term and the change in the standard Wiener process in discrete time. Eq. (3) reveals that σ_t follows a unit-root process with constant conditional mean, that is

$$E[\sigma_{t+\tau}|\mathcal{F}_t] = \sigma_t + E\left[\sum_{i=t}^{t+\tau-1}\frac{\alpha}{\beta}\sigma_i\Delta W_i\,|\mathcal{F}_t\right] = \sigma_t + \\ + \frac{\alpha}{\beta}\sum_{i=t}^{t+\tau-1}E[\sigma_i|\mathcal{F}_t]E[\Delta W_i|\mathcal{F}_t] = \sigma_t + \frac{\alpha}{\beta}\Delta W_t, \quad (4)$$

where $\mathcal{F}_t$ stands for the filtration (information available) at time t. Here, the separation of contemporaneous volatility and noise requires the assumption that they are uncorrelated (only the delayed response yields a negative correlation). According to Eq. (4), the volatility process seems to be valid and realistic in the sense that periodical volatility tends to remain in a finite interval over a long horizon; it converges neither to infinity nor to zero.

Furthermore, Eq. (3) reveals another interesting pattern: it is very similar to the Threshold Generalized Autoregressive Conditional Heteroscedasticity (TGARCH) model introduced by Zakoian (1994) that is one of the most accurate heteroscedasticity models based on goodness-of-fit tests (Awartani and Corradi, 2005; Tavares et al., 2008). In particular, TGARCH models are defined as

$$\sigma_t = K + \delta\sigma_{t-1} + \alpha^+ {e_{t-1}}^+ + \alpha^- {e_{t-1}}^- \quad (5)$$

where ${e_{t-1}}^+ = \begin{cases} e_{t-1}\ if\ e_{t-1} > 0 \\ 0\ if\ e_{t-1} \leq 0 \end{cases}$ and ${e_{t-1}}^- = \begin{cases} e_{t-1}\ if\ e_{t-1} \leq 0 \\ 0\ if\ e_{t-1} > 0 \end{cases}$. Therefore, the special case of Eq. (3) implies that K=0, δ=1 and $\alpha^+ = \alpha^- = \frac{\alpha}{\beta}$. Effects of previous gains and losses could be handled separately in Eq. (3) as well by using different α^+ and α^-; however, as we show below, previous gains play only a much less significant role in the asymmetric effect on volatility. Nevertheless, distinct α^+ and α^- would also have a reasonable economic interpretation: considering that extreme gains do not cause a negative required return, that is, investors cannot and will not invest in assets with negative expected return irrespective of the previous outcomes, gains should have a less significant effect on the subsequently required return, therefore, α^+ should differ from α^-.

Another interpretation of Eq. (3) leads to another well-fitting, asymmetric GARCH model: the Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH) by Nelson, (1991). Dividing by σ_{t-1} and taking the natural logarithms of both sides yields

$$\ln\sigma_t = \ln\sigma_{t-1} + \ln\left(1 + \frac{\alpha}{\beta}\Delta W_{t-1}\right) \quad (6)$$

Taking the Taylor approximation around $\Delta W_{t-1} = 0$ then gives

$$\ln\sigma_t = \ln\sigma_{t-1} + \frac{\alpha}{\beta}\Delta W_{t-1} - \frac{1}{2}\left(\frac{\alpha}{\beta}\right)^2 {\Delta W_{t-1}}^2 + \\ + \sum_{n=3}^{\infty}\frac{(-1)^{n-1}}{n!}\left(\frac{\alpha}{\beta}\right)^n {\Delta W_{t-1}}^n. \quad (7)$$

Due to the well-known property of the Wiener process, as Δt approaches to zero (the continuous time version is considered) third and higher order polynomials of ΔW_t vanish and ${\Delta W_t}^2 = \Delta t$. Therefore, the continuous time version of Eq. (7) can be written as

$$\ln\sigma_t = \ln\sigma_{t-1} + \frac{\alpha}{\beta} dW_{t-1} - \frac{1}{2}\left(\frac{\alpha}{\beta}\right)^2 dt, \tag{8}$$

or by multiplying both sides by 2

$$\ln{\sigma_t}^2 = \ln{\sigma_{t-1}}^2 + 2\frac{\alpha}{\beta} dW_{t-1} - \left(\frac{\alpha}{\beta}\right)^2 dt. \tag{9}$$

The similarity to EGARCH comes from its definition of

$$\ln{\sigma_t}^2 = \omega + \beta_1[\theta dW_{t-1} + \lambda(|dW_{t-1}| - E|dW_{t-1}|)] + \\ +\alpha_1 \ln{\sigma_{t-1}}^2, \tag{10}$$

where $\omega = -\left(\frac{\alpha}{\beta}\right)^2 dt, 2\frac{\alpha}{\beta} = \beta_1\theta, \lambda = 0$ and $\alpha_1 = \alpha$ yields exactly Eq. (9). The unit-root, constant conditional mean property of Eq. (9) is again found by applying Itō's lemma for

$$x_t \equiv \ln{\sigma_t}^2, dx_t = 2\frac{\alpha}{\beta} dW_t - \left(\frac{\alpha}{\beta}\right)^2 dt. \tag{11}$$

Then the inverse function is defined as

$$\sigma_t = e^{0.5x_t}. \tag{12}$$

By Itō's lemma

$$d\sigma_t = \left[-\left(\frac{\alpha}{\beta}\right)^2 \frac{\partial\sigma_t}{\partial x_t} + \frac{1}{2}\left(2\frac{\alpha}{\beta}\right)^2 \frac{\partial^2\sigma_t}{\partial {x_t}^2}\right] + 2\frac{\alpha}{\beta}\frac{\partial\sigma_t}{\partial x_t} dW_t = \\ = \left[-\left(\frac{\alpha}{\beta}\right)^2 0.5\sigma_t + \frac{1}{2}\left(2\frac{\alpha}{\beta}\right)^2 0.25\sigma_t\right] + \\ +2\frac{\alpha}{\beta} 0.5\sigma_t dW_{t-1} = \frac{\alpha}{\beta}\sigma_t dW_t. \tag{13}$$

Again, the correlation between concurrent volatility and noise has zero expected value, therefore, the conditional mean is constant regardless of the length of delay. In conclusion, the TGARCH and EGARCH models are implications of prospect theory in a dynamic setting and they represent the underlying volatility process in discrete and continuous time respectively.

The dynamics of market volatility

We have derived so far the change of investors' risk attitude and the dynamics of the volatility of their portfolios. However, reasons behind the change of market volatility have not yet been covered. In this section, we propose an explanation for the positive relationship between the dynamics of market volatility and the riskiness of investors' portfolio based on a simple market microstructural idea.

As discussed above, mental accounting leads to a clear pattern in investors choice that depends on the previous unexpected price shock: losses increase the subsequent demand for risky assets, whereas, gains reduce their demand. If we stick to the idea that investors hold the market portfolio or at least a well-diversified one that is correlated with the market, one can clearly see the following market microstructural situation: in line with the model of Glosten and Milgrom (1985) we find informed and uninformed traders in the market with probabilities π and (1-π) that place market orders. In their model the informed investors know the exact value of an asset that can be either high (v^H) or low (v^L) and place their orders accordingly. Other participants of the market, such as the specialists that provide the liquidity by placing limit orders (thus define the spread) know only the probability of the true value that is P($v = v^H$)=θ and P($v = v^L$)=1-θ. Uninformed investors place their buy and sell orders completely randomly, hence, the probabilities of buy and sell orders coming from uninformed traders are equal (P=0.5).

Therefore, the profit of specialists is generated by the losses on transactions with informed investors and gains on transactions with uninformed investors. If we assume that the market is competitive, their zero expected profit criteria for transactions at the buy limit price and at the sell limit price yield the equilibrium ask and bid prices respectively (and the spread as their difference).

Moreover, if we introduce the pattern discussed in the previous sections, the spread changes in the following way: let us assume that, based on mental accounting heuristic, there is a new type of investors in addition to informed and uninformed traders, the heuristic-driven investor. This latter definition is not new in related literature: although, according to the pioneering papers of Glosten and Milgrom (1985) and Kyle (1985) uninformed traders are defined as those who do not possess fundamental information on assets, irrespective of their motives, a definition similar to our setting has already appeared in the paper of Bloomfield et al. (2009b), in which uninformed investors can have other trading motives than fundamental (e.g. behavioral). In their study the similar three-class distinction of investors is analyzed, where informed and uninformed investors and liquidity traders are present. The liquidity trader, however, may follow a behavioral pattern according to the dynamics of liquidity demand we have discussed so far, hence, we call this class the heuristic-driven trader.

Turning back to the introduction of such traders in the equilibrium criteria, let π and δ and (1-π-δ) stand for the shares of informed, heuristic-driven and uninformed traders (the probability of their trades). Then, subsequent to a negative market shock, the zero profit criteria of specialists at the ask and bid prices can be defined as

$$\pi(a - v^H) + \delta(a - v) + 0.5(1 - \pi - \delta)(a - v) = 0, \tag{14}$$

$$(1-\theta)\pi(v^L - b) + 0.5(1-\pi-\delta)(v-b) = 0. \quad (15)$$

Then the ask price is given as

$$\frac{\theta\pi v^H + 0.5(1-\pi+\delta)v}{\theta\pi + 0.5(1-\pi+\delta)} = v + \frac{\theta\pi(v^H - v)}{\theta\pi+0.5(1-\pi+\delta)} = v + \\ + \frac{\theta\pi(1-\theta)(v^H - v^L)}{\theta\pi+0.5(1-\pi+\delta)}, \quad (16)$$

whereas the bid price follows

$$\frac{(1-\theta)\pi v^L + 0.5(1-\pi-\delta)v}{(1-\theta)\pi + 0.5(1-\pi-\delta)} = v + \frac{(1-\theta)\pi(v^L - v)}{(1-\theta)\pi+0.5(1-\pi-\delta)} = \\ v - \frac{\theta\pi(1-\theta)(v^H - v^L)}{(1-\theta)\pi+0.5(1-\pi-\delta)} \quad (17)$$

One can clearly see the economic processes underlying in the aforementioned formulas: if herustic-driven traders are present the midprice differs from the expected value. Subsequent to a negative shock, the δ proportion of investors place buy orders at the ask price; however, they do not form supply at the bid price. Furthemore, their uninformed trades contribute positively to the profit; therefore, the equilibrium ask price declines as in Eq. (16). Still, their existence lowers the proportion of uninformed investors; hence, the equilibrium bid price declines as well as in Eq. (17). Although, both the ask and bid prices decline, the zero profit remains intact due to the modified probabilities of incoming buy and sell orders.
Then, the spread in competitive equilibrium can be defined as

$$S_- = \frac{\theta\pi(1-\theta)(v^H - v^L)}{\theta\pi+0.5(1-\pi+\delta)} + \frac{\theta\pi(1-\theta)(v^H - v^L)}{(1-\theta)\pi+0.5(1-\pi-\delta)} = \\ \frac{\theta\pi(1-\theta)(v^H - v^L)}{[\theta\pi+0.5(1-\pi+\delta)][(1-\theta)\pi+0.5(1-\pi-\delta)]} \quad (18)$$

where S_- stands for the spread subsequent to a negative market shock. The spread following positive market shocks is similar except for the sign of δ:

$$S_+ = \frac{\theta\pi(1-\theta)(v^H - v^L)}{[\theta\pi+0.5(1-\pi-\delta)][(1-\theta)\pi+0.5(1-\pi+\delta)]}. \quad (19)$$

Let the spread be defined as a function of Δ where

$$\Delta = \begin{cases} \delta \; for \; negative \; previous \; shocks \\ -\delta \; for \; positive \; previous \; shock \end{cases},$$

$$S(\Delta) = \frac{\theta\pi(1-\theta)(v^H - v^L)}{[\theta\pi+0.5(1-\pi+\Delta)][(1-\theta)\pi+0.5(1-\pi-\Delta)]}. \quad (20)$$

Then $S_- > S_+$ if and only if $S(|\Delta|) > S(-|\Delta|)$. As the numerator takes on a constant value in the function, we focus on the denominator value $f(\Delta)$. Then, $S_- > S_+$ if and only if $f(|\Delta|) < f(-|\Delta|)$, where

$$f(\Delta) = [\theta\pi + 0.5(1-\pi+\Delta)][(1-\theta)\pi + 0.5(1-\pi-\Delta)] \quad (21)$$

is a concave, second order polynomial function of Δ. If and only if the maximum place of this function is reached in its negative domain, then $f(|\Delta|) < f(-|\Delta|)$ is always true. Therefore, it is enough to test whether

$$\operatorname*{argmax}_{\Delta} f(\Delta) < 0. \quad (22)$$

According to the first order condition

$$0.5[(1-\theta)\pi + 0.5(1-\pi-\Delta)] \\ - 0.5[\theta\pi + 0.5(1-\pi+\Delta)] = 0,$$

$$\Delta = (1-2\theta)\pi. \quad (24)$$

Hence, if and only if $\theta > 0.5$, then $\operatorname*{argmax}_{\Delta} f(\Delta) < 0$, $f(|\Delta|) < f(-|\Delta|)$, $S(|\Delta|) > S(-|\Delta|)$ and $S_- > S_+$. In other words, if the probability of a subsequent higher value is greater than that of a lower value, then spread is greater subsequent to a negative shock than it is following a positive shock.
The economic intuition behind an average $\theta > 0.5$ is simple as the growth of value is one of the basic assumptions in analyzing capital markets. This greater probability of a higher value is confirmed by empirical studies as well: although the authors apply a bit different methodology, Easley et al. (2002) and Brennan et al. (2014) measure the probability of an increase in the value to be $P(v|v = v^H) = 0.67$ and 0.614.
In conclusion, we argue that, on average, the spread increases subsequent to losses and decreases subsequent to gains. Moreover, considering that continuous market orders at the ask and bid prices define the standard deviation of price changes, our explanation clearly implies that previous positive (negative) shocks decrease (increase) both the spread and the volatility accordingly.
Related literature provides further support to our aforementioned reasoning. Park and Sabourian (2011) analyze a similar setting based on the Glosten-Milgrom model and find that people act as contrarian if their information leads them to concentrate on middle values. Kaniel et al. (2008), Choe et al. (1999), Grinblatt and Keloharju (2000, 2001), Richards (2005), Bloomfield et al. (2009a) also confirm the existence of such contrarian traders. Moreover, according to Lof (2014), the introduction of contrarian trading in asset pricing models dramatically increases the predictive power of the models. Furthermore, our former, mental accounting-based explanation for the contrarian activity is supported

by Yao and Li (2013) who argue that prospect theory investors can behave as contrarian noise traders in a market, while Kadous et al. (2014) finds that investors act as contrarians if and only if they have held in the past the particular asset that they buy in the subsequent period; this latter provides evidence that mental accounting and prospect theory are indeed responsible for the negative feedback trading instead of an alternative exogenous factor. For the well-documented, significant, positive relationship between spread and price volatility see Hussain (2011) Wang and Yau (2000), Wyart et al. (2008).

CONCLUDING REMARKS

We find that asymmetric and autoregressive volatility measured in previous empirical studies in asset pricing can be derived from and attributed to intertemporal choice of investors, assuming that they behave according to prospect theory in a dynamic setting. We show that, in contrast to most of the studies on this topic, individuals should tend to become less risk-averse (or risk-seeking until a given point) and more risk-averse subsequent to losses and gains respectively, which leads to the rejection of the volatility feedback and BHS explanations for asymmetric volatility. Furthermore, we argue that the third existing explanation (the leverage effect) does not hold either, as we find a volatility decreasing effect of both previous gains and losses of a given asset when controlling for the market return.

However, our proposed model is based on a negative relationship between market returns and market volatility; and is thus able to capture the dynamics of volatility measured empirically. Combining the linear relationship between risk and return, as presented above in detail, and the aforementioned pattern in the intertemporal choice (i.e. the required return) yields the autoregressive conditional heteroscedasticity model presented in this paper. We show that the discrete and continuous time alternatives of the main equation result in the TGARCH and EGARCH models respectively, which in particular are measured to be two of the regressions with the highest goodness-of-fit in most of the empirical studies.

Potential ways of further research include various opportunities. First, an experimental analysis would be interesting to show whether these patterns are found in a laboratory environment as well if the focus is on the effect of breaking-even. Second, the influence of this behavior on asset liquidity and market microstructure could be analyzed in detail including an empirical analysis of the probability estimation of heuristic-driven traders. Third, the application of the proposed model in mathematical finance could reveal further interesting patterns; in particular, asymmetric stochastic volatility models (Heston and Nandi, 2000) in option pricing are found to provide better estimates on option prices and fit the "volatility smile" of the Black-Scholes implied volatilities, which regressions could be further improved by including the proposed model described in this paper. Finally, the introduction of cognitive research, such as the neuroeconomic approach, could reveal further underlying factors behind the behavioral patterns presented in this paper.

ACKNOWLEDGEMENTS

We gratefully acknowledge the help of Terry Odean, who provided us the individual trading dataset. We also would like to express our gratitude for the thoughtful remarks of Adam Zawadowski, which have significantly contributed to our paper. We thank Zsolt Bihary and Niklas Wagner for their comments and suggestions of at the 6th Annual Financial Market Liquidity Conference 2015 at Corvinus University of Budapest. Mihály Ormos acknowledges the support by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences. Dusán Timotity acknowledges the support by the Pallas Athéné Domus Scientiae Foundation. This research was partially supported by Pallas Athene Domus Scientiae Foundation.

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