IDENTIFICATION AND LQ DIGITAL CONTROL OF A SET OF EQUAL CYLINDER ATMOSPHERIC TANKS – SIMULATION STUDY

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ABSTRACT

Time-delays (dead time) are found in many processes in industrial practice. Time-delays are mainly caused by the time required to transport mass, energy and information. In many cases time-delay is caused by the effect produced by the accumulation of a large number of low-order systems. One of possibilities of control of such processes is their approximation by lower-order model with time-delay. The contribution is focused on the design of an algorithm for digital control of high-order process that is approximated by a second-order model with time-delay. The controller algorithms use the digital modification of the linear quadratic (LQ) Smith predictor (SP). The LQ criterion was combined with pole assignment principle. These algorithms were applied to the control of a set of equal liquid cylinder atmospheric tanks.

INTRODUCTION

Some technological processes in industry are characterized by high-order dynamic behaviour or large time constants and time-delays. For control engineering, such processes can often be approximated by the FOTD (first-order-time-delay) model. Time-delay in a process increases the difficulty of controlling it. However using the approximation of a high-order process by a lower-order model with time-delay provides simplification of the control algorithms.

Let us consider a continuous-time dynamical linear SISO (single input $u(t)$ – single output $y(t)$) system with time-delay L. The transfer function of a pure transportation lag is e^{-Ls} where s is a complex variable. Overall transfer function with time-delay is in the form

$$G_L(s) = G(s)e^{-Ls} \tag{1}$$

where $G(s)$ is the transfer function without time-delay.

Processes with time-delay are difficult to control using standard feedback controllers. When a high performance of the control process is desired or the relative time-delay is very large, a predictive control strategy must be used. The predictive control strategy includes a model of the process in the structure of the controller. The first time-delay compensation algorithm was proposed by (Smith 1957). This control algorithm known as the Smith predictor contained a dynamic model of the time-delay process and it can be considered as the first model predictive algorithm.

Historically first modifications of time-delay algorithms were proposed for continuous-time (analogue) controllers using various approaches. In industrial practice, the implementation of the time-delay compensation algorithms on continuous technique is difficult.

One of possible approaches to control of process with time-delay is digital Smith predictor based on polynomial theory.

Polynomial methods are design techniques for complex systems (including multivariable), signals and processes encountered in control, communications and computing that are based on manipulations and equations with polynomials, polynomial matrices and similar objects. Systems are described by input-output relations in fractional form and processed using algebraic methodology and tools (Šebek and Hromčík 2007). Controller design consists in solving polynomial (Diophantine) equations. This paper is oriented to design of a robust LQ control using polynomial theory. The Diophantine equations can be solved using the uncertain coefficient method – which is based on comparing coefficients of the same power. This is transformed into a system of linear algebraic equations (Kučera 1993).

The digital pole assignment Smith predictor was designed using a polynomial approach in (Bobál et al. 2011). The design of this controller was extended by a method for a choice of a suitable pole assignment of the characteristic polynomial. Because the classical analogue Smith predictor is not suitable for control of unstable and integrating time-delay processes, the polynomial digital LQ Smith predictor for control of unstable and integrating time-delay processes has been designed in (Bobál et al. 2014).

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It is obvious that the majority processes met in industrial practice are influenced by uncertainties. The uncertainties suppression can be solved either implementation adaptive control or robust control. Some adaptive (self-tuning) modifications of the digital Smith predictors are designed in (Hang et al. (1989; 1993); Bobál et al. 2011). Two versions of these controllers were implemented into MATLAB Toolbox (Bobál et al. 2012a; Bobál et al. 2012b).

The paper is organized in the following way. The general problem of a control of the time-delay systems with regard to polynomial approach is described in Section 1. The fundamental principle of digital Smith predictor is described in Section 2. The high-order system (a set of n equal liquid cylinder atmospheric tanks) is analysed in Section 3. Section 4 contains description of identification procedures. Two versions of the primary polynomial LQ controller, which are components of the digital Smith Predictor, are proposed in Section 5. The simulation verifications of individual control-loops with their results are presented in Section 6. Section 7 concludes this paper.

PRINCIPLE OF DIGITAL SMITH PREDICTOR

The discrete versions of the SP and its modifications are more suitable for time-delay compensation in industrial practice. The block diagram of a digital SP (see Hang et al. (1989, 1993)) is shown in Fig. 1. The function of the digital version is similar to the classical analogue version.

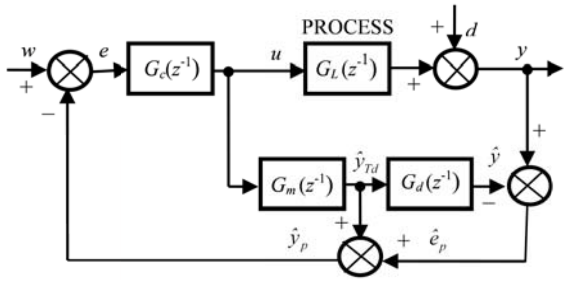

Figure 1: Block diagram of a digital Smith Predictor

Number of high-order industrial processes can be approximated by a reduced order model with a pure time-delay. In this paper, the following second-order linear model with a time-delay is considered

$$G_L\left(z^{-1}\right)=\frac{B\left(z^{-1}\right)}{A\left(z^{-1}\right)}z^{-d}=\frac{b_1z^{-1}+b_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}z^{-d} \qquad (1)$$

The term z^{-d} represents the pure discrete time-delay. The time-delay is equal to dT_0 where T_0 is the sampling period. The block $G_m(z^{-1})$ represents process dynamics without the time-delay and is used to compute an open-loop prediction. The numerator in transfer function $G_d(z^{-1})$ is replaced by its static gain $B(1)$, i.e. for $z = 1$. This is to avoid problem of controlling a model with a $B(z^{-1})$, which has non-minimum phase zeros caused by a high sampling period or fractional delay. Since $B(z^{-1})$ is not controllable as in the case of a time-delay, it is moved out of the prediction model $G_m(z^{-1})$ and is treated together with the time-delay block, as shown in Fig. 1. The difference between the output of the process y and the model including time-delay $\hat{y}$ is the predicted error $\hat{e}_p$ as shown in Fig. 1, whereas e and d are the error and the measured disturbance, w is the reference signal. The primary (main) controller $G_c(z^{-1})$ can be designed by different approaches (for example digital PID control or methods based on polynomial approach). The detailed description of the block diagram (Fig. 1) is in (Bobál et al. 2011).

SERIES OF EQUAL LIQUID TANKS

In many cases in industrial practice the time-delay is caused by the effect produced by the accumulation of a large number of low-order systems. Consider a set of n equal cylinder atmospheric tanks, where a single tank is shown in Fig. 2 (Torrico and Normey-Rico 2007) and the whole set is shown in Fig. 3. In this system, the output flow of tank i (q_{iO}) feeds tank $i + 1$; that is, the input flow tank $i + 1$ is $q_{(i+1)I}=q_{iO}$. If all the tanks have the same area (F) of crosscut and the individual tank levels are near to an operating point, then the dynamic behaviour of the level in each tank h_i can be modelled by a linear system

$$\begin{aligned} F\frac{dh_i}{dt}&=q_{iI}-q_{io} \\ q_{iO}&=K_1h_i \end{aligned} \qquad (2)$$

where K_1 is a constant that depends on the tank characteristics.

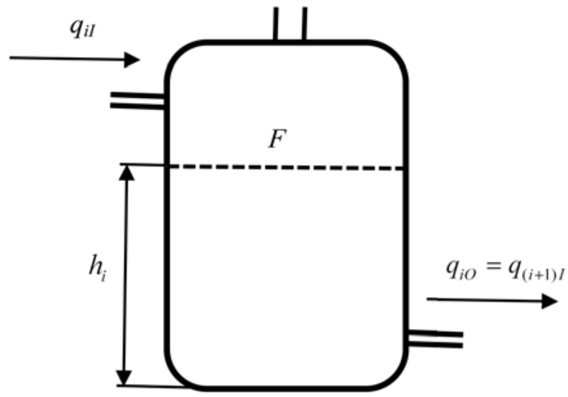

Figure 2: Schema of liquid cylinder tank

Consider a set of n tanks as shown in Fig. 3. Thus, the transfer function relating the input follow in tank i and its level is given by

$$h_i\left(s\right)=\frac{1/K_1}{Ts+1}q_{iI}\left(s\right) \qquad (3)$$

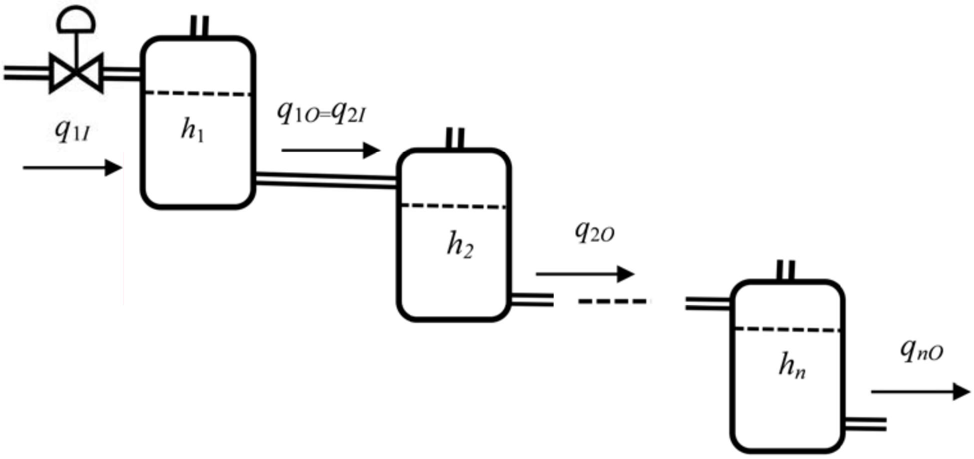

Figure 3: Series of liquid cylinder tanks

where $T = F / K_1$ is time constant.

For tank 1 is

$$h_1(s) = \frac{1/K_1}{Ts+1} q_{1I}(s) \tag{4}$$

and for tank 2 using the second equation of (2)

$$h_2(s) = \frac{1/K_1}{Ts+1} q_{2I}(s) = \frac{1/K_1}{Ts+1} q_{1O}(s) = \frac{1/K_1}{Ts+1} K_1 h_1(s) \tag{5}$$

Then, using the expression (4) it follows

$$h_n(s) = G(s) q_{1I}(s) = \frac{K_g}{(Ts+1)^n} q_{1I}(s) \tag{6}$$

and the transfer function of the series of tanks system is

$$G(s) = \frac{h_n(s)}{q_{1I}(s)} = \frac{K_g}{(Ts+1)^n} \tag{7}$$

where $K_g = 1/K_1$ is static gain of the system.

Consider for simulation experiments of control model (7) the eight – order system, i. e. n = 8. Following parameters of the individual liquid tanks are considered (see Fig. 1): high of tank h = 1.5 m; diameter of tank d_T = 1 m; tank area $F = \frac{\pi d_T}{4} = 0.785$ m^2; set point h_1 = 1 m; time constant T = 2 min;

constant $K_1 = \frac{F}{T} = \frac{0.785}{2} = 0.3925$ m^{2}min^{-1};

static gain $K_g = \frac{1}{K_1} = \frac{1}{0.3925} = 3.08$ m^{-2} min.

The resulting transfer function is given by

$$G(s) = \frac{h_8(s)}{q_{1I}(s)} = \frac{3.08}{(2s+1)^8} \tag{8}$$

If (8) is the transfer function of a continuous-time dynamic system, then the following expression for the discrete transfer function with zero-order holder and sampling period T_0 is valid

$$G(z^{-1}) = \frac{b_1 z^{-1} + b_2 z^{-2} + \ldots + b_8 z^{-8}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_8 z^{-8}} \tag{9}$$

The transfer function (9) was approximated by the discrete second-order model with time-delay (1).

It is obvious that linear model (8) was obtained without complying with valves contain hysteresis and other nonlinearities that the series liquid tanks system contains (Chalupa et al. 2011).

IDENTICATION OF SERIES LIQUID TANKS

Determination of number time-delay steps

In this paper, the number of time-delay steps is obtained using an off-line identification by the least squares method (LSM). The measured process output (liquid level $h_8(k)[\mathrm{m}]$ near operating flow) is influenced by input – generator of white noise which excites changes of flow rate $q_{1I}(k)\left[\mathrm{m}^3\,\mathrm{min}^{-1}\right]$. The non-measurable system disturbances cause errors $\boldsymbol{e}$ in the determination of model parameters and therefore real output vector is in the form

$$\boldsymbol{y} = \boldsymbol{F\Theta} + \boldsymbol{e} \tag{10}$$

The matrix $\boldsymbol{F}$ has dimension (N-n-d, 2n), the vector $\boldsymbol{y}$ (N-n-d) and the vector of parameter model estimates $\hat{\boldsymbol{\Theta}}$ (2n). N is the number of samples of measured input and output data, n is the model order. It is possible to obtain the LSM expression for calculation of the vector of the parameter estimates

$$\hat{\boldsymbol{\Theta}} = \left(\boldsymbol{F}^T \boldsymbol{F}\right)^{-1} \boldsymbol{F}^T \boldsymbol{y} \tag{11}$$

Equation (11), where n = 8, serves for calculation of the vector of the parameter estimates $\hat{\boldsymbol{\Theta}}$ using N samples of measured input-output data. The form of

individual vectors and matrices in equations (10) and (11) are introduced in (Bobál et al. 2013a,b).
Consider that model (1) is the deterministic part of the stochastic process described by the ARX (regression) model

$$\begin{aligned} y(k) = -a_1 y(k-1) - a_2 y(k-2) + \\ + b_1 y(k-1-d) + b_2 y(k-2-d) + e_s(k) \end{aligned} \tag{12}$$

where $e_s(k)$ is the random non-measurable component.
The vector of parameter model estimates is computed by solving equation (11)

$$\hat{\boldsymbol{\Theta}}^T(k) = \begin{bmatrix} \hat{a}_1 & \hat{a}_2 & \hat{b}_1 & \hat{b}_2 \end{bmatrix} \tag{13}$$

and is used for computation of the predicted output

$$\begin{aligned} \hat{y}(k) = -\hat{a}_1 y(k-1) - \hat{a}_2 y(k-2) + \\ \hat{b}_1 u(k-1-d) + \hat{b}_2 u(k-2-d) \end{aligned} \tag{14}$$

The quality of identification can be considered according to error, i.e. the deviation

$$\hat{e}(k) = y(k) - \hat{y}(k) \tag{15}$$

Continuous-time system (8) was identified by discrete model (1) using off-line LSM (11) for different time-delay dT_0; $T_0 = 1$ min . The White Noise Generator was used as excitation input signal. A criterion of the identification quality is based on sum of squares of error

$$J_{\hat{e}^2}(d) = \sum_{k=1}^{N} \hat{e}^2(k) \tag{16}$$

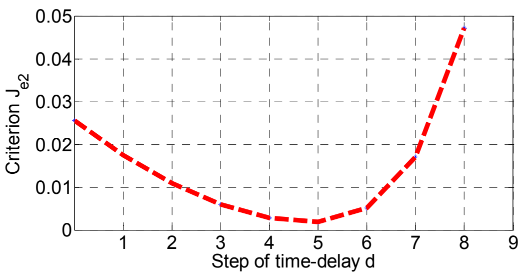

Figure 4: Criterion of Quality Identification for $d \in [0,8]$

This criterion represents accuracy of process identification. It is obvious from Fig. 4 that minimum value of the criterion (21) is reached when the number of time-delay steps d = 5. Then it is possible to use model

$$\hat{G}_L(z^{-1}) = \frac{\hat{b}_1 z^{-1} + \hat{b}_2 z^{-2}}{1 + \hat{a}_1 z^{-1} + \hat{a}_2 z^{-2}} z^{-5} \tag{17}$$

for an approximation of model (8).

Identification procedures

Two identification procedures were used for calculation of parameter estimates of model (17). Following individual parameters were used for off-line LSM (11): $n = 2$; $d = 5$; $N = 300$.

Beside LSM the MATLAB function from the Optimization Toolbox

$$x = \text{fminsearch}('name\_fce', x_0) \tag{18}$$

was also used for the off-line process identification. This function finds minimum of an unconstrained multivariable function using derivative-free method. Algorithm "fminsearch" uses the simplex search method of (Lagaris et al. 1998). This is a direct search method that does not use numerical or analytic gradients.
The difference between static gain $K_g = 3.08$ of the continuous-time transfer function (8) and estimation of the static gain of discrete transfer function (17) can serve as a good criterion for the quality of identification.

$$\hat{K}_g = \frac{\hat{b}_1 + \hat{b}_2}{1 + \hat{a}_1 + \hat{a}_2} \tag{19}$$

Identification using LSM

Discrete model for sampling period $T_0 = 1$ min

$$\hat{G}_{L1}(z^{-1}) = \frac{-0.0161 z^{-1} + 0.0798 z^{-2}}{1 - 1.7789 z^{-1} + 0.7996 z^{-2}} z^{-5} \tag{20}$$

was obtained using LSM method, $K_{g1} = 3.0733$.

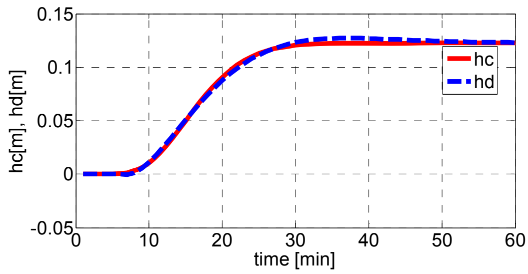

Figure 5: Comparison of step responses of models (8) and (20)

Comparison of step responses of continuous-time model (8) and discrete model (20) is shown in Fig. 5, where hc is the step response of the continuous-time model (8) and hd is step response of the discrete model (20). The input step signal $\Delta q_{1l} == 0.04\ \mathrm{m^3\,min^{-1}}$ was chosen so that tank level is near to an operating point. It is obvious from numerator of the transfer function (20) than this system is slightly non-minimum phase (this is incurred by an identification error).

Identification using algorithm "fminsearch"

Discrete model for sampling period $T_0 = 1$ min

$$\hat{G}_{L2}\left(z^{-1}\right)=\frac{0.0309z^{-1}+0.0286z^{-2}}{1-1.777z^{-1}+0.7964z^{-2}}z^{-5} \qquad (21)$$

was obtained using fminsearch method, $K_{g2}=3.08$.

Comparison of step responses of continuous-time (8) and discrete model (21) is shown in Fig. 6. The input step signal Δq_{1I} is the same as in a previous case.

Model (21) is more accurate than model (20'') and therefore was chosen for the design of two versions primary polynomial LQ controller for control of the series of liquid cylinder tanks.

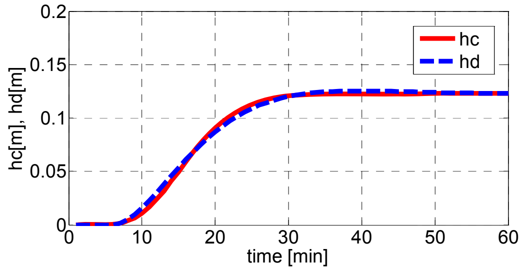

Figure 6: Comparison of step responses of models (8) and (21)

DESIGN OF PRIMARY POLYNOMIAL 2DOF LQ CONTROLLER

The design of the control algorithm is based on a general block scheme of a closed-loop with two degrees of freedom (2DOF) according to Fig. 7. The controller synthesis consists in the solving linear polynomial (Diophantine) equations. From first polynomial equation

$$A(z^{-1})K(z^{-1})P(z^{-1})+B(z^{-1})Q(z^{-1})=D(z^{-1}) \qquad (22)$$

it is possible to compute 7 feedback controller parameters – coefficients of the polynomials Q, P. Polynomial $D\left(z^{-1}\right)$ is the characteristic polynomial and $K\left(z^{-1}\right)=1-z^{-1}$.

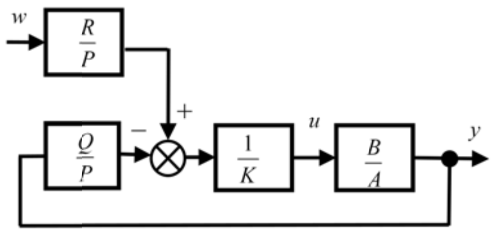

Figure 7: Block diagram of a closed loop 2DOF control system

Asymptotic tracking of the reference signal w is provided by the feedforward part of the controller which is given by solution of the following polynomial Diophantine equation

$$S(z^{-1})D_w\left(z^{-1}\right)+B(z^{-1})R(z^{-1})=D(z^{-1}) \qquad (23)$$

For a step-changing reference signal value, polynomial $D_w(z^{-1}) = 1 - z^{-1}$ and S is an auxiliary polynomial which does not enter into the controller design. Then it is possible to derive the polynomial R from equation (23) by substituting $z = 1$

$$R=r_0=\frac{D(1)}{B(1)} \qquad (24)$$

The 2DOF controller output is given by

$$u(k)=\frac{r_0}{K\left(z^{-1}\right)P\left(z^{-1}\right)}w(k)-\frac{Q\left(z^{-1}\right)}{K\left(z^{-1}\right)P\left(z^{-1}\right)}y(k) \qquad (25)$$

Two primary polynomial LQ controllers are derived in this paper using minimization of LQ criterion (Kučera 1991). Spectral factorization by means of the MATLAB Polynomial Toolbox 3.0 (Šebek 2014) is used for a minimization procedure.

The design of two LQ controllers for control of the second-order system with time-delay (1) is in detail derived in (Bobál et al. 2014; Bobál et al. 2015).

Minimization of LQ Criterion Using $u(k)$

In the first case the linear quadratic control methods try to minimize the quadratic criterion which uses penalization of the value of the controller output

$$J=\sum_{k=0}^{\infty}\left\{\left[w(k)-y(k)\right]^2+q_u\left[u(k)\right]^2\right\} \qquad (26)$$

where q_u is the so-called penalization constant, which gives the influence of the controller output to the value of the criterion. In this paper, criterion minimization (26) will be realized through the spectral factorization for an input-output description of the system

$$A(z)q_uA\left(z^{-1}\right)+B(z)B\left(z^{-1}\right)=D(z)\delta D\left(z^{-1}\right) \qquad (27)$$

where δ is a constant chosen so that $d_0 = 1$. $A(z)$, $B(z)$ are the second-order polynomials and $D(z)$ is also the second-order polynomial

$$D\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2} \qquad (28)$$

Spectral factorization of polynomials of the first and the second degree can be computed by analytical way; the procedure for higher degrees must be performed iteratively (Bobál et al. 2005). The MATLAB Polynomial Toolbox is used for a computation of spectral factorization (27) using file *spf.m* by command

d = spf(a*qu*a' + b*b') (29)

It is known that by using the spectral factorization (27), it is possible to compute only two suitable poles (α, β). It is obvious from equation (22) that in this case a choice of the fourth-degree polynomial $D(z)$ is optimal

$$D_4\left(z^{-1}\right)=1+d_1z^{-1}+d_2z^{-2}+d_3z^{-3}+d_4z^{-4} \qquad (30)$$

Therefore the other poles (γ, δ) are user-defined. A method for suitable pole assignment and computation of parameters of polynomial (30) was designed in

(Bobál et al. 2014). Then the primary digital 2DOF controller (25) can be expressed in the form

$$\begin{aligned} u(k) &= r_0 w(k) - q_0 y(k) - q_1 y(k-1) - q_2 y(k-2) \\ &+ (1-p_1) u(k-1) + p_1 u(k-2) \end{aligned} \qquad (31)$$

where

$$r_0 = \frac{1+d_1+d_2+d_3+d_4}{b_1+b_2} \qquad (32)$$

and parameters q_0, q_1, q_2, p_1 are computed from (22).

Minimization of LQ Criterion Using $\Delta u(k)$

In the second case the linear quadratic control methods try to minimize the quadratic criterion which uses penalization of the incremental value of controller output

$$J = \sum_{k=0}^{\infty} \left\{ \left[w(k) - y(k) \right]^2 + q_u \left[\Delta u(k) \right]^2 \right\} \qquad (33)$$

Equation (27) for computation of the spectral factorization changes into

$$\begin{aligned} &(1-z) A(z) q_u \left(1-z^{-1}\right) A\left(z^{-1}\right) \\ &+ B(z) q_u B\left(z^{-1}\right) = D(z) \delta D\left(z^{-1}\right) \end{aligned} \qquad (34)$$

It is obvious that the characteristic polynomial in (34) is the three-degree polynomial

$$D\left(z^{-1}\right) = 1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3} \qquad (35)$$

Spectral factorization of (34) gives three optimal poles. However for the 2DOF controller design it is possible to propose other three user-defined real poles of the polynomial

$$\begin{aligned} D_6\left(z^{-1}\right) &= 1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3} \\ &+ d_4 z^{-4} + d_5 z^{-5} + d_6 z^{-6} \end{aligned} \qquad (36)$$

The expressions for computation of individual parameters of polynomial (36) are derived in (Bobál et al. 2015).
Then the 2DOF controller design consists of determination of polynomial parameters (36) using command (29) from the Polynomial Toolbox and solution of the Diophantine equation for computation of feedback controller parameters

$$A_s(z^{-1}) K(z^{-1}) P(z^{-1}) + B(z^{-1}) Q(z^{-1}) = D_6(z^{-1}) \qquad (37)$$

where

$$A_s\left(z^{-1}\right) = 1 + a_{s1} z^{-1} + a_{s2} z^{-2} + a_{s3} z^{-3} \qquad (38)$$

$$a_{s1} = a_1 - 1; \quad a_{s2} = a_2 - a_1; \quad a_{s3} = -a_3 \qquad (39)$$

and

$$\begin{aligned} &K\left(z^{-1}\right) = 1 - z^{-1}; \quad P\left(z^{-1}\right) = 1 + p_1 z^{-1} + p_2 z^{-2}; \\ &Q\left(z^{-1}\right) = q_0 + q_1 z^{-1} + q_2 z^{-2} + q_3 z^{-3} \end{aligned} \qquad (40)$$

and from expression (24)

$$r_0 = \frac{1+d_1+d_2+d_3+d_4+d_5+d_6}{b_1+b_2} \qquad (41)$$

The primary 2DOF controller output is given by

$$\begin{aligned} u(k) &= r_0 w(k) - q_0 y(k) - q_1 y(k-1) - q_2 y(k-2) \\ &+ (p_1 - p_2) u(k-2) - p_2 u(k-3) \end{aligned} \qquad (42)$$

SIMULATION VERIFICATION AND RESULTS

A simulation verification of the designed control algorithms was performed in MATLAB/SIMULINK environment. The robustness of individual control loops was experimentally investigated by a change of the static gain K of the nominal process model. From the point of view of the robust theory it is possible to consider these experiments as the gain margin determination by the parametric uncertainty influence. The experimental process model (8) was used for simulation experiments.
The individual simulation experiments are realized subsequently: the static gain K_g = 3.08 was increased as far as the control closed-loop was in the stability boundary. The experiments are not realized when the static K_g = 3.08 was decreased.

Control Using Primary Controller (31)

Because the subject of this paper is oriented to design of the polynomial robust control, the following simulation experiments have been realized. The discrete transfer function (21)

$$\hat{G}_{L2}\left(z^{-1}\right) = \frac{0.0309 z^{-1} + 0.0286 z^{-2}}{1 - 1.777 z^{-1} + 0.7964 z^{-2}} z^{-5} \qquad (43)$$

with $K_{g2} = 3.08$ is the nominal model.

The penalization factor q_u = 2 was used for all experiments . The characteristic polynomial is given by $D_4(z) = z^4 - 3.3716 z^3 + 4.4271 z^2 - 2.4148 z + 0.5154$ with individual poles
$\alpha, \beta = 0.8358 \pm 0.1301 i; \quad \gamma = 0.8; \quad \delta = 0.9$.

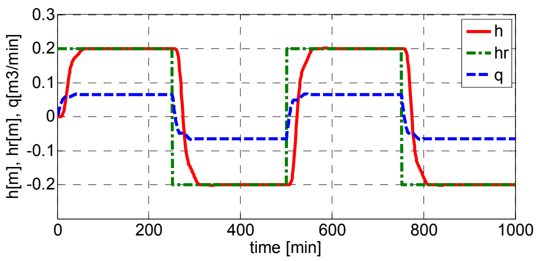

Figure 8: Control of nominal model $G_{L2}(z^{-1})$, $K_{g2} = 3.08$

The individual control parameters of controller (31):
$q_0 = 0.8832;\quad q_1 = -1.5652;\quad q_2 = 0.6968;$
$p_1 = -0.6218;\quad r_0 = 0.0147.$

The control courses of the process output and controller output for the nominal model $G_{L2}\left(z^{-1}\right)$ are shown in Fig. 8.

The discrete transfer function

$$\hat{G}_{P2}\left(z^{-1}\right) = \frac{0.0602z^{-1} + 0.0558z^{-2}}{1 - 1.777z^{-1} + 0.7964z^{-2}} z^{-5} \qquad (44)$$

with $K_{g2c} = 1.95 * 3.08 = 6$ is the perturbed model when the closed-loop control is on the stability boundary. The control courses of the process output and controller output for perturbed model (44) are shown in Fig. 9.

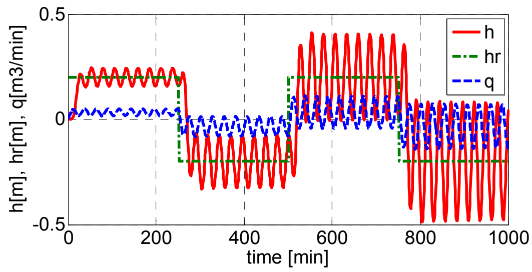

Figure 9: Control of perturbed model $G_{P2}\left(z^{-1}\right)$, $K_{g2c} = 6$

It is obvious from Figs. 8 and 9 that approximate interval of the robust stability of nominal model $G_{L2}\left(z^{-1}\right)$ by increase of the static gain is $K_{g2} = \in \left(3.08,\ 6\right)$.

Control Using Primary Controller (42)

The discrete model (43) was chosen also as the nominal model. The characteristic polynomial is given by

$$D_6\left(z^{-1}\right) = 1 - 3.2798z^{-1} + 4.1025z^{-2} - 2.3413z^{-3} + 0.5339z^{-4} - 0.0144z^{-5} + 0.0001z^{-6}$$

with individual poles
$\alpha, \beta = 0.7925 \pm 0.25321i;\quad \gamma = 0.6948;\quad \delta = 0.01;$
$\lambda = 0.02;\quad \mu = 0.97.$

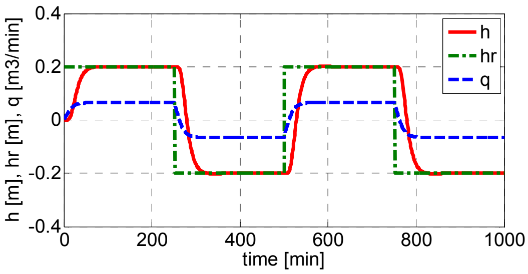

Figure 10: Control of nominal model $G_{\Delta L2}\left(z^{-1}\right)$, $K_{g2} = 3.08$

The individual control parameters of controller (42):
$q_0 = 0.4891;\quad q_1 = -1.115;\quad q_2 = 0.8771;\quad q_3 = -0.2352;$
$p_1 = -0.0151; p_2 = 1.9400e-04;\quad r_0 = 0.0160.$

The control courses of the process output and controller output for the nominal model $G_{\Delta L2}\left(z^{-1}\right)$ are shown in Fig. 10.

The discrete transfer function

$$\hat{G}_{\Delta P2}\left(z^{-1}\right) = \frac{0.0479z^{-1} + 0.0443z^{-2}}{1 - 1.777z^{-1} + 0.7964z^{-2}} z^{-5} \qquad (45)$$

with $K_{g2c} = 1.55 * 3.08 = 4.77$ is the perturbed model when the closed-loop control is on the stability boundary. The control courses of the process output and controller output for perturbed model (45) are shown in Fig. 11.

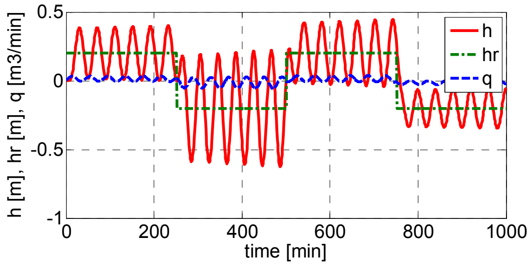

Figure 11: Control of perturbed model $G_{\Delta P2}\left(z^{-1}\right)$, $K_{g2c} = 4.77$

It is obvious from Figs. 10 and 11 that approximate interval of the robust stability of nominal model $G_{\Delta L2}\left(z^{-1}\right)$ by increase of the static gain is $K_{g2} = \in \left(3.08,\ 4.77\right)$. It is possible to improve the robust stability of the controller (42) by increase of the penalization constant q_u. For example, when $q_u = 5$ the interval of the robust stability is $K_{g2} = \in \left(3.08,\ 7.7\right)$.

CONCLUSION

Digital LQ Smith predictor algorithms for control of the high-order processes was designed. The high-order process, a set of equal liquid cylinder atmospheric tanks, was identified by the second-order model with five time- delay steps. The off-line least squares method was used for the identification of the number time-delay steps. The White Noise Generator was used as an excitation input signal. Two controller algorithms are based on polynomial design using the linear quadratic control method. This method minimizes the quadratic criterion by penalizing the value of the controller output $u\left(k\right)$ or its increment $\Delta u\left(k\right)$. The linear quadratic control method was combined with pole-assignment. Both designed controllers were derived to obtain algorithms with easy implementability in industrial practice. The control designs of both modifications were verified by simulation. The results of simulation verifications in both cases demonstrated very good control quality and robustness of designed digital LQ algorithms. It is possible to improve the robust stability by increasing the penalization constant q_u. The contribution of this paper is the fact that a high-order system, which is composed of a set of low-order systems, can be approximated by a low-order model with time-delay. For these approximated model it is possible to design relatively simple digital controllers.

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