LINEAR PREDICTIVE CONTROL OF NONLINEAR TIME-DELAYED MODEL OF LIQUID-LIQUID STIRRED HEAT EXCHANGER

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ABSTRACT

Many nonlinear processes in industry exhibit time delay in their dynamic behavior. Time delay is mainly caused by the time required to transport energy, information or mass, but it can be caused by processing time as well. There are also many cases when compensation of measurable disturbance is required. In case, when the nonlinearity of system is not significant, it can be controlled by linear control method, but accurate system identification in the operating area should be performed. Moreover the suitable control algorithm, which is able to handle these requirements, should be used. In light of these facts, the goal of this paper is to design predictive algorithm for control of nonlinear time-delayed systems with the possibility of measurable disturbance compensation. Basically this paper deals with the essential principles of model predictive control (MPC), design process of the predictive controller, calculation of control law and identification of control process. Identification of control process is also an important part of MPC. This paper describes simulation and verification of designed regulator which was verified in MATLAB/SIMULINK as well.

INTRODUCTION

Heat exchangers are an essential part of many technologies in energy and chemical industry, polymer manufacturing, petroleum refineries, and many others and they are typical examples of nonlinear system.

Nonlinear processes with time delay are difficult to control using standard feedback controllers. When a high performance of the control process is desired or the relative time delay is very large, we can choose another approach. One of the most known methods for control of processes with time delay is Smith predictor, however this predictor does not allow to control nonlinear processes. Another negative aspect is effect of measurable disturbance on the process output which is a very important issue that needs to be analyzed and considered in control problems where it is possible to be measured. Disturbances drive the system away from its desired operating point and require more sophisticated control strategies to minimize their influences. The most known solution to eliminate this problem is classical feedforward control of measurable disturbances. These types of compensators provide a possibility to take control actions before the disturbance affects the process output. On the other hand, for simple processes, where effect of disturbance is not too significant, using of classical feedforward control is sufficient. For more complex processes which are slightly nonlinear, exhibit time delay and with possibility of measuring disturbance, Model Predictive Control (MPC) strategy can be used. This paper deals with the use of MPC for slightly nonlinear processes with time delay with possibility of measuring disturbance compensation.

Strategy of MPC presents a series of advantages over other methods. The MPC can be used to control a great variety of processes, ranging from those with relatively simple dynamics to other more complex ones, including systems with long time-delay, unstable ones or non-minimum phase. The multivariable case can easily be dealt with. The additional advantage is that extension to the treatment of constraints is conceptually simple and these can be systematically included during the design process. This approach of control is a totally open methodology based on certain basic principles that is allowed for future extensions.

The MPC was mainly deployed on slower processes. It was caused by the large computational complexity of control algorithms. Over the years trends have expanded towards modifications of MPC, which can control very fast processes (e.g. explicit MPC can be used). In practice, MPC has proven to be a reasonable strategy for industrial control, and several reports indicate that it is the most used advanced control technology in industry (Camacho and Normey-Rico 2007; Rossiter 2003).

Extended version of Generalized Predictive Control (GPC) algorithm is used for design of predictive controller in this paper.

The paper is organized in the following way. The general principle of the MPC is described in the first section. The next section is devoted to explain the extended GPC algorithm. The GPC cost function and control law are introduced in following section. The identification of model of experimental laboratory liquid-liquid stirred heat exchanger is described in the next section. Verification of control method is shown next. Results evaluation is described in following section and the last section concludes the paper.

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MODEL PREDICTIVE CONTROL PRINCIPLES

The term MPC does not describe a specific control strategy but a very extensive range of control methods that make explicit use of a model of the process to obtain the control signal by minimizing an objective function. The essential ideas of the predictive control family are (Camacho and Normey-Rico 2007; Rossiter 2003):

- Explicit use of a model to predict the process output at future time instants (horizon).
- The trajectory of the reference signal is known for the time horizon of the prediction.
- Calculation of a control sequence minimizing an objective function, mostly quadratic.
- Only the first computed system input value is used for control and the calculation is repeated in the next sampling period (Camacho and Normey-Rico 2007).

Various discrete models can be used to represent the process behavior which is the main difference between the MPC methods. For example step response, transfer function, impulse response and other discrete models can be used.

Fig. 1 and Fig. 2 represent principle of MPC.

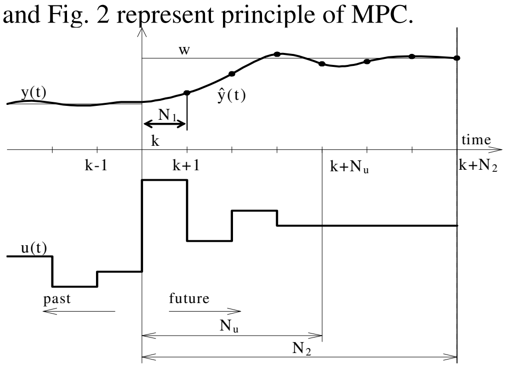

Figure 1 : The MPC Main Idea

- The future outputs for the prediction horizon N are predicted at each instant t using the process model. These predicted outputs $\hat{y}(k+j)$ for $j=1...N$ depend on the known values up to instant k (past inputs and outputs) and on the future control signals $u(k+j)$, $j=0...N-1$, which are to be sent to the system and to be calculated, but only the first calculated one will be implemented.
- The vector of future control signals is calculated by optimizing a determined criterion in order to keep the process as close as possible to the reference trajectory $w(k+j)$. This criterion usually takes the form of quadratic function of the errors between the predicted output signal and the predicted reference trajectory. The control effort is included in the objective function in most cases.
- The control signal $u(k)$ is sent to the process while the next control signals calculated are neglected, because at the next sampling instant $y(k+1)$ will already be known, and step 1 will be repeated with this new value and all the sequences will be brought up to date. Therefore the next control signal $u(k+1)$ is calculated using the receding horizon concept (Bars et al. 2011; Bobál 2008; Camacho and Normey-Rico 2007; Schwarz et al. 2012).

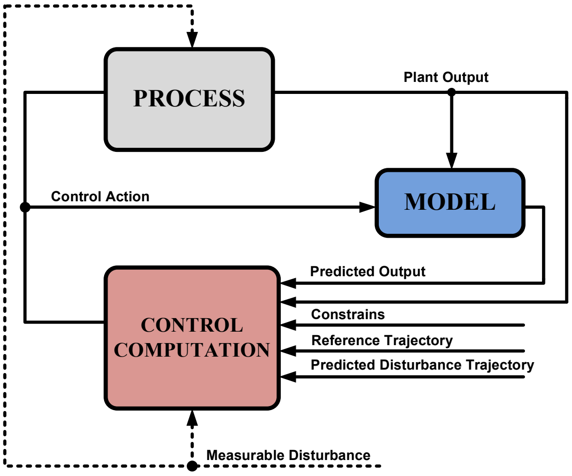

Figure 2 : Extended Structure of MPC

EXTENDED GENERALIZED PREDICTIVE CONTROL ALGORITHM

Principle scheme of extended MPC algorithm is depicted in Fig. 2, where dashed line represents measurable disturbance.

The GPC minimizes a cost function that can be rewritten as

$$\begin{aligned} J(N_1,N_2,N_u) = \sum_{i=N_1}^{N_2}\delta(i)\left[\hat{y}(k+i)-w(k+i)\right]^2 + \\ +\sum_{i=1}^{N_u}\lambda(i)\left[\Delta u(k+i-1)\right]^2 \end{aligned} \qquad (1)$$

where $\hat{y}(k+i)$ is an optimum prediction of the system output, N_1 and N_2 are the minimum and maximum costing horizons, N_u is the control horizon, $\delta(i)$ and $\lambda(i)$ are weighting coefficients and $w(k+i)$ is a vector of future reference sequence. The objective of predictive control is to compute the future incremental control sequence $\Delta u(k)$, $\Delta u(k+1)$,... This is accomplished by minimizing of cost function *J*. To minimize *J,* the predictions $\hat{y}(k+i)$ are first expressed as a function of the past data and the future control actions $\Delta u(k+i-1)$. Then, J can be considered as a function of the future control sequence and minimization is accomplished in order to obtain the optimal value. The weighting factors and horizons are the tuning parameters (Camacho and Normey-Rico 2007).

Without loss of generality and because of the time delay characteristics of the process, horizons $N_1$and $N_2$are computed as $N_1 = d+1$ and $N_2 = N_u + d$.

The extended mathematical model used by GPC to compute the predictions is a modified CARIMA model. It is a typical CARIMA model extended by the vector $v(k)$ which represents measurable disturbance.

$$A(z^{-1})y(k) = z^{-d}B(z^{-1})u(k-1) + z^{-dv}D(z^{-1})v(k) + \\ + \frac{C(z^{-1})}{\Delta} e_s(k) \quad (2)$$

where $\Delta = 1 - z^{-1}$, d and dv is number of steps of the time delay and $e_S(k)$ is the white noise. The polynomial $D(z^{-1})$ represents character of disturbance and the polynomial $C(z^{-1})$ describes character of the noise. This character is difficult to determine, therefore polynomial $C(z^{-1})$ was chosen to be equal to one. Vector $v(k)$ is discrete set of measurable disturbance (Camacho and Bordons 2004; Fikar and Mikleš 2008).

After application of following equation and multiplication equation (2) by Δ

$$\tilde{A}(z^{-1}) = (1-z^{-1})A(z^{-1}) = 1 - \tilde{a}_1 z^{-1} \cdots - \tilde{a}_{na+1} z^{-na-1} = \\ = 1 - (1-a_1)z^{-1} - (a_1 - a_2)z^{-2} \cdots - a_{na+1} z^{-na-1} \quad (3)$$

model should be represented in following way, where output should be predicted as

$$\hat{y}(k+1) = \sum_{i=1}^{na+1} \tilde{a}_i y(k+1-i) + \sum_{i=1}^{nb} b_i \Delta u(k-d-i) + \\ + \sum_{i=1}^{nd} d_i \Delta v(k+1-dv-i) \quad (4)$$

wherena, nb, nc a nd are degrees of polynomials $A(z^{-1})$, $B(z^{-1})$, $C(z^{-1})$ and $D(z^{-1})$. The white noise $e_S(k)$ and its future values are considered to be equal to zero for the prediction of the future output values.

In case where time delay is present, following equation should be used when equation (4) is applied recursively for i = 1, 2, ...,N_u

$$\begin{bmatrix} \hat{y}(k+d+1) \\ \hat{y}(k+d+2) \\ \vdots \\ \hat{y}(k+d+N_u) \end{bmatrix} = \boldsymbol{G} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_u-1) \end{bmatrix} + \boldsymbol{H} \begin{bmatrix} \Delta u(k-1) \\ \Delta u(k-2) \\ \vdots \\ \Delta u(k-nb) \end{bmatrix} + \\ + \boldsymbol{S} \begin{bmatrix} \hat{y}(k+d) \\ \hat{y}(k+d-1) \\ \vdots \\ \hat{y}(k+d-na) \end{bmatrix} + \boldsymbol{H_{VI}} \begin{bmatrix} \Delta v(k-1) \\ \Delta v(k-2) \\ \vdots \\ \Delta v(k-nd+1) \end{bmatrix} + \quad (5) \\ + \boldsymbol{H_{V2}} \begin{bmatrix} \Delta v(k) \\ \Delta v(k+1) \\ \vdots \\ \Delta v(k+N_u-1) \end{bmatrix}$$

Equation (5) should be written as

$$\boldsymbol{\hat{y}} = \boldsymbol{Gu} + \boldsymbol{Hu_1} + \boldsymbol{Sy_1} + \boldsymbol{H_{V1}v_1} + \boldsymbol{H_{V2}v_2} \quad (6)$$

Matrices **G**, **H** and **S** are constant matrices of dimensions $N_u \times N_u$, $N_u \times nb$ and $N_u \times (na+1)$. Matrices $\mathbf{H_{V1}}$ and $\mathbf{H_{V2}}$ are of dimensions $N_u \times (nd-1)$ and $N_u \times N_u$. Matrix $\mathbf{H}_{V1}$ can be used only in case when degree of polynomial $D(z^{-1})$ is equal to 2 or higher.

Following equation corresponds to the free response of the system that is the output that would be obtained if the control and disturbance signals were kept constant.

$$\boldsymbol{f} = \boldsymbol{H\,u_1} + \boldsymbol{S\,y_1} + \boldsymbol{H_{V1}v_1} \quad (7)$$

Forced response of system is represented by next equation

$$\boldsymbol{f_r} = \boldsymbol{Gu} + \boldsymbol{H_{V2}v_2} \quad (8)$$

Based on equation mentioned above (7) and (8), overall response of system is computed as sum of free and forced response.

$$\hat{y} = f + f_r \quad (9)$$

COST FUNCTION AND CONTROL LAW

If $\hat{y}$ is introduced in equation (1), it is evident that J is a cost function of $\mathbf{y_1}$, **u** and $\mathbf{u_1}$. Function should be written as

$$J = \boldsymbol{(Gu + Hu_1 + Sy_1 - w)^T Q_\delta (Gu + Hu_1 + Sy_1 - w)} + \\ + \boldsymbol{u^T Q_\lambda u} \quad (10)$$

where $\mathbf{Q_\delta}$ and $\mathbf{Q_\lambda}$ are the diagonal weighting matrices of size $N_u \times N_u$ with elements $\delta(j)$ and $\lambda(j)$, respectively. Although, in practice, the most common choice is to set $\delta(j)$ and $\lambda(j)$ constants on the horizon. In practice, the values of these weighting factors must be normalized in order to obtain a correct weighting of the different errors and controller outputs.

After some manipulations J is written as

$$J = \boldsymbol{u}^T (\boldsymbol{Q_\lambda} + \boldsymbol{G^T Q_\delta G})\boldsymbol{u} + 2(\boldsymbol{Hu_1} + \boldsymbol{Sy_1} - \boldsymbol{w})^T \boldsymbol{Q_\delta Gu} + \\ + (\boldsymbol{Hu_1} + \boldsymbol{Sy_1} - \boldsymbol{w})^T \boldsymbol{Q_\delta} (\boldsymbol{Hu_1} + \boldsymbol{Sy_1} - \boldsymbol{w}) \quad (11)$$

Minimizing J with respect to **u**, it means $\frac{\partial J}{\partial \mathbf{u}} = 0$, leads to

$$\mathbf{Mu} = \mathbf{P_0 y_1} + \mathbf{P_1 u_1} + \mathbf{P_2 w} \quad (12)$$

where $\mathbf{M} = \mathbf{G}^T \mathbf{Q}_\delta \mathbf{G} + \mathbf{Q}_\lambda$ is of dimension $N_u \times N_u$, $\mathbf{P}_0 = -\mathbf{G}^T \mathbf{Q}_\delta \mathbf{S}$ of dimension $N_u \times (na+1)$, $\mathbf{P}_1 = -\mathbf{G}^T \mathbf{Q}_\delta \mathbf{H}$ of dimension $N_u \times nb$ and $\mathbf{P}_2 = \mathbf{G}^T \mathbf{Q}_\delta$ of dimension $N_u \times N_u$, therefore

$$\boldsymbol{M} \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N_u-1) \end{bmatrix} = \boldsymbol{P_0} \begin{bmatrix} \hat{y}(k+d) \\ \hat{y}(k+d-1) \\ \vdots \\ \hat{y}(k+d-na) \end{bmatrix} + \\ + \boldsymbol{P_1} \begin{bmatrix} \Delta u(k-1) \\ \Delta u(k-2) \\ \vdots \\ \Delta u(k-nb) \end{bmatrix} + \boldsymbol{P_2} \begin{bmatrix} w(k+d+1) \\ w(k+d+2) \\ \vdots \\ w(k+d+N_u) \end{bmatrix} \quad (13)$$

In a receding horizon algorithm only the actual value $\Delta u(k)$ is computed, so if **m** is the first row of matrix $\mathbf{M}^{-1}$, then $\Delta u(k)$ is given by

$$\Delta u(k) = \boldsymbol{mP_0 y_1} + \boldsymbol{mP_1 u_1} + \boldsymbol{mP_2 w} \quad (14)$$

When compensation of measurable disturbance is required, $\Delta u(k)$ is given by extended form of control law

$$\Delta u(k) = \boldsymbol{mP_0 y_1} + \boldsymbol{mP_1 u_1} + \boldsymbol{mP_2 w} + \boldsymbol{mP_{V1} v_1} + \boldsymbol{mP_{V2} v_2} \quad (15)$$

where $\mathbf{P_{V1}} = -\mathbf{G}^T\mathbf{Q}_\delta\mathbf{H_{V1}}$ is of dimension $N_u \times (nd-1)$ and $\mathbf{P_{V2}} = -\mathbf{G}^T\mathbf{Q}_\delta\mathbf{H_{V2}}$ of dimension $N_u \times N_u$.

After introducing vectors $\mathbf{y_1}$, $\mathbf{u_1}$, $\mathbf{w}$, $\mathbf{v_1}$ and $\mathbf{v_2}$, final control law is defined as

$$\begin{aligned}\Delta u(k) = \boldsymbol{mP_0}&\begin{bmatrix}\hat{y}(k+d)\\ \hat{y}(k+d-1)\\ \vdots\\ \hat{y}(k+d-na)\end{bmatrix}+\boldsymbol{mP_1}\begin{bmatrix}\Delta u(k-1)\\ \Delta u(k-2)\\ \vdots\\ \Delta u(k-nb)\end{bmatrix}+\\ +\boldsymbol{mP_2}&\begin{bmatrix}w(k+d+1)\\ w(k+d+2)\\ \vdots\\ w(k+d+N_u)\end{bmatrix}+\boldsymbol{mP_{V1}}\begin{bmatrix}\Delta v(k-1)\\ \Delta v(k-2)\\ \vdots\\ \Delta v(k-nd+1)\end{bmatrix}+\\ +\boldsymbol{mP_{V2}}&\begin{bmatrix}\Delta v(k)\\ \Delta v(k+1)\\ \vdots\\ \Delta v(k+N_u-1)\end{bmatrix}\end{aligned} \qquad (16)$$

$\mathbf{H_{V1}}$ and $\mathbf{H_{V2}}$ are matrices including the coefficients of the system step response to the disturbance.

Future values of disturbance can be known in some cases, when disturbance is related to the process load. In other cases, it can be predicted by using means, trends or other information. If this is the case, the term corresponding to future deterministic disturbance can be computed (Pawlowskaa et al. 2012).

It is evident that matrices $\mathbf{H_{V1}}$ and $\mathbf{H_{V2}}$ are dependent on the relative difference between number of steps of time delay of input-output and disturbance-output which is defined as

$$\rho = d - dv \qquad (17)$$

This leads to three different structures for matrices $\mathbf{H_{V1}}$ and $\mathbf{H_{V2}}$ based on the sign of ρ:

- $\rho < 0$

$$\boldsymbol{H_{VX}} = \begin{bmatrix}0&0&0&0&0&0\\ 0&0&0&0&0&0\\ \cdots&\cdots&\cdots&\cdots&\cdots&\cdots\\ 0&0&0&0&0&0\\ h_1&0&0&0&0&0\\ \vdots&\ddots&0&0&0&0\\ h_{Nu+\rho}&\cdots&h_1&0&0&0\end{bmatrix}\begin{matrix}\left.\vphantom{\begin{matrix}0\\0\\\cdots\\0\end{matrix}}\right\}|\rho|\\ \\ \\ \\ \end{matrix} \qquad (18)$$

- $\rho = 0$

$$\boldsymbol{H_{VX}} = \begin{bmatrix}h_1&0&0&0&0&0\\ h_2&\ddots&0&0&0&0\\ \vdots&\ddots&\ddots&0&0&0\\ \vdots&\vdots&\ddots&h_1&0&0\\ \vdots&\vdots&\vdots&\ddots&\ddots&0\\ h_{Nu}&h_{Nu-1}&\cdots&\cdots&h_2&h_1\end{bmatrix} \qquad (19)$$

- $\rho = 0$

$$\boldsymbol{H_{VX}} = \begin{bmatrix}h_{\rho+1}&h_\rho&\cdots&h_1&0&0\\ h_{\rho+2}&h_{\rho+1}&\ddots&\ddots&\ddots&0\\ \vdots&\ddots&\ddots&\ddots&\ddots&h_1\\ \vdots&\ddots&\ddots&\ddots&\ddots&h_2\\ \vdots&\vdots&\vdots&\vdots&\ddots&\vdots\\ h_{N+\rho}&h_{N+\rho-1}&\cdots&\cdots&h_\rho&h_{\rho+1}\end{bmatrix}\begin{matrix}\\ \\ \left.\vphantom{\begin{matrix}h_1\\h_2\\\vdots\\h\end{matrix}}\right\}\rho\end{matrix} \qquad (20)$$

Where h_i are the coefficients of $\mathbf{H_{V1}}$ and $\mathbf{H_{V2}}$ matrices obtained from the delay free disturbance response shifted in accordance with ρ (Pawlowskaa et al. 2012).

IDENTIFICATION

Since MPC is based on an internal model of the system, a sufficient model is necessary therefore system identification is an important factor in MPC strategy.

Identification of control processes can be divided into two groups that are used most often. The first group is one-time (offline) method and the second is ongoing (online) methods. Online identification methods can be used for self-tuning controllers (STC).

Both types of identification can be used for estimation of the model parameters of the system. Online identification, offline identification and comparison with estimation by MATLAB function *fminsearch* are shown in following subsections.

Offline Identification Methods

The most used method for the identification of the parameters of discrete transfer function models is the least squares estimator (LSE) based on the idea of linear regression. These identification algorithms can be carried out in an online manner as well.

The least squares estimator is defined as the vector $\hat{\mathbf{\Theta}}$ that minimizes the quadratic error

$$\hat{\boldsymbol{\Theta}} = \left(\boldsymbol{F}^T\boldsymbol{F}\right)^{-1}\boldsymbol{F}^T\boldsymbol{y} \qquad (21)$$

Notice that $\hat{\mathbf{\Theta}}$ is a vector of estimated model parameters of dimension 2*n*, **F** is a matrix of dimension $N-n-d \times 2n$, $\boldsymbol{y}$ is a data vector of dimension $N-n-d$, where *N* is a number of measured data, *n* is an order of system and *d* is a number of steps of time delay (Camacho and Normey-Rico 2007).

MATLAB function *fminsearch* can find the minimum of a scalar function of several variables as well, starting at an initial estimate. This function uses the simplex search method for finding the minimum of a function.

$$x = \mathrm{fminsearch}(\mathit{fun}, x_0, \mathit{options}) \qquad (22)$$

Online Identification Methods

One of the advantages of the identification procedure based on the LSM is that it can be used recursively because the parameter vector estimated at time *t* can be computed as a function of the parameter vector estimated at $k-1$. The recursive least squares method (RLSM) is the most known method. This method uses ARX model (Bobál et al. 2012).

$$y(k) = \boldsymbol{\Theta}^T(k)\boldsymbol{\Phi}(k) + e_s(k) \qquad (23)$$

where $\boldsymbol{\Theta}$ is a vector of model parameters

$$\boldsymbol{\Theta}^T(k) = \left[a_1 \quad a_2 \ldots a_n \quad b1 \quad b_2 \ldots b_n\right] \tag{24}$$

and $\boldsymbol{\Phi}$ is a regression vector

$$\begin{aligned}\boldsymbol{\Phi}^T(k) = \big[&-y(k-1) \quad -y(k-2)\ldots -y(k-n) \\ &u(k-d-1) \quad u(k-d-2)\ldots u(k-d-n)\big]\end{aligned} \tag{25}$$

Identification of Model of Liquid-Liquid Stirred Heat Exchanger

Identification was performed on the SIMULINK model of stirred heat exchanger and the estimated mathematical model was used for verification purposes.

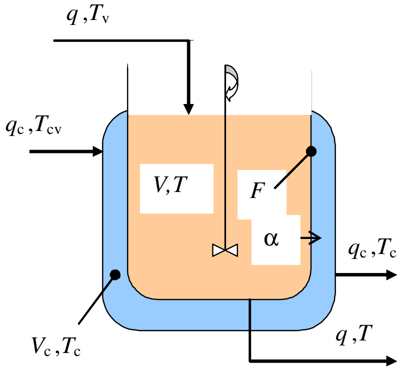

Figure 3 : Model of Stirred Flow Heat Exchanger

Model of this heat exchanger can be described by two differential equations of first order:

$$q\rho c_p T_v = q\rho c_p T + F\alpha(T - T_c) + V\rho c_p \frac{dT}{dt} \tag{26}$$

$$q_c \rho_c c_{pc} T_{cv} + F\alpha(T - T_c) = q_c \rho_c c_{pc} T_c + V_c \rho_c c_{pc} \frac{dT_c}{dt} \tag{27}$$

with initial conditions:

$$T(0) = T^s, T_c(0) = T_c^s \tag{28}$$

and boundary conditions:

$$0.02 \le q_c \le 0.6 \tag{29}$$

where t stands for the time, T for temperatures, q for flow of fluids, ρ for densities, c_p for specific heat capacities, α for heat transfer coefficients, V for volumes and F for heat exchanger area.

For the control purposes, the output temperature of the refrigerated fluid $T(t)$ is considered as the controlled output, and, the coolant flow $q_C(t)$ as the control input, while other inputs can enter into the process as disturbances.

Parameters of heat exchanger were chosen as is shown in the following table.

Table 1 : Parameters of Heat Exchanger

Parameter	**Value**
$V = 2.65$ m^3	$q = 0.2$ m^{3}min^{-1}
$V_c = 0.63$ m^3	$q_c = 0.4$ m^{3}min^{-1}
ρ = 985 kg m^{-3}	$c_p = 4.05$ kJ kg^{-1}K^{-1}
ρ$_c$ = 998 kg m^{-3}	$c_{pc} = 4.18$ kJ kg^{-1}K^{-1}
$F = 8.8$ m^2	$\alpha = 58$ kJ m^{-2}min^{-1}K^{-1}
$T_v = 370.0$ K	$T_c = 293.0$ K

Steady-State Characteristic of Model of Liquid-Liquid Stirred Heat Exchanger

The dependence of the refrigerated fluid output temperature on the coolant flow in the steady-state is in Figure. 4.

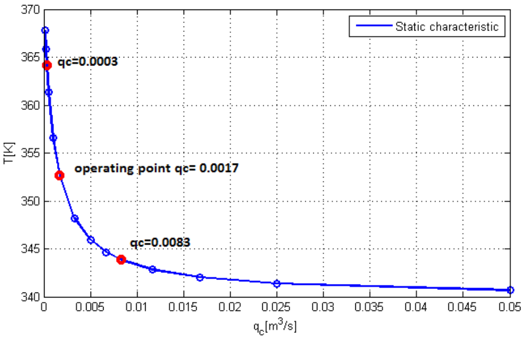

Figure 4 : Static Characteristic of Heat Exchanger

In subsequent control simulations, the operating interval for q_c has been determined as $0.0003 \le q_c \le 0.0083$, where the static characteristic is only slightly nonlinear as is shown in Figure 5.

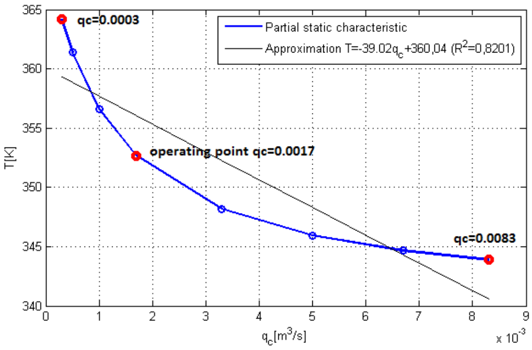

Figure 5 : Static Characterstic for Operating Interval

Choice of Input Sequence for Model of Liquid-Liquid Stirred Heat Exchanger Identification

Pseudorandom Binary Signal (PRBS), Random Binary Signal (RBS) and Random Gaussian Signal (RGS) were used for identification of laboratory model, see Table 6-8. Input signals were generated by MATLAB function *idinput*.

$$u = \text{idinput}(N, type, band, levels) \tag{30}$$

Function *idinput* generates input signals which are used for identification purposes, where u is returned as matrix or column vector. Parameter N determines the number of generated input data and parameter *type* defines the type of input signal to be generated.

Sum of squares subtraction of estimated outputs and measured data was used for analysis of quality of identificated models.

$$S_y = \frac{1}{N}\sum_{k=a}^{b}\left[\hat{y}(k) - y(k)\right]^2 \tag{31}$$

Where N represents number of measured data, $y(k)$ is measured output value and $\hat{y}(k)$ is estimated output value.

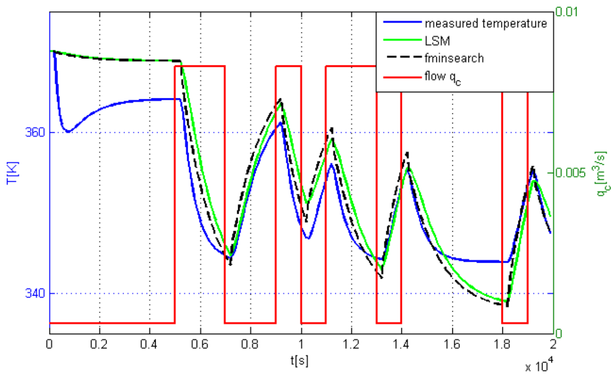

Figure 6 : Identification by PRBS Input Signal

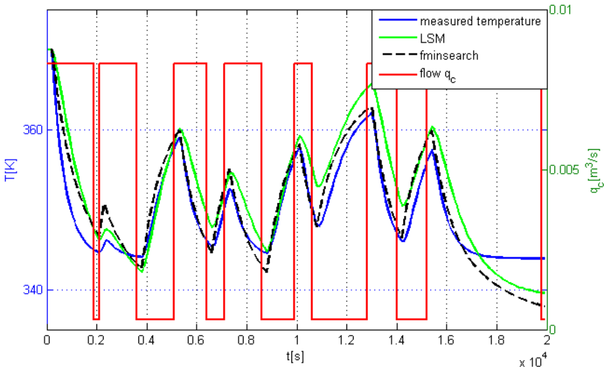

Figure 7 : Identification by RBS Input Signal

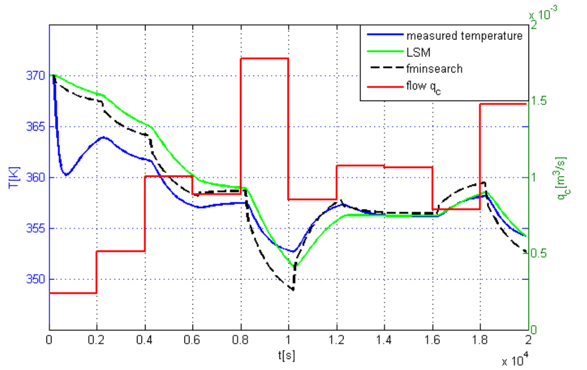

Figure 8 : Identification by RGS Input Signal

Following table shows results of laboratory model identification.

Table 2 : Evaluation of Identification Quality

Method	Parameter		PRBS	RBS	RGS
LSM	Coeffs.	a_1	-1.6166	-1.6059	-1.5631
		a_2	0.6416	0.6319	0.5839
		b_1	-149.9939	-126.3404	-276.0283
		b_2	54.6479	29.0058	6.4397
	Quality S_y		**18.9143**	**13.5954**	**8.1969**
RLSM	Coeffs	a_1	-1.3710	-1.2834	-1.5538
		a_2	0.4183	0.3368	0.5779
		b_1	-144.4788	-157.9094	-202.3707
		b_2	-40.0787	-47.1242	-109.0916
	Quality S_y		**17.6613**	**10.7138**	**7.7424**
fminsearch	Coeffs	a_1	-0.9825	-1.7038	-0.8247
		a_2	0.0487	0.7121	-0.0858
		b_1	-889.1335	-485.4889	-4823.0477
		b_2	630.3898	450.0139	3678.7293
	Quality S_y		**16.1456**	**6.8179**	**5.7447**

SIMULATION VERIFICATION

Parameters estimated by RLSM with RBS were chosen for verification purposes since the frequency spectrum of RBS is suitable for these types of systems. Discrete transfer function for exchanger has the following form:

$$G_S(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} z^{-d} = \frac{-157.9094 z^{-1} - 47.1242 z^{-2}}{1-1.2834 z^{-1} + 0.3368 z^{-2}} z^{-2} \qquad (32)$$

Discrete transfer function for disturbance is:

$$G_V(z^{-1}) = \frac{D(z^{-1})}{A(z^{-1})} z^{-dv} = \frac{-78.9547 z^{-1} - 23.5621 z^{-2}}{1-1.2834 z^{-1} + 0.3368 z^{-2}} z^{-2} \qquad (33)$$

where $T_S = 100s$, $d = 2$, $dv = 2$, $\delta = 1, \lambda = 20$ and $N_u = 10$.

Figures below show regulation processes for various types of disturbance compensation (predictive algorithm without compensation of measurable disturbance (A), with compensation (B) and the case when vector of measurable disturbance is known in time (C)).

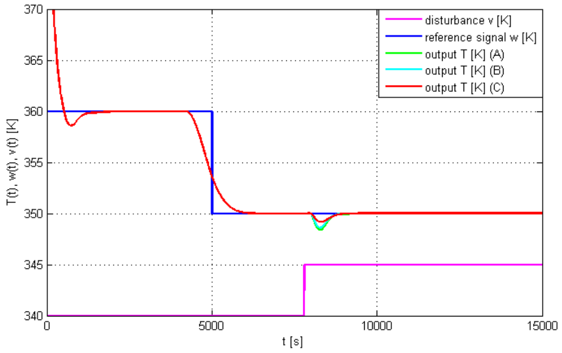

Figure 9 : Regulation Processes (A, B and C)

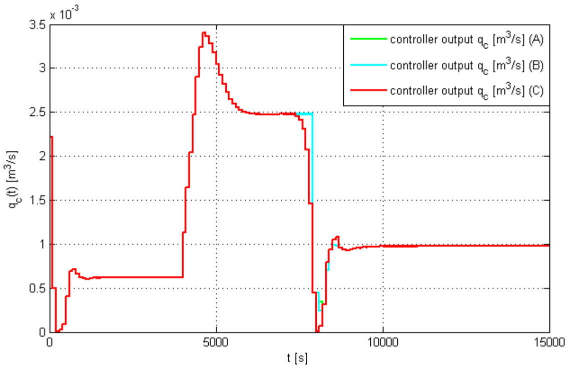

Figure 10 : Controller Outputs (A, B and C)

ANALYSIS OF RESULTS

The quadratic criterion of the measured error, the quadratic criterion of the system output increments and absolute values of these signals were chosen for a control quality analysis.

$$S_{e2} = \frac{1}{N}\sum_{k=a}^{b}[e(k)]^2, \quad S_{ea} = \frac{1}{N}\sum_{k=a}^{b}|e(k)|$$
$$S_{du2} = \frac{1}{N}\sum_{k=a}^{b}[\Delta u(k)]^2, \quad S_{dua} = \frac{1}{N}\sum_{k=a}^{b}|\Delta u(k)| \qquad (34)$$

Evaluation of control quality for various types of disturbance compensation is shown in the following table.

Table 3 :Evaluation for Simulation of Heat Exchanger

	Overshoot	S_{e2}	S_{ea}	S_{du2} $\cdot 10^{-8}$	S_{dua} $\cdot 10^{-5}$
Without com. (A)	2.11	13.22	2.59	6.02	4.00
With comp. (B)	1.52	13.14	2.56	6.02	4.00
Known dist. (C)	1.27	13.02	2.49	7.79	4.56

CONCLUSION

This paper dealt with an extended GPC design procedure to improve the measurable disturbance compensation. The extended GPC control law with implicit disturbance compensation is interpreted as classical feedback plus feedforward control scheme that is described by theoretical analysis. Functionality of designed algorithm was simulation verified on a mathematical model of liquid-liquid stirred heat exchanger.

In case, where impact of different types of disturbance compensation was verified, there is shown that integration of disturbance compensation to basic GPC algorithm improves regulation processes and control quality as well. Without compensation of measurable disturbance, i.e. standard version of GPC, overshoot of output signal is 2.11 K. When compensation of disturbance is used, but vector of disturbance is unknown (the case when only disturbance is measured), overshoot is reduced to 1.52 K. In situation, when vector of disturbance is known, compensation is more substantial (overshoot 1.27 K) and can have positive effect when it is measured and predicted simultaneously, e.g. sunrise or sunset and effect of sun can be predicted based on part of season; particular substance is inserted into chemical reactor at a certain time; vehicles can predict and adapt driving style based on type of turn on road and many other cases.

The simulation results demonstrate improved usability of extended GPC algorithm and results also showed that this algorithm is able to control slightly nonlinear processes.

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