STATE-SPACE PREDICTIVE CONTROL OF TWO LIQUID TANKS SYSTEM WITH CONSTRAINTS OF PROCESS VARIABLES

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KEYWORDS

predictive control, time-delay, constraints of process variables, state-space, two liquid tanks.

ABSTRACT

This paper presents a method called the predictive control used to control a nonlinear process about a selected operating point. The system of the two funnel liquid tanks in series is chosen as an exemplar process. The parameters of the tanks simulate a large industry tanks used in a chemical industry. The state-space CARIMA mathematical model is used for the output values prediction. This paper describes the linearization process of the nonlinear system at the operating point and a possibility of constraints of the process variables. The designed controller is verified on the process without and with a time-delay.

INTRODUCTION

Many processes in the real world are nonlinear, complex and they often include a time-delay. These processes are very difficult to control. The situation is more complicated when some process variables require some sort of a limitation. The basic control methods do not handle with this situation so we need a more advanced method. The predictive control is a great example of the modern control method capable of solving the complex control problem (Bobál 2008).

This method is based on the prediction of the output values on the chosen time horizon. This time horizon should be long enough to cover the step response of the controlled system and the prediction of the output values is based on the mathematical model of the controlled system. The predictive control in this paper uses state-space CARIMA model for multi-input multi-output (MIMO) system (Bars et al. 2011; Wang 2009).

The control signal is obtain by minimization of a cost function. This cost function has usually a quadratic form and it minimize the differences between the reference value and the output value and the control signal increments. We can also take into account the constraints of the process variables in the cost function minimization process. This is done by using a quadratic programming method (Camacho and Bordons 2004; Maciejowski 2002; Rossiter 2003).

However, the state-space CARIMA model is a linear mathematical model. So we have to do one more step to control the nonlinear system like the chosen system of the two funnel liquid tanks in series. This step is linearization of the nonlinear mathematical model in a selected operating point. The divergence linear model is result of the linearization. It means, that the selected operating point is a new origin state for the controller and the input and the output values are divergence from the equilibrium values (Albertos Peréz and Sala 20014; Hangos et al. 2004).

MATHEMATICAL MODEL OF THE CONTROLLED SYSTEM

The mathematical model of the chosen experimental system of the two liquid tanks system is taken from (Krhovják et al. 2015). This model represents a nonlinear system with two input variables and two output variables. Figure 1 shows a schematic diagram of the controlled process consists of two funnel liquid tanks in series. The first tank is filled by input stream q_{1f}, the second tank is filled by input stream q_{2f} and output stream from the first tank q_1.

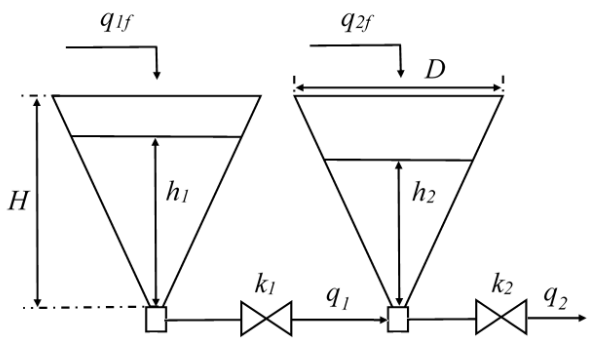

Figure 1 : Schematic diagram of the process

The mathematical model of this process can be obtained from balancing equations of the input and output mass streams. The equation (1) stand for the balancing equation of the input, output and accumulation of the first tank and the equation (2) stand for the balancing equation of the input, output and accumulation of the second tank (Richardson 1989).

$$\pi \frac{D^2}{4H^2} h_1^2 \frac{dh_1}{dt} + q_1 = q_{1f} \quad (1)$$

$$\pi \frac{D^2}{4H^2} h_2^2 \frac{dh_2}{dt} - q_1 + q_2 = q_{2f} \quad (2)$$

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In these equations, D is the maximum diameter, H is the total height and h_1 and h_2 are the liquid levels from the bottom of the tanks. The output liquid streams q_1 and q_2 depend on the valve constants k_1, k_2 and the liquid levels as well.

$$q_1 = k_1\sqrt{|h_1 - h_2|} \tag{3}$$

$$q_2 = k_2\sqrt{h_2} \tag{4}$$

However, the chosen predictive control method works only with linear mathematical models, so the model of the described process needs to be linearized about an operating point. First of all, the equations (1) and (2) have to be expressed as nonlinear state-space model by selecting the input variables as $u_1 = q_{1f}$ and $u_2 = q_{2f}$ and the output and state variables as $y_1 = x_1 = h_1$ and $y_2 = x_2 = h_2$. This state-space model has form

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u})\\ \boldsymbol{y} &= \boldsymbol{g}(\boldsymbol{x})\end{aligned} \tag{5}$$

where equations of single states and outputs are

$$\begin{aligned}\frac{dx_1}{dt} &= \frac{4H^2}{\pi D^2 x_1^2}\left(u_1 - k_1\sqrt{|x_1 - x_2|}\right)\\ \frac{dx_2}{dt} &= \frac{4H^2}{\pi D^2 x_2^2}\left(u_2 + k_1\sqrt{|x_1 - x_2|} - k_2\sqrt{x_2}\right)\\ y_1 &= x_1\\ y_2 &= x_2\end{aligned} \tag{6}$$

To linearized this nonlinear state-space model, we have to calculate the operating point as equilibrium point of the system. This means, that we are looking for the state, where are no changes of the liquid levels in the tanks and steady input streams to the tanks.

$$\begin{aligned}0 &= \frac{4H^2}{\pi D^2 x_1^2}\left(u_1 - k_1\sqrt{|x_1 - x_2|}\right)\\ 0 &= \frac{4H^2}{\pi D^2 x_2^2}\left(u_2 + k_1\sqrt{|x_1 - x_2|} - k_2\sqrt{x_2}\right)\end{aligned} \tag{7}$$

The linearization about the operating point means substitution of the absolute value of the input, output and state variables by its divergence from the steady state.

$$\begin{aligned}\boldsymbol{x}_\delta(t) &= \boldsymbol{x}(t) - \overline{\boldsymbol{x}}\\ \boldsymbol{u}_\delta(t) &= \boldsymbol{u}(t) - \overline{\boldsymbol{u}}\\ \boldsymbol{y}_\delta(t) &= \boldsymbol{y}(t) - \overline{\boldsymbol{y}}\end{aligned} \tag{8}$$

Where $\overline{\boldsymbol{x}}$ is a vector of the equilibrium state variables, $\overline{\boldsymbol{u}}$ is a vector of the equilibrium input variables, $\overline{\boldsymbol{y}}$ is a vector of the equilibrium output variables, $\boldsymbol{x}_\delta, \boldsymbol{u}_\delta, \boldsymbol{y}_\delta$ are divergences from equilibrium values.

The linearized state-space model can be expressed in form

$$\begin{aligned}\dot{\boldsymbol{x}}_\delta &= \boldsymbol{A}\boldsymbol{x}_\delta + \boldsymbol{B}\boldsymbol{u}_\delta\\ \boldsymbol{y}_\delta &= \boldsymbol{C}\boldsymbol{x}_\delta\end{aligned} \tag{9}$$

where matrices $\boldsymbol{A}$, $\boldsymbol{B}$ are

$$\begin{aligned}\boldsymbol{A} &= \left.\frac{\partial f}{\partial \boldsymbol{x}}\right|_{x=\bar{x},u=\bar{u}}\\ \boldsymbol{B} &= \left.\frac{\partial f}{\partial \boldsymbol{u}}\right|_{x=\bar{x},u=\bar{u}}\end{aligned} \tag{10}$$

and $\boldsymbol{C}$ is an identity matrix (Albertos Peréz and Sala 2004; Hangos et al. 2004).

At this point we transfer this linear state-space model into linear continuous-time input-output model

$$\boldsymbol{A}(s)\boldsymbol{y}(t) = \boldsymbol{B}(s)\boldsymbol{u}(t)$$

$$\begin{bmatrix} s + a_{01} & a_{02} \\ a_{03} & s + a_{04}\end{bmatrix}\begin{bmatrix} y_1(t) \\ y_2(t)\end{bmatrix} = \begin{bmatrix} b_{01} & 0 \\ 0 & b_{04}\end{bmatrix}\begin{bmatrix} u_1(t) \\ u_2(t)\end{bmatrix} \tag{11}$$

where

$$\begin{aligned}a_{01}(\boldsymbol{h}) &= \frac{4H^2 k_1\sqrt{h_1 - h_2}}{2D^2\pi h_1^2(h_1 - h_2)}\\ a_{02}(\boldsymbol{h}) &= -\frac{4H^2 k_1\sqrt{h_1 - h_2}}{2D^2\pi h_1^2(h_1 - h_2)}\\ a_{03}(\boldsymbol{h}) &= -\frac{4H^2 k_1\sqrt{h_1 - h_2}}{2D^2\pi h_2^2(h_1 - h_2)}\\ a_{04}(\boldsymbol{h}) &= \frac{4H^2}{2D^2\pi h_2^2}\left[\frac{k_1\sqrt{h_1 - h_2}}{h_1 - h_2} + \frac{k_2\sqrt{h_2}}{h_2}\right]\\ b_{01}(\boldsymbol{h}) &= \frac{4H^2}{D^2\pi h_1^2}, b_{04}(\boldsymbol{h}) = \frac{4H^2}{D^2\pi h_2^2}\end{aligned} \tag{12}$$

This continuous-time model needs to be transferred into a discrete-time input-output model with the sampling period T_0.

$$\tilde{\boldsymbol{A}}(z^{-1})\boldsymbol{y}(k) = \boldsymbol{B}(z^{-1})z^{-d}\Delta\boldsymbol{u}(k) \tag{13}$$

where the polynomial matrix $\tilde{\boldsymbol{A}}(z^{-1})$ is

$$\tilde{\boldsymbol{A}}(z^{-1}) = (1 - z^{-1})\boldsymbol{A}(z^{-1}) \tag{14}$$

STATE-SPACE PREDICTIVE CONTROL

The mathematical model used for the prediction of the output values is based on the state-space CARIMA model. It can be expressed as

$$\begin{aligned}\boldsymbol{x}(k+1) &= \tilde{\boldsymbol{A}}\boldsymbol{x}(k) + \boldsymbol{B}\Delta\boldsymbol{u}(k-d)\\ \boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k)\end{aligned} \tag{15}$$

where the vector of state variables has form

$$\begin{aligned}\boldsymbol{x}(k) = [&\boldsymbol{y}(k), \boldsymbol{y}(k-1), \cdots, \boldsymbol{y}(k-na),\\ &\Delta\boldsymbol{u}(k-d-1), \cdots, \Delta\boldsymbol{u}(k-d-nb+1)]^T\end{aligned} \tag{16}$$

and the vectors of the outputs variables and the input control increments are

$$\boldsymbol{y}(k) = \begin{bmatrix} y_1(k) & y_2(k) & \cdots & y_n(k)\end{bmatrix}^T \tag{17}$$

$$\Delta\boldsymbol{u}(k-d) = \begin{bmatrix}\Delta u_1(k-d) & \cdots & \Delta u_m(k-d)\end{bmatrix}^T \tag{18}$$

where n is number of outputs and m is number of inputs (Bars et al. 2011).

The matrices $\tilde{\boldsymbol{A}}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ can be expressed as

$$\tilde{\boldsymbol{A}} = \begin{bmatrix} -\tilde{\boldsymbol{A}}_1 & \cdots & -\tilde{\boldsymbol{A}}_{na} & -\tilde{\boldsymbol{A}}_{na+1} & \boldsymbol{B}_2 & \cdots & \boldsymbol{B}_{nb-1} & \boldsymbol{B}_{nb} \\ \boldsymbol{I} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{I} & \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_1 & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}^T$$

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \tag{19}$$

where $\boldsymbol{I}$ is an identity matrix and $\boldsymbol{0}$ is a zeros matrix. The matrices $-\tilde{\boldsymbol{A}}_i$ for $i = 1,\ldots,na+1$ and $\boldsymbol{B}_j$ for $j = 1,\ldots,nb$ consist of the coefficients of the polynoms of the polynomial matrices $\tilde{\boldsymbol{A}}(z^{-1})$ and $\boldsymbol{B}(z^{-1})$.

The output values prediction can be calculated recursively using state-space model in equation (15) and it can be expressed in a matrix form

$$\hat{\boldsymbol{y}} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{H}_p \Delta\boldsymbol{u}_p + \boldsymbol{H}_f \Delta\boldsymbol{u}_f \tag{20}$$

where $\hat{\boldsymbol{y}}$ is the vector of the predicted output values, $\Delta\boldsymbol{u}_p$ is the vector of the past control increments and $\Delta\boldsymbol{u}_f$ is the vector of the future control increments. These vectors are

$$\hat{\boldsymbol{y}} = \begin{bmatrix} \hat{\boldsymbol{y}}(k+d+1) \\ \hat{\boldsymbol{y}}(k+d+2) \\ \vdots \\ \hat{\boldsymbol{y}}(k+d+N) \end{bmatrix}$$

$$\Delta\boldsymbol{u}_p = \begin{bmatrix} \Delta\boldsymbol{u}(k-d) \\ \Delta\boldsymbol{u}(k-d+1) \\ \vdots \\ \Delta\boldsymbol{u}(k-1) \end{bmatrix}$$

$$\Delta\boldsymbol{u}_f = \begin{bmatrix} \Delta\boldsymbol{u}(k) \\ \Delta\boldsymbol{u}(k+1) \\ \vdots \\ \Delta\boldsymbol{u}(k+N) \end{bmatrix} \tag{21}$$

where d is number of steps of the time-delay and N is the prediction time horizon (Camacho and Normey-Rico 2007). This time horizon should be long enough to cover the step response of the controlled system.

This output values prediction can be substituted into the quadratic cost function that the chosen predictive control method minimizes.

$$J = \left(\boldsymbol{w} - \hat{\boldsymbol{y}}\right)^T \boldsymbol{Q}_\delta \left(\boldsymbol{w} - \hat{\boldsymbol{y}}\right) + \Delta\boldsymbol{u}_f^T \boldsymbol{Q}_\lambda \Delta\boldsymbol{u}_f \tag{22}$$

where $\boldsymbol{w}$ is a vector of the future reference values, $\hat{\boldsymbol{y}}$ is the vector of the predicted outputs values, $\boldsymbol{Q}_\lambda$ and $\boldsymbol{Q}_\delta$ are the diagonal weighting matrices. The vector $\Delta\boldsymbol{u}_f$ is unknown vector of the future control increments (Fikar and Mikleš 2008).

When the constraints of the process variables are not required, the control signal is obtain by minimizing this cost function. But when the constraints of the process variables are necessary, the cost function needs to be modified into form suitable for quadratic programming method

$$J = \frac{1}{2}\boldsymbol{u}^T \boldsymbol{H}_c \boldsymbol{u} + \boldsymbol{g}^T \boldsymbol{u} \tag{23}$$

where

$$\begin{aligned} \boldsymbol{H}_c &= 2\left(\boldsymbol{Q}_\lambda + \boldsymbol{H}_f^T \boldsymbol{Q}_\delta \boldsymbol{H}_f\right) \\ \boldsymbol{g}^T &= 2\left(\boldsymbol{H}_p \Delta\boldsymbol{u}_p + \boldsymbol{F}\boldsymbol{x} - \boldsymbol{w}\right)^T \boldsymbol{Q}_\delta \boldsymbol{H}_f \end{aligned} \tag{24}$$

CONSTRAINTS OF THE PROCESS VARIABLES

The constraints of the process variables mean limitation of the input, output and state values. The most common limitations are

- Limitation of control increments: $\Delta u_{\min} \le \Delta u(k) \le \Delta u_{\max}$
- Limitation of the absolute control input signal: $u_{\min} \le u(k) \le u_{\max}$
- Limitation of the output value: $y_{\min} \le y(k) \le y_{\max}$

All of the constraints need to be expressed as control increments constraints. It means, that it is possible to constrict every variable that depends on the control signal (Camacho and Bordons 2004; Maciejowski 2002). All of these constrains can be joined into one equation for purposes of the quadratic programming

$$\boldsymbol{A}\boldsymbol{u} \le \boldsymbol{b} \tag{25}$$

where $\boldsymbol{u}$ is a vector of the future control increments, $\boldsymbol{A}$ is a corresponding matrix and $\boldsymbol{b}$ is a vector of the constricted values. Their dimensions depend on desired constraints and number of the inputs and outputs.

The constraints of the control increments can be expressed as

$$\begin{aligned} \Delta\boldsymbol{u}(k) &\le \Delta\boldsymbol{u}_{\max} \\ \boldsymbol{I}\Delta\boldsymbol{u}_f &\le \Delta\boldsymbol{u}_{\max} \end{aligned} \tag{26}$$

$$\begin{aligned} \Delta\boldsymbol{u}(k) &\ge \Delta\boldsymbol{u}_{\min} \\ -\Delta\boldsymbol{u}(k) &\le -\Delta\boldsymbol{u}_{\min} \\ -\boldsymbol{I}\Delta\boldsymbol{u}_f &\le -\Delta\boldsymbol{u}_{\min} \end{aligned} \tag{27}$$

The constraints of the absolute value of the control signals can be expressed as

$$\boldsymbol{u}(k) \le \boldsymbol{u}_{\max}$$
$$\boldsymbol{u}(k-1) + \Delta\boldsymbol{u}(k) \le \boldsymbol{u}_{\max}$$
$$\Delta\boldsymbol{u}(k) \le \boldsymbol{u}_{\max} - \boldsymbol{u}(k-1)$$
$$\boldsymbol{T}\Delta\boldsymbol{u}_f \le \boldsymbol{u}_{\max} - \boldsymbol{u}_{k-1} \tag{28}$$

$$\boldsymbol{u}(k) \ge \boldsymbol{u}_{\min}$$
$$-\boldsymbol{u}(k-1) - \Delta\boldsymbol{u}(k) \le -\boldsymbol{u}_{\min}$$
$$-\Delta\boldsymbol{u}(k) \le -\boldsymbol{u}_{\min} + \boldsymbol{u}(k-1)$$
$$-\boldsymbol{T}\Delta\boldsymbol{u}_f \le -\boldsymbol{u}_{\min} + \boldsymbol{u}_{k-1} \tag{29}$$

The constraints of the output values can be expressed as

$$\boldsymbol{y}(k) \le \boldsymbol{y}_{\max}$$
$$\boldsymbol{H}_f\Delta\boldsymbol{u}_f + \boldsymbol{y}_{free} \le \boldsymbol{y}_{\max}$$
$$\boldsymbol{H}_f\Delta\boldsymbol{u}_f \le \boldsymbol{y}_{\max} - \boldsymbol{y}_{free} \tag{30}$$

$$\boldsymbol{y}(k) \ge \boldsymbol{y}_{\min}$$
$$-\boldsymbol{H}_f\Delta\boldsymbol{u}_f - \boldsymbol{y}_{free} \le -\boldsymbol{y}_{\min}$$
$$-\boldsymbol{H}_f\Delta\boldsymbol{u}_f \le -\boldsymbol{y}_{\min} + \boldsymbol{y}_{free} \tag{31}$$

If we join these constraints into equation (25), we get

$$\boldsymbol{Au} \le \boldsymbol{b}$$

$$\begin{bmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \\ \boldsymbol{T} \\ -\boldsymbol{T} \\ \boldsymbol{H}_f \\ -\boldsymbol{H}_f \end{bmatrix} \boldsymbol{u} \le \begin{bmatrix} \boldsymbol{\Delta u}_{\max} \\ -\boldsymbol{\Delta u}_{\min} \\ \boldsymbol{u}_{\max} - \boldsymbol{u}_{k\text{-}1} \\ -\boldsymbol{u}_{\min} + \boldsymbol{u}_{k\text{-}1} \\ \boldsymbol{y}_{\max} - \boldsymbol{y}_{free} \\ -\boldsymbol{y}_{\min} + \boldsymbol{y}_{free} \end{bmatrix} \tag{32}$$

where $\boldsymbol{I}$ is an identity matrix, $\boldsymbol{T}$ is a lower triangular matrix and $\boldsymbol{H}_f$ is a square matrix of dimension $\left[(N\cdot n)\times(N\cdot n)\right]$. The vector $\boldsymbol{b}$ on the right side of the equation contains column vector of length ($N.m$) and ($N.n$). The vector $\boldsymbol{y}_{free}$ is the free response of the controlled system

$$\boldsymbol{y}_{free} = \boldsymbol{H}_p\Delta\boldsymbol{u}_p + \boldsymbol{Fx} \tag{33}$$

RESULTS

This section shows the process control simulation results. The simulations demonstrate functionality and possibilities of the designed state-space predictive controller. All of the simulations were done for the system parameters shown in Table 1. Table 2 shows the controller parameters in case without a time-delay and in case with a time-delay. The presence of the time-delay can pretends a situation when a pipeline is attached between the valve changing the input stream and the tank. The process control simulation was done about one operating point with values: $\overline{h}_1 = 7\text{m}$, $\overline{h}_2 = 5\text{m}$, $\overline{q}_{1f} = 0.4469\text{m}^3/\text{min}$, $\overline{q}_{2f} = 0.2150\text{m}^3/\text{min}$.

The control simulations were also compared by two quadratic criterions for analysis of the control quality. The first criterion, described in equation (34), compares the control increments made in every step and the second criterion, described in equation (35), compares a difference between the reference value and the output value.

$$S_u = \frac{1}{N}\sum_{k=1}^{N}\Delta u^2(k) \tag{34}$$

$$S_e = \frac{1}{N}\sum_{k=1}^{N}\left[w(k) - y(k)\right]^2 \tag{35}$$

Table 1 :System Parameters

Tank	D[m]	H[m]	k[m^3/min]
1	4	10	0.316
2	4	10	0.296

Table 2 : Controller Parameters

d[steps]	T_0[min]	N[steps]	λ	δ
0	0.25	20	1	1
10	0.25	20	3	0.1

Figure 2 and Figure 3 show the simulation results when the change of both tank liquid levels is desired.
Constraints setup for Figure 2 and Figure 3:

- Case 1: no constraints.
- Case 2:
  - $u_{1\max} = 1$ m^3/min, $u_{1\min} = 0$ m^3/min
  - $u_{2\max} = 1$ m^3/min, $u_{2\min} = 0$ m^3/min
- Case 3:
  - $u_{1\max} = 1$ m^3/min, $u_{1\min} = 0$ m^3/min
  - $u_{2\max} = 1$ m^3/min, $u_{2\min} = 0$ m^3/min
  - $y_{1\max} = 7.5$ m, $y_{1\min} = 6.5$ m
  - $y_{2\max} = 5.5$ m, $y_{2\min} = 4.5$ m

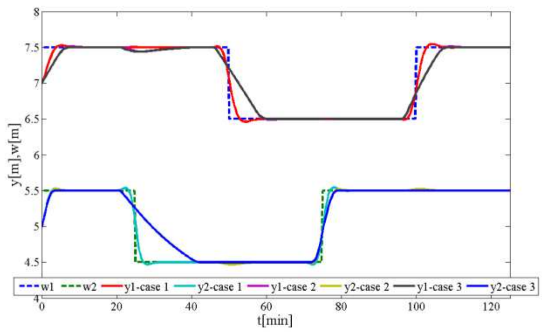

Figure 2 : System output signals

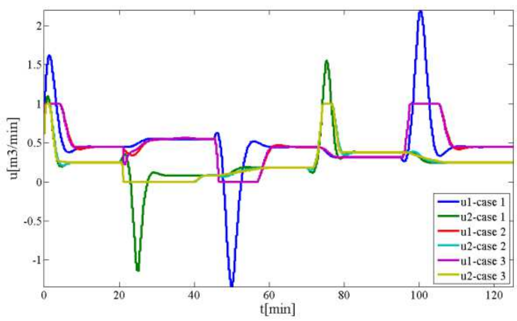

Figure 3 : System control signals

Table 3 : Control simulation results

	Case 1	Case 2	Case 3
$Se_1[m^2]$	0.007	0.021	0.021
$Se_2[m^2]$	0.005	0.023	0.023
$Su_1[(m^3/min)^2]$	$8.61 \cdot 10^{-4}$	$3.85 \cdot 10^{-4}$	$4.03 \cdot 10^{-4}$
$Su_2[(m^3/min)^2]$	$5.24 \cdot 10^{-4}$	$2.83 \cdot 10^{-4}$	$2.85 \cdot 10^{-4}$

Figure 4 and Figure 5 show the simulation results when liquid level of tank 2 is changing and liquid level of tank 1 should stay constant.

Constraints setup for Figure 4 and Figure 5:

- Case 1: no constraints.
- Case 2:
  - $u_{1max} = 1$ m^3/min, $u_{1min} = 0$ m^3/min
  - $u_{2max} = 1$ m^3/min, $u_{2min} = 0$ m^3/min
- Case 3:
  - $u_{1max} = 1$ m^3/min, $u_{1min} = 0$ m^3/min
  - $u_{2max} = 1$ m^3/min, $u_{2min} = 0$ m^3/min
  - $y_{1max} = 7$ m, $y_{1min} = 7$ m

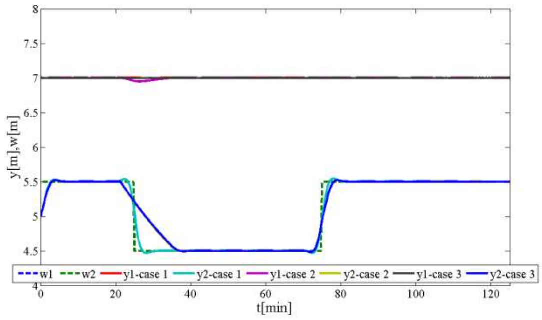

Figure 4 : System output signals

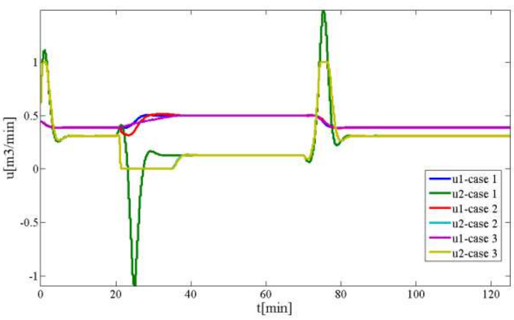

Figure 5 : System control signals

Table 4 :Control simulation results

	Case 1	Case 2	Case 3
$Se_1[m^2]$	$1.91 \cdot 10^{-7}$	$8.89 \cdot 10^{-5}$	$2.26 \cdot 10^{-13}$
$Se_2[m^2]$	0.005	0.016	0.017
$Su_1[(m^3/min)^2]$	$8.26 \cdot 10^{-5}$	$8.37 \cdot 10^{-5}$	$8.30 \cdot 10^{-5}$
$Su_2[(m^3/min)^2]$	$5.21 \cdot 10^{-4}$	$2.89 \cdot 10^{-4}$	$2.89 \cdot 10^{-4}$

Figure 6 and Figure 7 show the simulation results when the 10 steps time-delay is present.

Constraints setup for Figure 6 and Figure 7:

- Case 1: no constraints.
- Case 2:
  - $u_{1max} = 1$ m^3/min, $u_{1min} = 0$ m^3/min
  - $u_{2max} = 1$ m^3/min, $u_{2min} = 0$ m^3/min

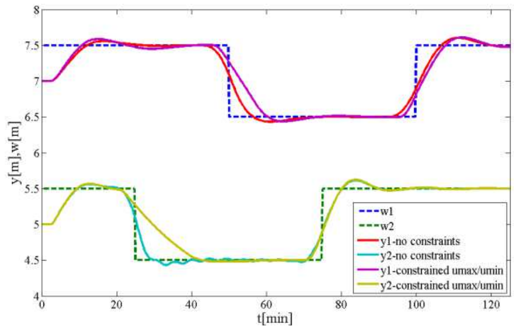

Figure 6 : System output signals with time-delay

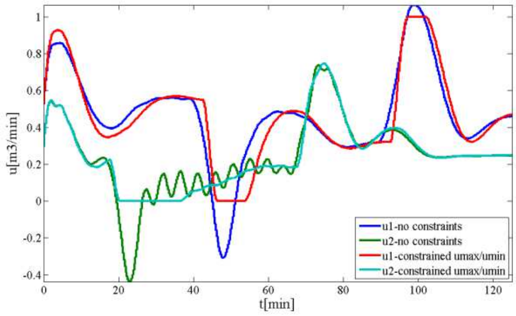

Figure 7 : System control signals with time-delay

Table 5 : Control simulation results

	No constraints	Constrained
$Se_1[m^2]$	0.022	0.031
$Se_2[m^2]$	0.017	0.033
$Su_1[(m^3/min)^2]$	$1.56 \cdot 10^{-4}$	$1.59 \cdot 10^{-4}$
$Su_2[(m^3/min)^2]$	$7.88 \cdot 10^{-5}$	$5.57 \cdot 10^{-5}$

The presented figures show functionality of the constraints of the process variables in the predictive control. The predictive controller without constraints tries to minimize the difference between the reference signal and the output signal and the result can be negative value of the input signal. In this case it means negative liquid stream to the tank. Which is not possible. The possibility of process variables constraints brings a huge advantage into this case. This possibility is directly implemented into the input signal calculation, so there is no need of an additional limitation method. It allows us to keep the input liquid streams between 0 m^3/min and 1 m^3/min. The constraint of the process variables is not restricted only at one kind of constraint, but they can be combined as Figures 2 to 5 show as case 3. The Figures 4 and 5 show the advantage of the combined constraints when the constant liquid level of the tank 1 is required and the tank 2 liquid level changes. If there is only the absolute control signal value limitation, the liquid level of the first tank drops under the reference value. However, this liquid level stays at the reference value when the hard output signal constraint is combined with the control signal value constraint. The Figures 6 and 7 show another possibility of control process influence. The weighting coefficients λ and δ are also an option to affect this process. As can be seen, it necessary to slow down the control process by changing these coefficients to handle it stable.

CONCLUSION

In this paper, the predictive controller based on the state-space CARIMA model with additional possibility of the process variables constraints was presented. This method was used to control the nonlinear process about the selected operating point. The multi-input multi-output system of the two funnel liquid tanks on series was chosen as the exemplar process. The parameters of the tanks simulate large tanks used in an industry. However, the chosen predictive control method works only with linear processes, so the linearization of the nonlinear process is also described in the mathematical model of the controlled system section. One of the main problems of controlling any system is possibility that some process variables can reach a value that is not physically or technologically achievable. The predictive control is a great method to implement the desired process variables constraints directly into the control signal calculation. The results section demonstrates this feature. There are two options how to influence the control process. The first is direct process variables constraints and the second is change of the weighting coefficients λ and δ. This method is also able to control this system with a time-delay with a proper setting of the controller.

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