CONTINUOUS-TIME VS. DISCRETE-TIME IDENTIFICATION MODELS USED FOR ADAPTIVE CONTROL OF NONLINEAR PROCESS

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ABSTRACT

An adaptive control is a technique where the controller adopts a structure or parameters somehow to the control conditions and the state of the controlled system. One way how we can fulfil the adaptivity of the controller is a recursive identification of the controlled system which satisfies that parameters of the controller changes according to parameters of the controlled system during the whole control process. The goal of this contribution is to compare identification models that work in continuous and discrete time. The control synthesis uses polynomial approach that satisfies basic control requirements such as a stability, a disturbance attenuation and a reference signal tracking. The control response could be tuned by the choice of the root position in the Pole-placement method. Moreover, this control method could be easily programmable that is big advantage while we use this method in simulation software such as Matlab etc.

INTRODUCTION

The adaptive control (Åström and Wittenmark, 1989) is not new control approach but it is still used because it produces good control results. Advantage of this method can be found in very good theoretical background and variety of modifications (Bobal *et al.*, 2005).

The approach used here is based on the choice of the External Linear Model (ELM) which describes controlled, originally nonlinear, process in the linear way for example by the discrete or the continuous transfer function (TF) (Bobal *et al.*, 2005). Parameters of this ELM are then identified recursively during the control and parameters of the controller are recomputed according to them. Results of control synthesis are the structure and relations for computing controller's parameters that reflect identified parameters of ELM.

The recursive identification of the continuous-time (CT) model (Wahlberg, 1990) is a bit more complicated than identification of the discrete-time (DT) model where the computation uses measured or simulated values of input and output variables in discrete time intervals. This approach could be inaccurate for bigger values of the sampling period. One solution can be found in the use of so called delta-models (Middleton and Goodwin, 2004) that are special types of DT models where parameters of input and output variables are related to the sampling period. It was proved that parameters of the delta-model approach to parameters of the CT model for sufficiently small sampling period (Stericker and Sinha, 1993). This combination of the continuous-time control synthesis with the discrete-time identification is called "Hybrid adaptive control" and some applications can be found for example in (Vojtesek and Dostal, 2005) and (Vojtesek and Dostal, 2011).

The second way is to use the CT control synthesis and also the CT recursive identification. The CT online estimation is not as simple as a DT estimation because derivatives of the input and output variable are immeasurable. This negative feature could be solved for example with the use of differential filters (Dostal *et al.,* 2001).

The control synthesis uses polynomial approach which satisfies basic control system requirements such as a stability of the control loop, a reference signal tracking and a disturbance attenuation. Moreover, the two degrees-of-freedom (2DOF) configuration has good results in the reference signal tracking (Kucera, 1993).

The continuous stirred-tank reactor (CSTR) is typical nonlinear equipment used in the chemical and biochemical industry for production of various chemicals (Ingham *et al.*, 2000). The mathematical model of this nonlinear system is described by the set of nonlinear ordinary differential equations (ODEs) which can be solved mathematically for example by the Runge-Kutta's method. This mathematical model than serves as a testing model for simulation analyses proposed in the theoretical part.

All results in this paper are simulations made in the mathematical software Matlab, version 7.0.1.

ADAPTIVE CONTROL

The adaptive approach (Åström and Wittenmark, 1989) takes its philosophy in the nature, where plants, animals or even human beings "adapt" their behavior to the actual conditions and environment they live in. There could be various adaptive control techniques but the one

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which is used in this work adapt parameters of the controller to actual state of the controlled system. This done via recursive identification of the system's ELM and parameters of the controller are then recomputed according to identified parameters of the ELM.
The design of the controller starts with the choice of the ELM. We can use for example transfer functions (TF) that are generally described in the CT form:

$$G(s)=\frac{b(s)}{a(s)} \tag{1}$$

where polynomials $a(s)$ and $b(s)$ will be later used in the computation of controller's parameters.
It is good to do the static and dynamic analysis of the controlled system before the design of the controller. The static analysis helps with the choice of the optimal working point where we can obtain for example the best concentration of the product or minimal costs. On the other hand, the dynamic analysis of the system can be used for example for the choice of the ELM's order.

Continuous-Time Identification Model

As $G(s)$ is also relation of the Laplace transform of the output variable, $Y(s)$, to the input variable, $U(s)$, the ELM in the (1) could be also rewritten to the form

$$a(\sigma)\cdot y(t)=b(\sigma)\cdot u(t) \tag{2}$$

where $u(t)$ denotes the input variable, $y(t)$ is the output variable and σ is the differentiation operator.
The identification of CT model in (2) is problem because the derivatives of the input and the output variables are immeasurable. If we replace these derivations by the filtered ones denoted by u_f and y_f and computed from

$$\begin{aligned} c(\sigma)\cdot u_f(t)&=u(t)\\ c(\sigma)\cdot y_f(t)&=y(t)\end{aligned} \tag{3}$$

for a new stable polynomial $c(\sigma)$ that fulfils condition $\deg c(\sigma)\geq \deg a(\sigma)$, the Laplace transform of (3) is then

$$\begin{aligned} c(s)\cdot U_f(s)&=U(s)+o_1(s)\\ c(s)\cdot Y_f(s)&=Y(s)+o_2(s)\end{aligned} \tag{4}$$

where polynomials $o_1(s)$ and $o_2(s)$ includes initial conditions of filtered variables. If we substitute (4) into the Laplace transform of the Equation (2), the relation for the Laplace transform of the filtered output variable, $Y_f(s)$ is

$$Y_f(s)=\frac{b(s)}{a(s)}U_f(s)+\Psi(s) \tag{5}$$

and $\Psi(s)$ is a rational function which contains initial conditions of both filtered and unfiltered variables.

The dynamics of the differential filters $c(s)$ in (4) must be faster than the dynamics of the controlled system (Dostal *et al.*, 2001). It is good to choose the parameters of this polynomial sufficiently small.
The values of filtered values are taken in the discrete time moment $t_k = k \cdot T_v$ for $k = 0, 1, 2, \ldots N$. T_v is sampling period and the regression vector has $n+m$ parts where deg $a = n$ and deg $b = m$, i.e.

$$\begin{aligned}\boldsymbol{\varphi}_{CT}(t_k)=\Big[&-y_f(t_k),-y_f^{(1)}(t_k),\ldots,-y_f^{(n-1)}(t_k),\ldots\\ &\ldots,u_f(t_k),u_f^{(1)}(t_k),\ldots,u_f^{(m)}(t_k),1\Big]^T\end{aligned} \tag{6}$$

The vector of parameters

$$\boldsymbol{\theta}_{CT}(t_k)=\left[a_0,a_1,\ldots,a_{n-1},b_0,b_1,\ldots,b_m\right]^T \tag{7}$$

is computed from the differential equation

$$y_f^{(n)}(t_k)=\boldsymbol{\theta}_{CT}^T(t_k)\cdot\boldsymbol{\varphi}_{CT}(t_k)+\Psi(t_k) \tag{8}$$

where $\Psi(t_k)$ includes immeasurable errors.

Discrete-Time Identification Model

The second approach used for example in (Vojtesek and Dostal, 2011) uses so called delta-models for identification. The delta-models are special types of DT models where input and output variables are related to the sampling period.
A new complex variable γ is defined generally as (Mukhopadhyay *et al.*, 1992)

$$\gamma=\frac{z-1}{\beta\cdot T_v\cdot z+(1-\beta)\cdot T_v} \tag{9}$$

where T_v denotes a sampling period and β is an optional parameter and it holds $0\leq\beta\leq 1$. It is clear, that there could be an infinite number of delta-models but so called *Forward delta-model* for $\beta = 0$ was used here.
The complex variable γ is then

$$\gamma=\frac{z-1}{T_v} \tag{10}$$

Some works compares parameters CT vs. delta-model and it was proved for example in (Stericker and Sinha, 1993), that parameters of the delta-model approaches to the CT ones for sufficiently small sampling period T_v.
The CT model (2) can be rewritten to

$$a'(\delta)y(t')=b'(\delta)u(t') \tag{11}$$

where $a'(\delta)$ and $b'(\delta)$ are discrete polynomials and their coefficients are different from those in CT model but we suppose, that they are close to them.
The regression vector is in this case

$$\begin{aligned}\boldsymbol{\varphi}_\delta(k-1)=[&-y_\delta(k-1),\ldots,-y_\delta(k-n),\\ &u_\delta(k+m-n),\ldots,u_\delta(k-n)]^T\end{aligned} \tag{12}$$

The vector of parameters is generally

$$\boldsymbol{\theta}_\delta(k) = [a'_{n-1}, ..., a'_0, b'_m, ..., b'_0]^T \tag{13}$$

and its parameters are computed again from the differential equation

$$y_\delta(k) = \boldsymbol{\theta}_\delta^T(k) \cdot \boldsymbol{\varphi}_\delta(k-1) + e(k) \tag{14}$$

for $e(k)$ as a general random immeasurable component. Both identification methods with the CT model and the delta-model was discussed in this work.

Recursive Identification

Vectors of CT and delta parameters must be identified recursively to satisfy the adaptivity condition. This could be done for example by the Recursive Least-Squares (RLS) method (Fikar and Mikles 1999) which is simple, easily programmable method that could be modified with exponential, directional etc. forgetting factors. These forgetting factors helps with the accuracy in the more complex systems. The RLS method used for estimation of vectors of parameters $\hat{\boldsymbol{\theta}}_{CT}^T$ or $\hat{\boldsymbol{\theta}}_\delta^T$ in (7) and (14) could be described generally by the set of equations:

$$\begin{aligned}
\varepsilon(k) &= y(k) - \boldsymbol{\varphi}^T(k) \cdot \hat{\boldsymbol{\theta}}(k-1) \\
\gamma(k) &= \left[1 + \boldsymbol{\varphi}^T(k) \cdot \mathbf{P}(k-1) \cdot \boldsymbol{\varphi}(k)\right]^{-1} \\
\boldsymbol{L}(k) &= \gamma(k) \cdot \mathbf{P}(k-1) \cdot \boldsymbol{\varphi}(k) \\
\mathbf{P}(k) &= \frac{1}{\lambda_1(k-1)}\left[\mathbf{P}(k-1) - \frac{\mathbf{P}(k-1) \cdot \boldsymbol{\varphi}(k) \cdot \boldsymbol{\varphi}^T(k) \cdot \mathbf{P}(k-1)}{\frac{\lambda_1(k-1)}{\lambda_2(k-1)} + \boldsymbol{\varphi}^T(k) \cdot \mathbf{P}(k-1) \cdot \boldsymbol{\varphi}(k)}\right] \\
\hat{\boldsymbol{\theta}}(k) &= \hat{\boldsymbol{\theta}}(k-1) + \boldsymbol{L}(k)\varepsilon(k)
\end{aligned} \tag{15}$$

where $\boldsymbol{\phi}$ is regression vector, ε denotes a prediction error, $\mathbf{P}$ is a covariance matrix and λ_1 and λ_2 are forgetting factors. For example constant exponential forgetting (Fikar and Mikles 1999) uses $\lambda_2 = 1$ and

$$\lambda_1(k) = 1 - K \cdot \gamma(k) \cdot \varepsilon^2(k) \tag{16}$$

where K is a very small value (e.g. $K = 0.001$). This RLS modification was used in this work for the online estimation.

DESIGN OF THE CONTROLLER

The controller is designed with the use of the polynomial synthesis (Kucera, 1993). There are several advantages of this approach. At first, they can work with the controller in the polynomial description, for example in the form of the transfer function (1). The result of the synthesis is not only the structure, but also the relations for computing of controller's parameters. Moreover, this method satisfies basic control requirements.

The control scheme with two degrees-of-freedom (2DOF) (Grimble, 1994) is shown in Figure 1.

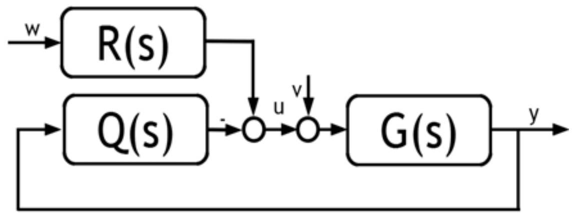

Figure 1: 2DOF control configuration

The signal w is reference signal (i.e. wanted value), v denotes disturbance, u is an input and y an output variable. The block $G(s)$ in Figure 1 represents the controlled system described by the TF (1), blocks $Q(s)$ and $R(s)$ are feedback and feedforward parts of the controller again in the form of TF, generally:

$$Q(s) = \frac{q(s)}{p(s)};\ R(s) = \frac{r(s)}{p(s)} \tag{17}$$

Degrees of polynomials $p(s)$, $q(s)$ and $r(s)$ must hold properness condition:

$$\deg q(s) \le \deg p(s);\ \deg r(s) \le \deg p(s) \tag{18}$$

The condition for the reference signal tracking is satisfied if the polynomial $p(s)$ in the denominator of the controller's transfer functions (17) is divided into

$$p(s) = f(s) \cdot \tilde{p}(s) \tag{19}$$

where $f(s)$ is a least common divisor of the reference and the disturbance transfer functions. If we have these TF in the form of the step function, $f(s) = s$ and (17) could be rewritten into

$$Q(s) = \frac{q(s)}{s \cdot \tilde{p}(s)};\ R(s) = \frac{r(s)}{s \cdot \tilde{p}(s)} \tag{20}$$

Parameters of controller's polynomials are computed from the set of polynomial equations

$$\begin{aligned}
a(s) \cdot s \cdot \tilde{p}(s) + b(s) \cdot q(s) &= d(s) \\
t(s) \cdot s + b(s) \cdot r(s) &= d(s)
\end{aligned} \tag{21}$$

that are in the literature called *Diophantine equations* (Kucera, 1993) and they can be solved by the Method of uncertain coefficients. Polynomial $t(s)$ in equation (21) is an auxiliary stable polynomial and coefficients of this polynomial are not used for computing of coefficients of the polynomial $r(s)$.

Polynomials $a(s)$ and $b(s)$ in (21) are known from the recursive identification and the polynomial $d(s)$ on the right side of Diophantine equations (21) is stable optional polynomial which could affect the quality of the control.

Degrees of controller's polynomials $\tilde{p}(s)$, $q(s)$ and $r(s)$ and the degree of the stable polynomial $d(s)$ are

$$\begin{aligned}
&\deg \tilde{p}(s) = \deg a(s) - 1 \qquad \deg r(s) = 0 \\
&\deg q(s) = \deg a(s) \qquad \deg d(s) = 2 \cdot \deg a(s)
\end{aligned} \tag{22}$$

The simplest way how to choose the stable optional polynomial $d(s)$ define the *Pole-placement method* The polynomial $d(s)$ is then divided into

$$d(s) = \prod_{i=1}^{\deg d(s)} (s + s_i) \tag{23}$$

where roots s_i are generally in the complex form $s_i = \alpha_i + \omega_i \cdot j$ and the stability is satisfied for $\alpha_i < 0$. If we want to obtain an aperiodic output response, ω_i must be $\omega_i = 0$ and (23) is then

$$d(s) = (s + \alpha)^{\deg d} \tag{24}$$

One disadvantage of this method is that it is very general and it provides for example for deg $d(s)$ = 4 four simple roots, two double roots, one single and one triple root but no recommendation for the choice of these roots.

Our previous experiment (Vojtesek and Dostal, 2011) have shown, that it is good co connect the choice of this polynomial somehow with the controlled system. The *Spectral factorization* could be used for this task and it means that the polynomial $d(s)$ is divided into two parts

$$d(s) = n(s) \cdot (s + \alpha)^{\deg d - \deg n} \tag{25}$$

where one part is classic pole-placement method and $n(s)$ comes from the Spectral factorization of the polynomial $a(s)$ in the denominator of the controlled system's transfer function (1):

$$n^*(s) \cdot n(s) = a^*(s) \cdot a(s) \tag{26}$$

The use of Spectral factorization satisfies that the polynomial $n(s)$ is always stable even if the polynomial $a(s)$ is unstable. This could happen for example by inaccurate estimation at the beginning of the control when an estimator does not have enough information about the system.

SIMULATION EXPERIMENT

The adaptive approach was tested by simulations on the mathematical model of the Continuous Stirred-Tank Reactor (CSTR) with so called Van der Vusse reaction inside (Chen *et al.*, 1995). This reaction can be described by the following scheme:

$$\begin{aligned} &A \xrightarrow{k_1} B \xrightarrow{k_2} C \\ &2A \xrightarrow{k_3} D \end{aligned} \tag{27}$$

and the mathematical model of this system comes from material and heat balances inside the reactor. The result is the set of four nonlinear ordinary differential equations (ODE):

$$\begin{aligned} \frac{dc_A}{dt} &= \frac{q_r}{V_r}(c_{A0} - c_A) - k_1 c_A - k_3 c_A^2 \\ \frac{dc_B}{dt} &= -\frac{q_r}{V_r} c_B + k_1 c_A - k_2 c_B \end{aligned} \tag{28}$$

$$\begin{aligned} \frac{dT_r}{dt} &= \frac{q_r}{V_r}(T_{r0} - T_r) - \frac{h_r}{\rho_r c_{pr}} + \frac{A_r U}{V_r \rho_r c_{pr}}(T_c - T_r) \\ \frac{dT_c}{dt} &= \frac{1}{m_c c_{pc}}\left(Q_c + A_r U (T_r - T_c)\right) \end{aligned} \tag{28}$$

State variables are in this case concentrations c_A, c_B and temperatures of the reactant T_r and the cooling T_c. There could be theoretically four input variables – a volumetric flow rate of the reactant, q_r, a heat removal of the cooling, Q_c, an input concentration c_{A0} and an input temperature of the reactant, T_{r0}. The last two are only theoretical and could not be used as an input variable from the practical point of view.

The scheme of this chemical reactor is in Figure 2.

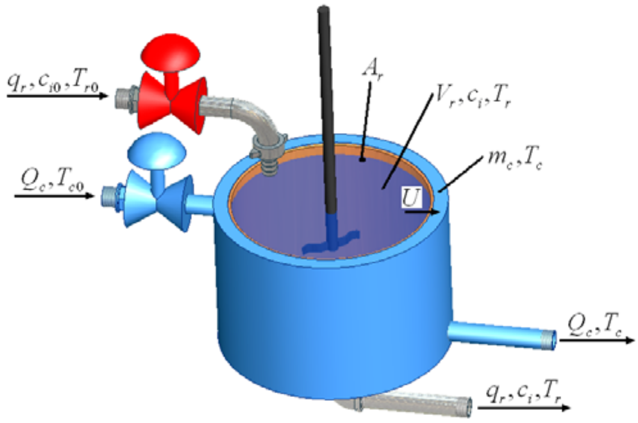

Figure 2: Continuous Stirred-tank Reactor (CSTR) with Van der Vusse reaction inside

Other variables are supposed to be constant during the control because of the simplification. The volume of the reactor is denoted as V_r, A_r is the heat exchange surface, ρ_r is used for the density of the reactant, U is the heat transfer coefficient, c_{pc} and c_{pr} are specific heat capacities of the cooling and the reactant a m_c is the weight of the cooling mass. Values of these fixed parameters are in Table 1 (Chen *et al.*, 1995).

Table 1: Parameters of the reactor

$k_{01} = 2.145 \cdot 10^{10}$ min^{-1}	$k_{02} = 2.145 \cdot 10^{10}$ min^{-1}
$k_{03} = 1.5072 \cdot 10^{8}$ $min^{-1}mol^{-1}$	E_1/R = 9758.3 K
E_2/R = 9758.3 K	E_3/R = 8560 K
h_1 =-4200 $kJ.kmol^{-1}$	h_2 = 11000 $kJ.kmol^{-1}$
h_3 = 41850 $kJ.kmol^{-1}$	
V_r = 0.01 m^3	ρ_r = 934.2 $kg.m^{-3}$
c_{pr} = 3.01 $kJ.kg^{-1}.K^{-1}$	c_{pc} = 2.0 $kJ.kg^{-1}.K^{-1}$
U = 67.2 $kJ.min^{-1}m^{-2}K^{-1}$	A_r = 0.215 m^2
c_{A0} = 5.1 $kmol.m^{-3}$	c_{B0} = 0 $kmol.m^{-3}$
T_{r0} = 387.05 K	m_c = 5 kg

The steady-state analysis (Vojtesek and Dostal, 2005) has shown that the optimal working point is in this case defined by the volumetric flow rate of the reactant $q_r^s = 2.4 \cdot 10^{-3}$ $m^3.min^{-1}$ and heat removal of the coolant Q_c^s = -18.56 $kJ.min^{-1}$.

The input variable for the dynamic study was the change of the heat removal of the coolant, ΔQ_c, and the output variable was the change of reactant's temperature, T_r,

$$u(t)=\frac{Q_c(t)-Q_c^s}{Q_c^s}\cdot 100\,[\%],\quad y(t)=T(t)-T^s\,[K] \quad (29)$$

The dynamic behavior was observed for various step changes of the input variable from the range $\Delta u(t)$ = <-100%; +100%> and results are in Figure 3.

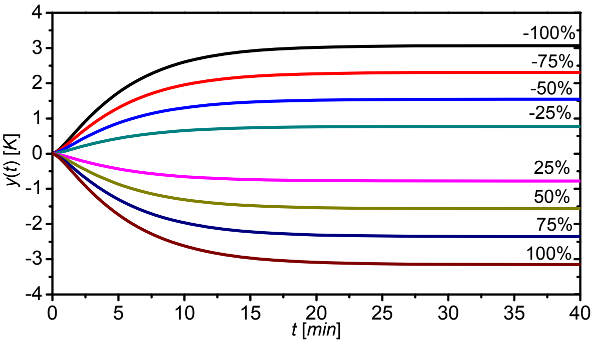

Figure 3: Results of the dynamic analysis for the various changes of the input variable $\Delta u(t)$

It was already mentioned, that the dynamic analysis could help us with the choice of the ELM. It can be seen, that resulted step responses could be approximated by the second order TF with relative order one.
The TF (1) is then

$$G(s)=\frac{b(s)}{a(s)}=\frac{b_1 s+b_0}{s^2+a_1 s+a_0} \quad (30)$$

As the ELM (30) is of the second order, the TF of the controller for both identification methods are according to (20) and (22)

$$Q(s)=\frac{q_2 s^2+q_1 s+q_0}{s\cdot(p_1 s+p_0)};\; R(s)=\frac{r_0}{s\cdot(p_1 s+p_0)} \quad (31)$$

and the stable polynomial $d(s)$ on the right side of (21) is of the fourth degree, i.e.

$$d(s)=n(s)\cdot(s+\alpha)^2 \quad (32)$$

where $n(s)$ comes from the Spectral factorization of (26)
Finally, we have one tuning parameter – the position of the root α.
All simulations have same parameters – the sampling period was T_v = 0.3 *min*, the initial covariance matrix **P**(0) has on the diagonal $1\cdot 10^6$ and starting vectors of parameters for the identification was chosen $\hat{\boldsymbol{\theta}}_{CT}(0)=\hat{\boldsymbol{\theta}}_{\delta}(0)=[0.1,0.1,0.1,0.1]^T$. The simulation took 750 *min* and there were done 5 changes of the reference signal $w(t)$. Our previous experiments have shown that we can obtain better control results if the first change of the reference signal is exponential function instead of the step function. The input signal $u(t)$ was limited to the values $u(t)$ = <-75%; +75%> due to physical limitations.

Control with CT Identification Model

The first simulation experiment was done for CT identification model. The degree of the polynomial $c(s)$ was chosen as deg $c(s)$ = deg $a(s)$ = 2 and

$$c(s)=s^2+c_1 s+c_0=s^2+1.4s+0.49 \quad (33)$$

Filtered input and output variables are then

$$\begin{aligned} y_f^{(2)}(t)+c_1 y_f^{(1)}(t)+c_0 y_f(t)&=y(t)\\ u_f^{(2)}(t)+c_1 u_f^{(1)}(t)+c_0 u_f(t)&=u(t)\end{aligned} \quad (34)$$

The vector of parameters and the regression vector are for ELM (30)

$$\begin{aligned}\boldsymbol{\varphi}_{CT}(t_k)&=\left[-y_f(t_k),-y_f^{(1)}(t_k),u_f(t_k),u_f^{(1)}(t_k)\right]^T\\ \boldsymbol{\theta}_{CT}(t_k)&=\left[a_0,a_1,b_0,b_1\right]^T\end{aligned} \quad (35)$$

where parameters $\boldsymbol{\theta}_{CT}(t_k)$ are estimated recursively by RLS method with constant exponential forgetting described in the theoretical part.
There were done three simulation studies for different α and results are shown in Figure 4 and Figure 5.

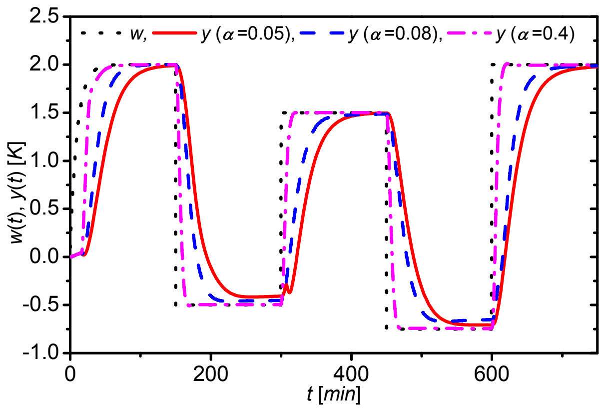

Figure 4: Courses of the reference signal, $w(t)$, and the output variable, $y(t)$, for various values of the parameter α, results for the CT identification model

Figure 4 clearly shows that increasing value of the parameter α affects mainly the speed of the output response – an increasing value of α produces quicker output response. It is worth to notice, that the change of the reference signal from the positive to the negative value causes problems for smaller values of α. The output response then do not reach the reference signal. On the other hand, the control with the biggest value of α = 0.4 produces very good results also with this negative step changes of the reference signal.
Figure 5 shows that the controller computes also very smooth course of the action value, $u(t)$, what is also important from the practical point of view. The action signal is represented by some action of the actuators and

quick changes of the input variable could affect the lifetime of them.

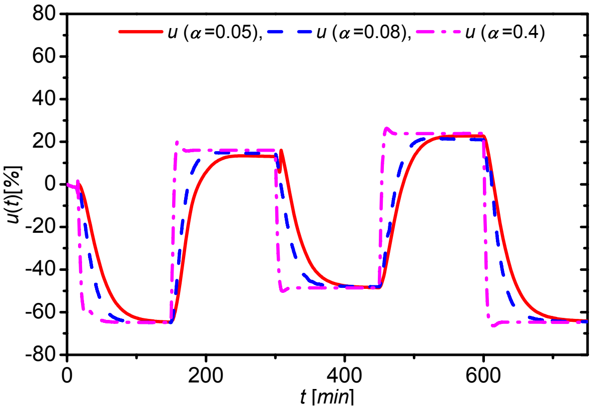

Figure 5: The course of the input variable, $u(t)$, for various values of the parameter α, results for the CT identification model

Control with Delta-model Identification

The second approach uses delta-models for identification which means that vector of parameters and data vector are

$$\boldsymbol{\varphi}_\delta(k-1) = \left[-y_\delta(k-1), -y_\delta(k-2), u_\delta(k-1), u_\delta(k-2)\right]^T$$
$$\boldsymbol{\theta}_\delta(k) = \left[a_1', a_0', b_1', b_0'\right]^T \quad (36)$$

where δ-values of the input and the output variables are

$$y_\delta(k) = \frac{y(k) - 2y(k-1) + y(k-2)}{T_v^2}$$
$$y_\delta(k-1) = \frac{y(k-1) - y(k-2)}{T_v}$$
$$y_\delta(k-2) = y(k-2) \quad (37)$$
$$u_\delta(k-1) = \frac{u(k-1) - u(k-2)}{T_v}$$
$$u_\delta(k-2) = u(k-2)$$

Parameters of $\hat{\boldsymbol{\theta}}_\delta(k)$ are again estimated recursively by the RLS method with constant exponential forgetting.

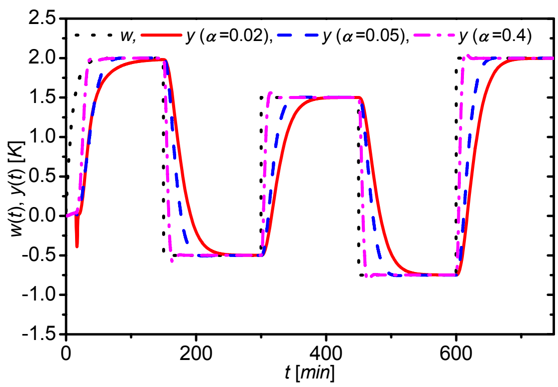

Figure 6: Courses of the reference signal, $w(t)$, and the output variable, $y(t)$, for various values of the parameter α, results for the delta identification model

There were done simulation experiments for the same values of the parameter α = 0.05, 0.08 and 0.4 and the same changes of the reference signal as in previous case due to comparability. Results are shown in Figure 6 and Figure 7.

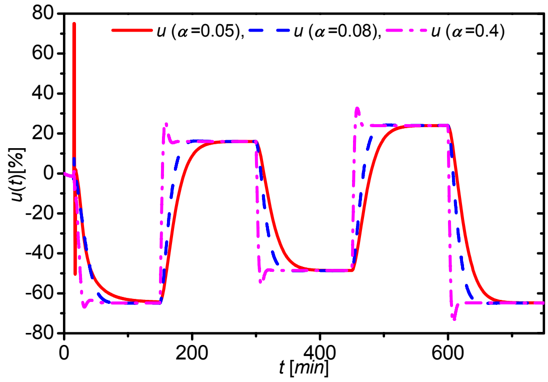

Figure 7: The course of the input variable, $u(t)$, for various values of the parameter α, results for the delta identification model

It can be seen that the use of delta-models for the identification can also produce good control results. The effect of α is the same as in previous case, i.e. quicker output response can be obtained for bigger values of α. The output response in this case have a very small overshoots for the biggest value of $\alpha = 0.4$ but does not have problem with negative changes of the reference signal compared with the CT model. The course of the input variable, $u(t)$, is very similar to the previous case. We can see only some problems at the very beginning of the control which is typical for the adaptive controll that starts from the general vector of parameters $\hat{\boldsymbol{\theta}}_\delta(0)$. It takes some time to approach to right values, but once they are reached, results are good.

The quality of the control for both control techniques was evaluated by the control quality criteria S_u that displays how big are changes of the input variable $u(t)$ and the output criteria S_y that sums the square of the control error $e = w - y$, i.e.

$$S_u = \sum_{i=2}^{N} \left(u(i) - u(i-1)\right)^2 \; [-]$$
$$S_y = \sum_{i=1}^{N} \left(w(i) - y(i)\right)^2 \; \left[K^2\right] \qquad \text{for } N = \frac{T_f}{T_v} \quad (38)$$

where T_f is final time which is in this case $T_f = 450$ *min*. Values of these quality criteria was computed for each simulation study and results are shown in Table 2.

Table 2: Values of quality criteria S_u and S_y

	CT model		*Delta model*	
	S_u [-]	S_y [K^2]	S_u [-]	S_y [K^2]
$\alpha = 0.02$	278	1 254	23 939	1 733
$\alpha = 0.05$	374	886	390	1 233
$\alpha = 0.4$	2 203	312	1 704	453

The choice of the optimal value of the tuning parameter α in both strategies depends what is important for us from the control point of view. Table 2 shows that if the output variable is more important, the bigger value of α is better. This can be also clearly seen from graphs in Figure 4 and Figure 6. Oppositely, if we want the less changes of the input variable, the control with lower value of α is good choice.

As all results in this paper comes from the simulation it is worth to mention the computation requirements. The simulation of the control with the delta identification model takes in Matlab about 10 *seconds*. On the other hand, the CT identification model is more computationally demanding and the simulation for the same parameters took 1.5 *minutes*. As a result, computation with CT model is nine times more demanding than control with delta identification model. In fact it is not big problem because the sampling period was T_v = 0.3 *min*, which is 20 *seconds* that is enough time for the identification and the computation of new parameters of the controller even for CT model.

CONCLUSIONS

The goal of this paper was to show two on-line recursive identification methods used in the adaptive control. The first one is continuous-time identification that uses differential filters. This method is more computationally demanding but offers more accurate results. The next identification method is based on delta-models that are special types of DT models where input and output variables are related to the sampling period which could shift parameters of the delta-model close to parameters of the CT model. As a result, this method is quicker and the output responses are very close to those from the CT model. Used adaptive approach uses polynomial approach with the Pole-placement method and the Spectral factorization that satisfies basic control requirements. Moreover, this adaptive controller could be tuned by the choice of the position of the root in the Pole-placement method and the main effect is in the speed of the control. All approaches were tested on the mathematical model of the CSTR as a typical member of the nonlinear systems with lumped parameters. The future work will head to the verification of simulated results on the real model of this or similar system.

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