STATE-SPACE PREDICTIVE CONTROL OF INVERTED PENDULUM MODEL

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ABSTRACT

This paper presents a possible way to control the a very fast nonlinear systems. The system of the inverted pendulum was chosen as an exemplar process. This is an example of the nonlinear single-input multi-output process with a sampling period in order of milliseconds. The state-space predictive control was chosen as a control method and the system is described by CARIMA model. The whole process of the controller design is described in this paper. That includes a description of the inverted pendulum nonlinear mathematical model and its linearization, the inference of the output values prediction and the control signal calculation. The control signal is calculated by predictor-corrector method. The results compare several optimization methods to achieve the fastest calculation of the control signal. All of the simulation was done in Matlab.

INTRODUCTION

In real life we can come across with many types of processes. Many of them are nonlinear and their mathematical models are very complex. Even the sampling period can be very different. This paper focuses on the very fast processes with a sampling period in the order of milliseconds. The basic control methods may not handle with this situation with required precision so we need a more advanced method. The predictive control is a great example of the modern control method that can be used to solve the complex control problems (Bobál 2008).

This method belongs to the model based control methods and the mathematical model is used for the output values prediction. This prediction is determine on the chosen time horizon that should be long enough to cover the step response of the controlled system. The model of the inverted pendulum is described by the state-space CARIMA mathematical model for the single-input multi-output (SIMO) system (Bars et al. 2011; Wang 2009).

The control signal calculated by the predictive control ensures the desired output values in the near future time horizon. This is achieved by minimization of the cost function that usually has a quadratic form and it minimize the differences between the reference value and the output value and the control signal increments. If the process require some kind of the process variable constraints, several method such as quadratic programming method, fast-gradient method, predictor-corrector method etc. can be used to minimize the cost function (Camacho and Bordons 2004; Maciejowski 2002; Rossiter 2003).

However, the chosen CARIMA mathematical model used to the prediction of the output values works only for the linear models so the nonlinear mathematical model of the inverted pendulum needs to be linearized.

This paper is divided into the following sections. The model of the inverted pendulum is described in the first section. The predictive control and the calculation of the control signal are described next. The final sections shows the results of the research and the conclusion (Albertos Peréz and Sala 20014; Hangos et al. 2004).

MATHEMATICAL MODEL OF THE CONTROLLED SYSTEM

The Amira PS600 inverted pendulum system was used as the exemplar model. The photo of this system is shown at figure 1. The main parts of the system are cart driven by servo amplifier and the pendulum rod attached to the cart (Amira 2000; Chalupa and Bobál 2008).

Figure 1 : Amira PS600 Inverted Pendulum system

The inverted pendulum system is an example of the single-input two-output system. The force produced by the DC motor that moves with the cart is the input variable and the cart position and the angle of the pendulum rod are the output variables. The figure 2 shows the analysis of the forces acting in the system (Amira 2000; Chalupa and Bobál 2008).

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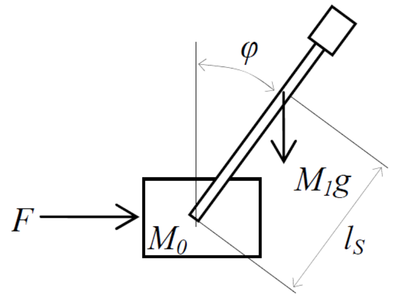

Figure 2 : Analysis of the inverted pendulum

The variables in the figure 2 are following. The angle of pendulum rod is φ, M_0 and M_1 stands for the weight of the cart and pendulum respectively, ls is a distance between centre of gravity of the pendulum and the centre of rotation of the pendulum and g is the gravity acceleration constant. Symbol F represents the force produced by the DC motor.

The affect of the pendulum on the cart can be expressed as a horizontal and a vertical forces described by the equations (1) and (2)

$$H = M_1 \frac{d^2\left(r + l_s \sin\varphi\right)}{dt^2} \qquad (1)$$

$$V = M_1 \frac{d^2\left(l_s \cos\varphi\right)}{dt^2} \qquad (2)$$

where r is the position of the cart.

The equation (3) describe a motion equation of the cart.

$$M_0 \frac{d^2 r}{dt^2} = F - H - F_r \frac{dr}{dt} \qquad (3)$$

where F_r is the constant of a velocity proportional friction of the cart. The rotary motion of the rod about its centre is derived according to the angular conversation law and described by the equation (4).

$$\Theta_s \frac{d^2\varphi}{dt^2} = Vl_s \sin\varphi - Hl_s \cos\varphi - C\frac{d\varphi}{dt} \qquad (4)$$

where Θ_s represents the inertia moment of the pendulum rod with respect to the centre of gravity and C denotes the friction constant of the pendulum.

If we substitute the equations (1) and (2) into the equations (3) and (4) we get the nonlinear equations (5) and (6) describing the behavior of the inverted pendulum system.

$$Mr'' + F_r r' + M_1 l_s \varphi'' \cos\varphi - M_1 l_s \left(\varphi'\right)^2 \sin\varphi = F \qquad (5)$$

$$\Theta\varphi'' + C\varphi' - M_1 l_s g \sin\varphi + M_1 l_s r'' \cos\varphi = 0 \qquad (6)$$

where following abbreviations were used:

$$\Theta = \Theta_s + M_1 l_s^2 \qquad (7)$$

$$M = M_0 + M_1 \qquad (8)$$

The nonlinear state-space model of this system can be obtain by choosing the state vector as shown in the equation (9).

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} r \\ r' \\ \varphi \\ \varphi' \end{bmatrix} \qquad (9)$$

The dynamics of the system in the state-space model representation can be described as the equation (10)

$$\boldsymbol{x}' = \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} x_2 \\ f_1(\boldsymbol{x}.u) \\ x_4 \\ f_2(\boldsymbol{x}.u) \end{bmatrix} \qquad (10)$$

where the functions f_1 and f_2 are derived from the equations (5) and (6).

$$f_1(\boldsymbol{x}.u) = \frac{1}{M_1^2 l_s^2 \cos^2 x_3 - M\Theta}\Big[-CM_1 l_s x_4 \cos x_3 + \\ + M_1^2 l_s^2 g \sin x_3 \cos x_3 + \Theta F_r x_2 - \Theta M_1 l_s x_4^2 \sin x_3 - \Theta u\Big] \qquad (11)$$

$$f_2(\boldsymbol{x}.u) = \frac{1}{M_1^2 l_s^2 \cos^2 x_3 - M\Theta}\Big[MCx_4 - MM_1 l_s g \sin x_3 - \\ - M_1 l_s F_r x_2 \cos x_3 + M_1^2 l_s^2 x_4^2 \sin x_3 \cos x_3 + M_1 l_s u \cos x_3\Big] \qquad (12)$$

The cart position r and the angle of the pendulum φ are the output variables (Chalupa and Bobál 2008).

$$\boldsymbol{y} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}\begin{bmatrix} r \\ \varphi \end{bmatrix} \qquad (13)$$

The described nonlinear model has to be linearized around some operating point. The linearization about the operating point means substitution of the absolute value of the input, output and state variables by its divergence from the steady state.

$$\begin{aligned} \boldsymbol{x}_\delta(t) &= \boldsymbol{x}(t) - \bar{\boldsymbol{x}} \\ \boldsymbol{u}_\delta(t) &= \boldsymbol{u}(t) - \bar{\boldsymbol{u}} \\ \boldsymbol{y}_\delta(t) &= \boldsymbol{y}(t) - \bar{\boldsymbol{y}} \end{aligned} \qquad (14)$$

Where $\bar{\boldsymbol{x}}$ is a vector of the equilibrium state variables, $\bar{\boldsymbol{u}}$ is a vector of the equilibrium input variables, $\bar{\boldsymbol{y}}$ is a vector of the equilibrium output variables, $\boldsymbol{x}_\delta, \boldsymbol{u}_\delta, \boldsymbol{y}_\delta$ are divergences from equilibrium values.

The linearized state-space model can be expressed in form

$$\begin{aligned} \dot{\boldsymbol{x}}_\delta &= \boldsymbol{A}\boldsymbol{x}_\delta + \boldsymbol{B}\boldsymbol{u}_\delta \\ \boldsymbol{y}_\delta &= \boldsymbol{C}\boldsymbol{x}_\delta \end{aligned} \qquad (15)$$

where matrices $\boldsymbol{A}$, $\boldsymbol{B}$ are

$$\boldsymbol{A} = \left.\frac{\partial f}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}=\bar{\boldsymbol{x}},\boldsymbol{u}=\bar{\boldsymbol{u}}} \qquad \boldsymbol{B} = \left.\frac{\partial f}{\partial \boldsymbol{u}}\right|_{\boldsymbol{x}=\bar{\boldsymbol{x}},\boldsymbol{u}=\bar{\boldsymbol{u}}} \qquad (16)$$

The precise form of the $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ matrices are shown in the equation (17)

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (17)$$

where the inner components of the matrices are derived for the equilibrium point when the pendulum is in the upright position (Chalupa and Bobál 2008).

$$a_{22} = \frac{\Theta F_r}{R}, a_{23} = \frac{M_1^2 l_s^2 g}{R}, a_{24} = \frac{-CM_1 l_s}{R}$$
$$a_{42} = \frac{-M_1 l_s F_r}{R}, a_{43} = \frac{-MM_1 l_s g}{R}, a_{44} = \frac{CM}{R}$$
$$b_2 = \frac{-\Theta}{R}, b_4 = \frac{M_1 l_s}{R}, R = M_1^2 l_s^2 - M\Theta \quad (18)$$

However, this is still a continuous-time model and it needs to be transferred into a discrete-time form suitable for the chosen predictive control method. It can be done by transferring the state-space model into the input-output model

$$A(s)y(t) = B(s)u(t) \quad (19)$$

and then into its discrete representation

$$\tilde{A}(z^{-1})y(k) = B(z^{-1})\Delta u(k) \quad (20)$$

where the polynom $\tilde{A}(z^{-1})$ is

$$\tilde{A}(z^{-1}) = (1 - z^{-1})A(z^{-1}) \quad (21)$$

STATE-SPACE PREDICTIVE CONTROL

The chosen predictive control method uses the state-space CARIMA (Controlled Auto-Regresive and Integrated Moving Average) model for prediction of the output values. This model is described by equation (22).

$$\boldsymbol{x}(k+1) = \tilde{\boldsymbol{A}}\boldsymbol{x}(k) + \boldsymbol{B}\Delta u(k)$$
$$y(k) = \boldsymbol{C}\boldsymbol{x}(k) \quad (22)$$

Where the vector of state variables has form

$$\boldsymbol{x}(k) = [y(k), y(k-1), \cdots, y(k-na),$$
$$\Delta u(k-1), \cdots, \Delta u(k-nb+1)]^T \quad (23)$$

The matrices $\tilde{\boldsymbol{A}}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ from the model (22) can be expressed as

$$\tilde{\boldsymbol{A}} = \begin{bmatrix} -\tilde{a}_1 & \cdots & -\tilde{a}_{na} & -\tilde{a}_{na+1} & b_2 & \cdots & b_{nb-1} & b_{nb} \\ 1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} b_1 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \end{bmatrix}^T$$
$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \quad (24)$$

The values $-\tilde{a}_i$ for $i = 1, \ldots, na+1$ and b_j for $j = 1, \ldots, nb$ consist of the coefficients of the polynoms $\tilde{A}(z^{-1})$ and $B(z^{-1})$ from the equation (21) (Bars et al. 2011; Camacho and Bordons 2004).

The prediction of the output values is obtained recursively using the CARIMA model represented by equation (22). The final matrix form of this prediction is

$$\hat{\boldsymbol{y}} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{H}_f \Delta \boldsymbol{u}_f \quad (25)$$

where $\hat{\boldsymbol{y}}$ is the vector of the predicted output values and $\Delta \boldsymbol{u}_f$ is the vector of the future control increments

$$\hat{\boldsymbol{y}} = \begin{bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \vdots \\ \hat{y}(k+N) \end{bmatrix}$$

$$\Delta \boldsymbol{u}_f = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N) \end{bmatrix} \quad (26)$$

where N is the chosen time horizon for prediction.

The aim of the predictive control is minimize the difference between the future reference values and the predicted output values and also minimize the control signal demand. The quadratic cost function in the equation (27) is used to this optimization problem.

$$J = (\boldsymbol{w} - \hat{\boldsymbol{y}})^T \boldsymbol{Q}_\delta (\boldsymbol{w} - \hat{\boldsymbol{y}}) + \Delta \boldsymbol{u}_f^T \boldsymbol{Q}_\lambda \Delta \boldsymbol{u}_f \quad (27)$$

where $\boldsymbol{w}$ is a vector of the future reference values, $\hat{\boldsymbol{y}}$ is the vector of the predicted outputs values, $\boldsymbol{Q}_\lambda$ and $\boldsymbol{Q}_\delta$ are the diagonal weighting matrices containing the weighting coeffitients λ and δ. The vector $\Delta \boldsymbol{u}_f$ is unknown vector of the future control increments (Camacho and Bordons 2004; Fikar and Mikleš 2008).

Because of the chosen optimization method, this cost function needs to be modified into the form of the equation (28).

$$J = \frac{1}{2}\boldsymbol{u}^T \boldsymbol{H}_c \boldsymbol{u} + \boldsymbol{g}^T \boldsymbol{u} \quad (28)$$

where

$$\boldsymbol{H}_c = 2\left(\boldsymbol{Q}_\lambda + \boldsymbol{H}_f^T \boldsymbol{Q}_\delta \boldsymbol{H}_f\right)$$
$$\boldsymbol{g}^T = 2\left(\boldsymbol{Fx} - \boldsymbol{w}\right)^T \boldsymbol{Q}_\delta \boldsymbol{H}_f \tag{29}$$

PREDICTOR-CORRECTOR METHOD

The predictor-corrector method is one of the primal-dual interior-point methods using to solve the inequality constrained convex quadratic problems

$$f(x) = \frac{1}{2}\boldsymbol{x}^T \boldsymbol{G}\boldsymbol{x} + \boldsymbol{g}^T \boldsymbol{x}$$
$$\boldsymbol{A}^T \boldsymbol{x} \geq \boldsymbol{b} \tag{30}$$

which is exactly the problem that the predictive control solves. The equation (30) represents the general formulation of the constrained quadratic problem. The aim is to find the unknown vector $\boldsymbol{x}$ with respect to the chosen constrains representing the future values of the control signal increments (Nocedal and Wright 2000; Wright 1997).

This is the iterative method and we have to set the starting points of the unknown vector $\boldsymbol{x}_0$, the vector of the Lagrange multipliers $\boldsymbol{\lambda}_0$ and the slackvector $\boldsymbol{s}_0$ where $\boldsymbol{s} = \boldsymbol{A}^T\boldsymbol{x} - \boldsymbol{b}, \boldsymbol{s} \geq 0$. These starting points serves to calculate the initial residual vectors $\boldsymbol{r}_d$, $\boldsymbol{r}_s$ and $\boldsymbol{r}_{s\lambda}$

$$\boldsymbol{r}_d = \boldsymbol{G}\boldsymbol{x}_0 + \boldsymbol{g} - \boldsymbol{A}\boldsymbol{\lambda}_0$$
$$\boldsymbol{r}_p = \boldsymbol{s}_0 - \boldsymbol{A}^T\boldsymbol{x}_0 + \boldsymbol{b}$$
$$\boldsymbol{r}_{s\lambda} = \boldsymbol{S}_0 \boldsymbol{\Lambda}_0 \boldsymbol{e} \tag{31}$$

where $\boldsymbol{S}_0$ and $\boldsymbol{\Lambda}_0$ are the diagonal matrices containing the elements of the $\boldsymbol{s}_0$ and $\boldsymbol{\lambda}_0$. The $\boldsymbol{e}$ is vector of ones (Nocedal and Wright 2000; Wright 1997).

There is also need to calculate the initial complementarity measure μ which is need for centering parameter σ

$$\mu = \frac{\boldsymbol{s}_0^T \boldsymbol{\lambda}_0}{m} \tag{32}$$

where m is the number of the inequality constraints.

The whole algorithm can be divided into two parts. The first is the calculation of the predictor step and the second is the calculation of the corrector step. The predictor step is calculated by applying the Newton's method around the current point on the equations (31).

$$\begin{bmatrix} \boldsymbol{G} & -\boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{A}^T & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{S} & \boldsymbol{\Lambda} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{x}^{aff} \\ \Delta\boldsymbol{\lambda}^{aff} \\ \Delta\boldsymbol{s}^{aff} \end{bmatrix} = -\begin{bmatrix} \boldsymbol{r}_d \\ \boldsymbol{r}_p \\ \boldsymbol{r}_{s\lambda} \end{bmatrix} \tag{33}$$

The affine scaling direction $\left(\Delta\boldsymbol{x}^{aff}, \Delta\boldsymbol{\lambda}^{aff}, \Delta\boldsymbol{s}^{aff}\right)$ is obtained by solving these equations. Then the scaling parameter α^{aff} for the predictor step is chosen. This parameter have to satisfy the conditions in the equations (34).

$$\boldsymbol{\lambda} + \alpha_\lambda^{aff} \Delta\boldsymbol{\lambda}^{aff} \geq 0$$
$$\boldsymbol{s} + \alpha_s^{aff} \Delta\boldsymbol{s}^{aff} \geq 0 \tag{34}$$

The final scaling parameter is chosen in the following way:

$$\alpha_\lambda^{aff} = \min_{i:\Delta\lambda_i < 0}\left(1, \min \frac{-\lambda_i}{\Delta\lambda_i^{aff}}\right)$$
$$\alpha_s^{aff} = \min_{i:\Delta s_i < 0}\left(1, \min \frac{-s_i}{\Delta s_i^{aff}}\right)$$
$$\alpha^{aff} = \min\left(\alpha_\lambda^{aff}, \alpha_s^{aff}\right) \tag{35}$$

Now the complementarity measure μ^{aff} of the predictor step and the centering parameter σ can be calculated.

$$\mu^{aff} = \frac{\left(\boldsymbol{s} + \alpha^{aff}\Delta\boldsymbol{s}^{aff}\right)^T \left(\boldsymbol{\lambda} + \alpha^{aff}\Delta\boldsymbol{\lambda}^{aff}\right)}{m} \tag{36}$$

$$\sigma = \left(\frac{\mu^{aff}}{\mu}\right)^3 \tag{37}$$

Now we can move to the calculation of the corrector step. This is done by adjusting the right hand side of the equation (33) by computed affine scaling direction and the centering parameter. The resulting equation system is shown as equation (38) (Nocedal and Wright 2000; Wright 1997).

$$\begin{bmatrix} \boldsymbol{G} & -\boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{A}^T & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{S} & \boldsymbol{\Lambda} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{x} \\ \Delta\boldsymbol{\lambda} \\ \Delta\boldsymbol{s} \end{bmatrix} = -\begin{bmatrix} \boldsymbol{r}_d \\ \boldsymbol{r}_p \\ \boldsymbol{r}_{s\lambda} + \Delta\boldsymbol{S}^{aff}\Delta\boldsymbol{\Lambda}^{aff}\boldsymbol{e} - \sigma\mu\boldsymbol{e} \end{bmatrix} \tag{38}$$

Solving this system gives us the final scaling direction $\left(\Delta\boldsymbol{x}, \Delta\boldsymbol{\lambda}, \Delta\boldsymbol{s}\right)$. The step length is chosen in the same way it was in the predictor step calculation in the equations (35).

$$\boldsymbol{\lambda} + \alpha_\lambda \Delta\boldsymbol{\lambda} \geq 0$$
$$\boldsymbol{s} + \alpha_s \Delta\boldsymbol{s} \geq 0 \tag{39}$$

Now we can update the unknown vector $\boldsymbol{x}$, the vector of the Lagrange multipliers $\boldsymbol{\lambda}$ and the slackvector $\boldsymbol{s}$.

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha\Delta\boldsymbol{x}$$
$$\boldsymbol{\lambda}_{k+1} = \boldsymbol{\lambda}_k + \alpha\Delta\boldsymbol{\lambda}$$
$$\boldsymbol{s}_{k+1} = \boldsymbol{s}_k + \alpha\Delta\boldsymbol{s} \tag{40}$$

The final step of this algorithm is updating the residuals vectors $\boldsymbol{r}_d$, $\boldsymbol{r}_s$ and $\boldsymbol{r}_{s\lambda}$ and the complementarity measure μ.

$$\boldsymbol{r}_d = \boldsymbol{G}\boldsymbol{x} + \boldsymbol{g} - \boldsymbol{A}\boldsymbol{\lambda}$$
$$\boldsymbol{r}_p = \boldsymbol{s} - \boldsymbol{A}^T\boldsymbol{x} + \boldsymbol{b} \tag{41}$$
$$\boldsymbol{r}_{s\lambda} = \boldsymbol{S}\boldsymbol{\Lambda}\boldsymbol{e}$$

$$\mu = \frac{\boldsymbol{s}^T\boldsymbol{\lambda}}{m} \tag{42}$$

RESULTS

This section shows the results of the process simulation for the step reference signal and the ramp reference signal. Both of the simulation were done with same controller parameters N = 20 steps, λ = 0.001, δ = 10 and the sampling period T_0 = 40 ms. The only difference between done simulations is in the control signal calculation method. The first method is quadprog function built-in in the Matlab and the second one is the presented predictor-corrector method. The results compare the computation time of one control step. The time was measured by the Matlab tic() ... toc() function. The simulations were also compared by two quadratic criterions for analysis of the control quality. The first criterion, described in equation (43), compares the control increments made in every step and the second criterion, described in equation (44), compares a difference between the reference value and the output value.

$$S_u = \frac{1}{N}\sum_{k=1}^{N}\Delta u^2(k) \tag{43}$$

$$S_e = \frac{1}{N}\sum_{k=1}^{N}\left[w(k) - y(k)\right]^2 \tag{44}$$

Table 1 shows the system parameters used for the mathematical model of the system.

Table 1 : System parameters

Symbol	Value	Meaning
M_0 [kg]	4	Cart weight
M_1 [kg]	0.36	Pendulum weight
l_s [m]	0.42	Pendulum length
Θ [kg.m^2]	0.08433	Pendulum inertia moment
F_r [kg/s]	6.5	Cart friction
C [Kg.m^2/s]	0.00652	Pendulum friction
k_a [N/V]	7.5	Servo amplifier gain
g [m/s^2]	9.81	Gravity constant

The figure 3 shows the pulse response of the system for the input pulse $F = 2$ N for 1s.

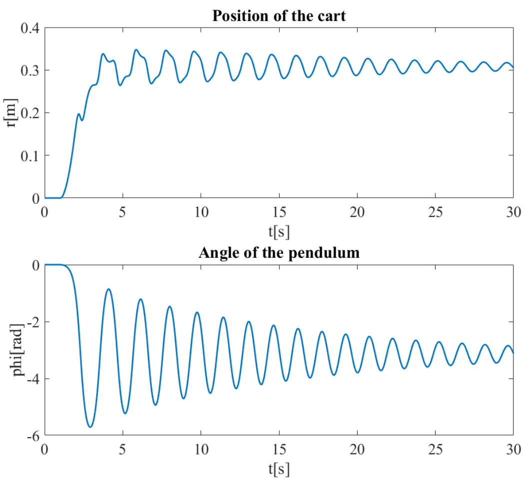

Figure 3 : Pulse response

As we can see, the angle of the pendulum dropped on the value of -π rad. That means the zero value of the pendulum angle is the upward position of the pendulum. Figures 5 and 6 represents the simulations result for the step reference signal.

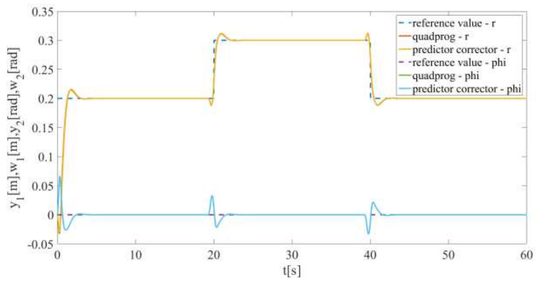

Figure 4 : Simulation outputs

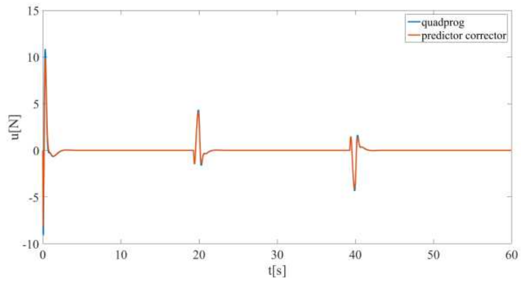

Figure 5 : Simulation inputs

Table 2 :Simulation results

	S_{e1}[m^2]	S_{e2}[rad^2]	S_u[N^2]
Predictor-corrector	5.28 . 10^{-4}	4.68 . 10^{-5}	0.102
quadprog	5.13 . 10^{-4}	4.95 . 10^{-5}	0.130

Computation time:

- Predictor-corrector - 7.73 ms
- Quadprog - 24 ms

Figures 6 and 7 represents the simulation result for the ramp reference signal.

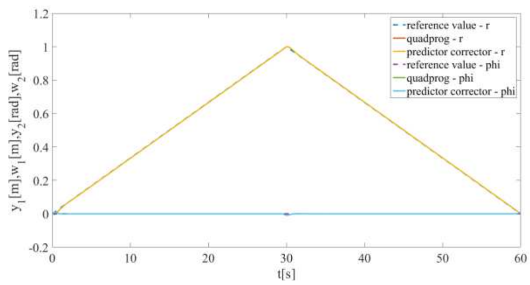

Figure 6 : Simulation outputs

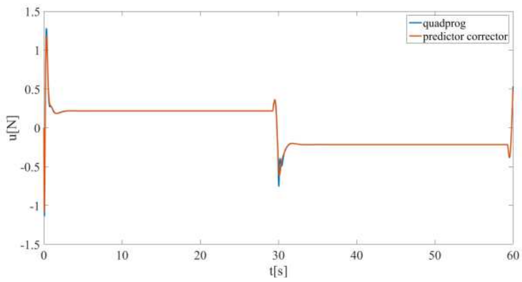

Figure 7 : Simulation inputs

Table 3 : Simulation results

	S_{e1}[m^2]	S_{e2}[rad^2]	S_u[N^2]
Predictor-corrector	$4.94 \cdot 10^{-6}$	$1.63 \cdot 10^{-6}$	$1.51 \cdot 10^{-3}$
quadprog	$4.87 \cdot 10^{-6}$	$1.66 \cdot 10^{-6}$	$1.78 \cdot 10^{-3}$

Computation time:

- Predictor-corrector - 7.99 ms
- Quadprog - 24.5 ms

CONCLUSION

In this paper, the predictive controller based on the state-space CARIMA was presented. The controller was tested on the inverted pendulum system which is an example of the nonlinear single-input two-output system. The goal of the control of this system is to keep the pendulum rod at the upward position while the cart is moving. The movement of the pendulum acting like a disturbance in the system. This system is also relatively fast with chosen sampling period T_0 = 40 ms. The mathematical model of the inverted pendulum was made according to the real laboratory model of the inverted pendulum Amira PS600. The aim of this paper is to present a complex procedure of the creation of the predictive controller which is capable to control such process. The presented predictive controller works with a linear models while the inverted pendulum model is nonlinear. Therefore the linearization of the nonlinear model is also presented besides only the inverted pendulum mathematical model explanation. The calculation of the input signal is done by the minimization of the cost function that minimize the differences between the output and the reference signals and the control signal increments. This minimization is achieved by two different methods. The first one is the quadprog function built-in the Matlab and the second one is the presented predictor-corrector method. The result section compares these two methods. Both methods have almost the same results according to the examined criterions Se and Su. The major difference between them is in the computation time. The mean computation time of the quadprog function is three times longer than the time of the predictor-corrector method.

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