MULTIMODEL APPROACH IN STATE-SPACE PREDICTIVE CONTROL

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KEYWORDS
predictive control, state-space, multimodel, inverted pendulum, predictor-corrector.

ABSTRACT

This paper presents a multimodel approach to control nonlinear systems. The system of the inverted pendulum which has one control input and two measured outputs was chosen as an exemplar process. This system is an example of the nonlinear process with a sampling period in order of milliseconds. The state-space predictive control of the system described by CARIMA model was chosen as a control method. This paper presents a description of the inverted pendulum nonlinear mathematical model, its linearization and the control signal calculation using predictor-corrector method. The results compare three methods of linearized model combination. All of the simulations were done in Matlab.

INTRODUCTION

Process control area offers a variety of different processes with different level of complexity. Even the sampling period can be very different. This paper focuses on nonlinear processes with fast sampling period. The complex and fast processes need a modern control method to control them effectively such as model predictive control (Bobál 2008).

This method predicts the output values on the chosen time interval based on the mathematical model of the controlled process. The model of the inverted pendulum is described by the state-space CARIMA mathematical model for the single-input multi-output (SIMO) system (Bars et al. 2011; Wang 2009).

The predictive control method uses a minimization of the cost function, which is usually in quadratic form, to calculate the control signal. The quadratic form of the cost function minimize the differences between the reference value and the output value and the control signal increments. The predictive control method offers a possibility to constrain the process variables then the chosen predictor-corrector minimization method can be used to minimize the cost function (Camacho and Bordons 2004; Maciejowski 2002; Rossiter 2003).

However, the chosen state-space predictive control method uses a linear CARIMA mathematical model for prediction of the output values but the chosen process of the inverted pendulum has nonlinear behaviour. That means we have to linearize the nonlinear model first. the nonlinear behaviour of the process can be described with a set of the linear models. The final linear model used to output values prediction is obtained by combination of two or more linear models out of the set of the linearized models according to the current output value (Albertos Peréz and Sala 20014; Hangos et al. 2004).

This paper is divided into the following sections. The model of the inverted pendulum is described in the first section. The predictive control and the calculation of the control signal are described next. The final sections shows the results of the research and the conclusion.

MATHEMATICAL MODEL OF THE CONTROLLED SYSTEM

The controlled system in this paper is represented by Amira PS600 inverted pendulum system which is shown at figure 1. The pendulum rod of this system is attached to the cart which is driven by a servo motor (Amira 2000; Chalupa and Bobál 2008).

Figure 1 : Amira PS600 Inverted Pendulum system

The servo motor produces the input force (input variable) that move with the cart. Position of the cart is the first measured output variable and the pendulum angle is the second measured output variable.. The figure 2 shows the analysis of the forces acting in the system (Amira 2000; Chalupa and Bobál 2008).

Proceedings 32nd European Conference on Modelling and Simulation ©ECMS Lars Nolle, Alexandra Burger, Christoph Tholen, Jens Werner, Jens Wellhausen (Editors)
ISBN: 978-0-9932440-6-3/ ISBN: 978-0-9932440-7-0 (CD)

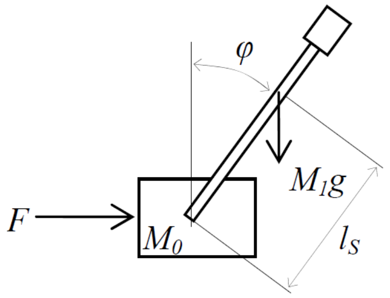

Figure 2 : Analysis of the inverted pendulum

The angle of the pendulum rod is expressed as φ, the input force produced by DC motor is symbol F, symbol ls means distance between pendulum gravity centre and rotation centre of the pendulum, weight of the cart is expressed as M_0, weight of the pendulum is expressed as M_1 and g is gravity acceleration constant.

The equations (1) and (2) describe the horizontal and the vertical forces that the pendulum causes on cart.

$$H = M_1 \frac{d^2\left(r + l_s \sin\varphi\right)}{dt^2} \tag{1}$$

$$V = M_1 \frac{d^2\left(l_s \cos\varphi\right)}{dt^2} \tag{2}$$

where r is the position of the cart.

The equation (3) describe a motion equation of the cart and the equation (4) represents the rotary motion of the pendulum rod about its centre.

$$M_0 \frac{d^2 r}{dt^2} = F - H - F_r \frac{dr}{dt} \tag{3}$$

$$\Theta_s \frac{d^2\varphi}{dt^2} = Vl_s \sin\varphi - Hl_s \cos\varphi - C\frac{d\varphi}{dt} \tag{4}$$

where F_r is the constant of a velocity proportional friction of the cart, Θ_s is the inertia moment of the pendulum rod with respect to the centre of gravity and C is the friction constant of the pendulum.

Substitution of the equations (1) and (2) into the equations (3) and (4) creates the nonlinear equations (5) and (6) describing the behaviour of the inverted pendulum system.

$$Mr'' + F_r r' + M_1 l_s \varphi'' \cos\varphi - M_1 l_s \left(\varphi'\right)^2 \sin\varphi = F \tag{5}$$

$$\Theta\varphi'' + C\varphi' - M_1 l_s g \sin\varphi + M_1 l_s r'' \cos\varphi = 0 \tag{6}$$

where following abbreviations were used:

$$\Theta = \Theta_s + M_1 l_s^2 \tag{7}$$

$$M = M_0 + M_1 \tag{8}$$

The position of the cart r, the angle of the pendulum φ and their derivations r' and φ' are chosen as state variables as is shown in the equation (9).

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} r \\ r' \\ \varphi \\ \varphi' \end{bmatrix} \tag{9}$$

And the output variables are the position of the cart r, the angle of the pendulum φ.

$$\boldsymbol{y} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} r \\ \varphi \end{bmatrix} \tag{10}$$

The nonlinear state-space model is described in the equation (11).

$$\boldsymbol{x}' = \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} x_2 \\ f_1(\boldsymbol{x}.u) \\ x_4 \\ f_2(\boldsymbol{x}.u) \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} \tag{11}$$

where the functions f_1 and f_2 are derived from the equations (5) and (6).

$$f_1(\boldsymbol{x}.u) = \frac{1}{M_1^2 l_s^2 \cos^2 x_3 - M\Theta}\Big[-CM_1 l_s x_4 \cos x_3 + \\ + M_1^2 l_s^2 g \sin x_3 \cos x_3 + \Theta F_r x_2 - \Theta M_1 l_s x_4^2 \sin x_3 - \Theta u\Big] \tag{12}$$

$$f_2(\boldsymbol{x}.u) = \frac{1}{M_1^2 l_s^2 \cos^2 x_3 - M\Theta}\Big[MCx_4 - MM_1 l_s g \sin x_3 - \\ - M_1 l_s F_r x_2 \cos x_3 + M_1^2 l_s^2 x_4^2 \sin x_3 \cos x_3 + M_1 l_s u \cos x_3\Big] \tag{13}$$

The described nonlinear model has to be linearized around an operating point. The linearization about the operating point is done with partial derivation of the nonlinear model which is shown in equations (14) and (15).

$$\begin{aligned} \dot{\boldsymbol{x}}_\delta &= \boldsymbol{A}\boldsymbol{x}_\delta + \boldsymbol{B}\boldsymbol{u}_\delta \\ \boldsymbol{y}_\delta &= \boldsymbol{C}\boldsymbol{x}_\delta \end{aligned} \tag{14}$$

where

$$
\boldsymbol{A}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{\partial f_1(\boldsymbol{x},u)}{\partial x_1} & \frac{\partial f_1(\boldsymbol{x},u)}{\partial x_2} & \frac{\partial f_1(\boldsymbol{x},u)}{\partial x_3} & \frac{\partial f_1(\boldsymbol{x},u)}{\partial x_4} \\ 0 & 0 & 0 & 1 \\ \frac{\partial f_2(\boldsymbol{x},u)}{\partial x_1} & \frac{\partial f_2(\boldsymbol{x},u)}{\partial x_2} & \frac{\partial f_2(\boldsymbol{x},u)}{\partial x_3} & \frac{\partial f_2(\boldsymbol{x},u)}{\partial x_4} \end{bmatrix}
$$

$$
\boldsymbol{B}=\begin{bmatrix} 0 \\ \frac{\partial f_1(\boldsymbol{x},u)}{\partial u} \\ 0 \\ \frac{\partial f_2(\boldsymbol{x},u)}{\partial u} \end{bmatrix}
$$

$$
\boldsymbol{C}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{15}
$$

In case of using multimodel approach, this linearization is done within multiple operation points. Every linearization creates a new linear model in the set of linear models. The operating points of the linearization should be chosen generally from the most nonlinear area of the system behaviour.

However, this is still a continuous-time model and it needs to be transferred into a discrete-time model. The chosen predictive control method using a specific form of the state-space model for prediction of the output values. The transformation of the continuous-time model into the suitable discrete-time model is done by transferring the state-space model (14) into the input-output model

$$
A(s)\,y(t)=B(s)\,u(t) \tag{16}
$$

and then into its discrete representation

$$
\tilde{A}(z^{-1})\,y(k)=B(z^{-1})\,\Delta u(k) \tag{17}
$$

where the polynom $\tilde{A}(z^{-1})$ is

$$
\tilde{A}(z^{-1})=(1-z^{-1})\,A(z^{-1}) \tag{18}
$$

STATE-SPACE PREDICTIVE CONTROL

The output values are predicted using the state-space CARIMA (Controlled Auto-Regressive and Integrated Moving Average) model (19)

$$
\begin{aligned} \boldsymbol{x}(k+1) &= \tilde{\boldsymbol{A}}\boldsymbol{x}(k)+\boldsymbol{B}\Delta u(k) \\ y(k) &= \boldsymbol{C}\boldsymbol{x}(k) \end{aligned} \tag{19}
$$

where the vector of state variables has form

$$
\begin{aligned} \boldsymbol{x}(k) = [\, & y(k), y(k-1), \cdots, y(k-na), \\ & \Delta u(k-1), \cdots, \Delta u(k-nb+1)]^T \end{aligned} \tag{20}
$$

The matrices $\tilde{\boldsymbol{A}}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ from the model (19) can be expressed as

$$
\tilde{\boldsymbol{A}}=\begin{bmatrix} -\tilde{a}_1 & \cdots & -\tilde{a}_{na} & -\tilde{a}_{na+1} & b_2 & \cdots & b_{nb-1} & b_{nb} \\ 1 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \cdots & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}
$$

$$
\boldsymbol{B}=\begin{bmatrix} b_1 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \end{bmatrix}^T
$$

$$
\boldsymbol{C}=\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{21}
$$

The values $-\tilde{a}_i$ for $i=1,\ldots,na+1$ and b_j for $j=1,\ldots,nb$, where na is order of the polynom $\tilde{A}(z^{-1})$ and nb is the order of the polynom $B(z^{-1})$, consist of the coefficients of the polynoms $\tilde{A}(z^{-1})$ and $B(z^{-1})$ from the equation (18) (Bars et al. 2011; Camacho and Bordons 2004).

In case of using multimodel approach, the state-space model used to output values prediction is calculated as a combination of two or more models from the set of the linearized models. The choice of the models is determined by current state.

The final model is calculated according to the equation (22)

$$
\begin{aligned} \tilde{\boldsymbol{A}} &= w_m \cdot \tilde{\boldsymbol{A}}_n + (1-w_m)\cdot \tilde{\boldsymbol{A}}_{n+1} \\ \boldsymbol{B} &= w_m \cdot \boldsymbol{B}_n + (1-w_m)\cdot \boldsymbol{B}_{n+1} \\ \boldsymbol{C} &= w_m \cdot \boldsymbol{C}_n + (1-w_m)\cdot \boldsymbol{C}_{n+1} \end{aligned} \tag{22}
$$

where the w_m is the weighting coefficient of the linear model from the set of the linear models and n is the model number.

This weighting coefficient can be calculated as a linear or nonlinear function or it can be equal 1 in case of an edge transition between models.

The equation for the output values prediction is obtained by recursive substitution of the state equation of the equation (19). The final matrix form of this prediction is

$$
\hat{\boldsymbol{y}} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{H}_f \Delta \boldsymbol{u}_f \tag{23}
$$

where $\hat{\boldsymbol{y}}$ is the vector of the predicted output values and $\Delta \boldsymbol{u}_f$ is the vector of the future control increments

$$
\hat{\boldsymbol{y}}=\begin{bmatrix} \hat{y}(k+1) \\ \hat{y}(k+2) \\ \vdots \\ \hat{y}(k+N) \end{bmatrix}
\qquad
\Delta \boldsymbol{u}_f=\begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ \vdots \\ \Delta u(k+N) \end{bmatrix} \tag{24}
$$

where N is the chosen time horizon for prediction.

The aim of the predictive control is minimize the difference between the future reference values and the predicted output values and minimize the control signal demand. The quadratic cost function witch satisfied this requirement is the equation (25).

$$J = \left(\boldsymbol{w} - \hat{\boldsymbol{y}}\right)^T \boldsymbol{Q}_\delta \left(\boldsymbol{w} - \hat{\boldsymbol{y}}\right) + \Delta\boldsymbol{u}_f^T \boldsymbol{Q}_\lambda \Delta\boldsymbol{u}_f \tag{25}$$

where $\boldsymbol{w}$ is a vector of the future reference values, $\hat{\boldsymbol{y}}$ is the vector of the predicted outputs values, $\boldsymbol{Q}_\lambda$ and $\boldsymbol{Q}_\delta$ are the diagonal weighting matrices containing the weighting coefficients λ and δ. The vector $\Delta\boldsymbol{u}_f$ is unknown vector of the future control increments (Camacho and Bordons 2004; Fikar and Mikleš 2008).

Because of the optimization method calculates with the possibility of the process variables constraints, this cost function needs to be modified into the form of the equation (26).

$$J = \frac{1}{2}\boldsymbol{u}^T \boldsymbol{H}_c \boldsymbol{u} + \boldsymbol{g}^T \boldsymbol{u} \tag{26}$$

where

$$\boldsymbol{H}_c = 2\left(\boldsymbol{Q}_\lambda + \boldsymbol{H}_f^T \boldsymbol{Q}_\delta \boldsymbol{H}_f\right)$$
$$\boldsymbol{g}^T = 2\left(\boldsymbol{Fx} - \boldsymbol{w}\right)^T \boldsymbol{Q}_\delta \boldsymbol{H}_f \tag{27}$$

PREDICTOR-CORRECTOR METHOD

The predictor-corrector method is using to solve the inequality constrained convex quadratic problems

$$f(x) = \frac{1}{2}\boldsymbol{x}^T \boldsymbol{G}\boldsymbol{x} + \boldsymbol{g}^T \boldsymbol{x}$$
$$\boldsymbol{A}^T \boldsymbol{x} \geq \boldsymbol{b} \tag{28}$$

which is exactly the problem that the predictive control solves. The equation (28) represents the general formulation of the constrained quadratic problem. The aim is to find the unknown vector $\boldsymbol{x}$ with respect to the chosen constrains representing the future values of the control signal increments (Nocedal and Wright 2000; Wright 1997).

The predictor-corrector is an iterative method so the starting points of the unknown vector $\boldsymbol{x}_0$, the vector of the Lagrange multipliers $\boldsymbol{\lambda}_0$ and the slackvector $\boldsymbol{s}_0$ where $\boldsymbol{s} = \boldsymbol{A}^T\boldsymbol{x} - \boldsymbol{b}, \boldsymbol{s} \geq 0$ have to be set first. These starting points are used to calculate the initial residual vectors $\boldsymbol{r}_d$, $\boldsymbol{r}_s$ and $\boldsymbol{r}_{s\lambda}$

$$\boldsymbol{r}_d = \boldsymbol{G}\boldsymbol{x}_0 + \boldsymbol{g} - \boldsymbol{A}\boldsymbol{\lambda}_0$$
$$\boldsymbol{r}_p = \boldsymbol{s}_0 - \boldsymbol{A}^T\boldsymbol{x}_0 + \boldsymbol{b}$$
$$\boldsymbol{r}_{s\lambda} = \boldsymbol{S}_0\boldsymbol{\Lambda}_0\boldsymbol{e} \tag{29}$$

where $\boldsymbol{S}_0$ and $\boldsymbol{\Lambda}_0$ are the diagonal matrices containing the elements of the $\boldsymbol{s}_0$ and $\boldsymbol{\lambda}_0$. The $\boldsymbol{e}$ is vector of ones (Nocedal and Wright 2000; Wright 1997).

The initial complementarity measure μ has to be calculated as next step which is needed for centering parameter σ

$$\mu = \frac{\boldsymbol{s}_0^T\boldsymbol{\lambda}_0}{m} \tag{30}$$

where m is the number of the inequality constraints.

The whole algorithm can be divided into two parts. The first is the calculation of the predictor step and the second is the calculation of the corrector step. The predictor step is calculated by applying the Newton's method around the current point on the equations (29).

$$\begin{bmatrix} \boldsymbol{G} & -\boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{A}^T & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{S} & \boldsymbol{\Lambda} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{x}^{aff} \\ \Delta\boldsymbol{\lambda}^{aff} \\ \Delta\boldsymbol{s}^{aff} \end{bmatrix} = -\begin{bmatrix} \boldsymbol{r}_d \\ \boldsymbol{r}_p \\ \boldsymbol{r}_{s\lambda} \end{bmatrix} \tag{31}$$

Solving these equations will give us the affine scaling direction $\left(\Delta\boldsymbol{x}^{aff}, \Delta\boldsymbol{\lambda}^{aff}, \Delta\boldsymbol{s}^{aff}\right)$. Then the scaling parameter α^{aff} for the predictor step witch satisfy the conditions in the equations (32) is chosen.

$$\boldsymbol{\lambda} + \alpha_\lambda^{aff}\Delta\boldsymbol{\lambda}^{aff} \geq 0$$
$$\boldsymbol{s} + \alpha_s^{aff}\Delta\boldsymbol{s}^{aff} \geq 0 \tag{32}$$

The final scaling parameter is chosen as follows:

$$\alpha_\lambda^{aff} = \min_{i:\Delta\lambda_i<0}\left(1, \min\frac{-\lambda_i}{\Delta\lambda_i^{aff}}\right)$$
$$\alpha_s^{aff} = \min_{i:\Delta s_i<0}\left(1, \min\frac{-s_i}{\Delta s_i^{aff}}\right)$$
$$\alpha^{aff} = \min\left(\alpha_\lambda^{aff}, \alpha_s^{aff}\right) \tag{33}$$

The predictor step is finished with the calculation of the complementarity measure μ^{aff} and the centering parameter σ.

$$\mu^{aff} = \frac{\left(\boldsymbol{s} + \alpha^{aff}\Delta\boldsymbol{s}^{aff}\right)^T\left(\boldsymbol{\lambda} + \alpha^{aff}\Delta\boldsymbol{\lambda}^{aff}\right)}{m} \tag{34}$$

$$\sigma = \left(\frac{\mu^{aff}}{\mu}\right)^3 \tag{35}$$

The adjustment of the right hand side of the equation (31) by the computed affine scaling direction and the centering parameter will prepare the equation for the corrector step (36) (Nocedal and Wright 2000; Wright 1997).

$$\begin{bmatrix} \boldsymbol{G} & -\boldsymbol{A} & \boldsymbol{0} \\ -\boldsymbol{A}^T & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{S} & \boldsymbol{\Lambda} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{x} \\ \Delta\boldsymbol{\lambda} \\ \Delta\boldsymbol{s} \end{bmatrix} = -\begin{bmatrix} \boldsymbol{r}_d \\ \boldsymbol{r}_p \\ \boldsymbol{r}_{s\lambda} + \Delta\boldsymbol{S}^{aff}\Delta\boldsymbol{\Lambda}^{aff}\boldsymbol{e} - \sigma\mu\boldsymbol{e} \end{bmatrix} \tag{36}$$

Solving this system gives us the final scaling direction $\left(\Delta\boldsymbol{x}, \Delta\boldsymbol{\lambda}, \Delta\boldsymbol{s}\right)$. The step length is chosen in the same way it was in the predictor step calculation in the equations (33).

$$\boldsymbol{\lambda} + \alpha_\lambda\Delta\boldsymbol{\lambda} \geq 0$$
$$\boldsymbol{s} + \alpha_s\Delta\boldsymbol{s} \geq 0 \tag{37}$$

The last calculation of this method is the update of the unknown vector $\boldsymbol{x}$, the vector of the Lagrange multipliers $\boldsymbol{\lambda}$ and the slackvector $\boldsymbol{s}$

$$x_{k+1} = x_k + \alpha \Delta x$$
$$\lambda_{k+1} = \lambda_k + \alpha \Delta \lambda$$
$$s_{k+1} = s_k + \alpha \Delta s \quad (38)$$

and the residuals vectors r_d, r_s and $r_{s\lambda}$ and the complementarity measure μ.

$$r_d = Gx + g - A\lambda$$
$$r_p = s - A^T x + b \quad (39)$$
$$r_{s\lambda} = S\Lambda e$$

$$\mu = \frac{s^T \lambda}{m} \quad (40)$$

RESULTS

This section shows the results of the process simulation of controlled fall of the inverted pendulum from up to down position. The controller parameters N = 20 steps, λ = 0.001, δ = 10 and the sampling period T_0 = 40 ms were set for all of the simulations. The presented simulation results are differ in calculation of the model weighting coefficient. The simulations were compared by two quadratic criterions for analysis of the control quality. The first criterion, described in equation (41), compares the control increments made in every step and the second criterion, described in equation (42), compares a difference between the reference value and the output value.

$$S_u = \frac{1}{N} \sum_{k=1}^{N} \Delta u^2(k) \quad (41)$$

$$S_e = \frac{1}{N} \sum_{k=1}^{N} \left[w(k) - y(k) \right]^2 \quad (42)$$

Table 1 shows the system parameters used for the mathematical model of the system.

Table 1 : System parameters

Symbol	Value	Meaning
M_0 [kg]	4	Cart weight
M_1 [kg]	0.36	Pendulum weight
l_s [m]	0.42	Pendulum length
Θ [kg.m^2]	0.08433	Pendulum inertia moment
F_r [kg/s]	6.5	Cart friction
C [Kg.m^2/s]	0.00652	Pendulum friction
k_a [N/V]	7.5	Servo amplifier gain
g [m/s^2]	9.81	Gravity constant

The figure 3 shows the pulse response (uncontrolled fall) of the system for the input pulse F = -2 N for 1s.

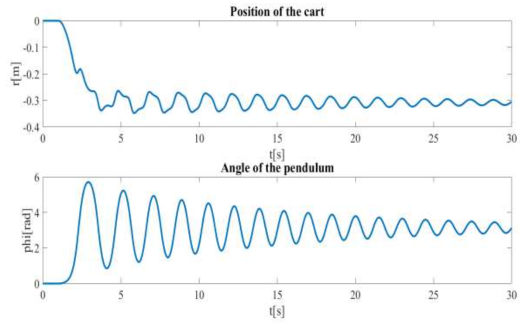

Figure 3 : Uncontrolled fall

The matrices $\tilde{A}$, B and C from the model (19) are calculated according the equation (43)

$$\tilde{A} = w_m \cdot \tilde{A}_n + (1 - w_m) \cdot \tilde{A}_{n+1}$$
$$B = w_m \cdot B_n + (1 - w_m) \cdot B_{n+1} \quad (43)$$
$$C = w_m \cdot C_n + (1 - w_m) \cdot C_{n+1}$$

where w_m is the weighting coefficient and n=1...12 is the model number. The matrices $\tilde{A}_n, B_n$ and C_n are matrices $\tilde{A}$, B and C of the n-th model The nonlinear model was linearized in 12 operating points of the pendulum angle. These operating points have equidistant range from 0 to 2π rad. Therefore, the angle distance between operating points is $\frac{1}{6}\pi$ rad.

The figures 4 and 5 show the simulation results of edge transition between models where w_m=1.

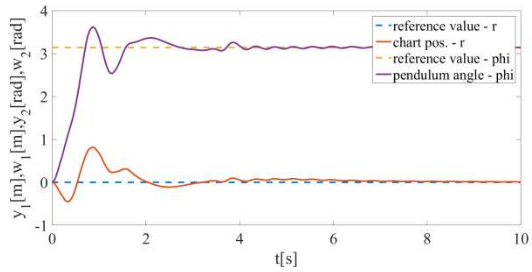

Figure 4 : System outputs - the edge transition

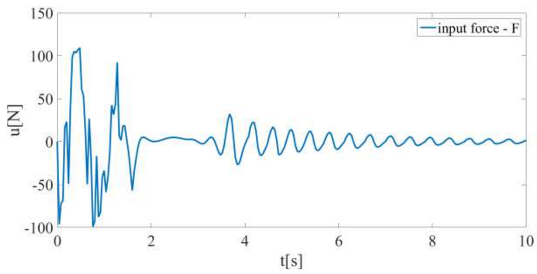

Figure 5 : System input - the edge transition

The figures 7 and 8 show the simulation results of the linear transition between models where w_m is calculated according to equation (44)

$$w_m = a x_3(k) + n \quad (44)$$

where $a = -\frac{6}{\pi}$ and n=1...12 is the model number. An example of this linear transition between models is shown in figure 6.

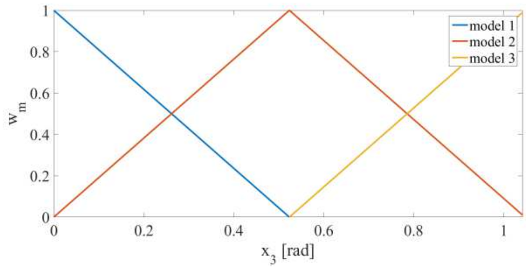

Figure 6 : Linear transition between models

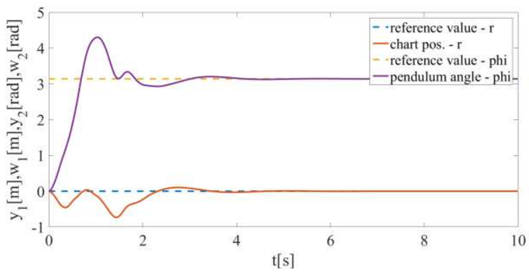

Figure 7 : System outputs - the linear transition

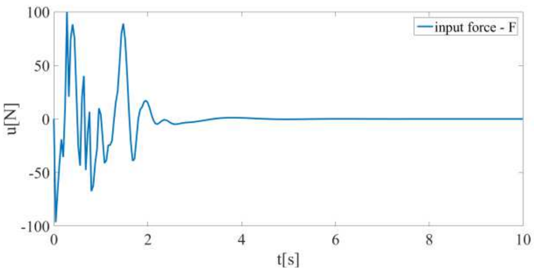

Figure 8 : System input - the linear transition

The figures 10 and 11 show the simulation results of the nonlinear transition between model where w_m is calculated according equation (45)

$$w_m = e^{-\frac{(x_3(k)-\mu)^2}{2\sigma^2}} \tag{45}$$

where $\mu = (n-1)\frac{1}{6}\pi; n = 1 \ldots 12$ and $\sigma = 0.2224$. An example of nonlinear transition between models is shown in figure 9.

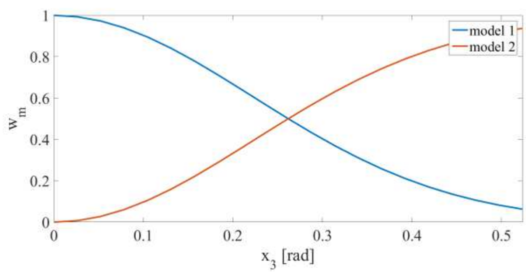

Figure 9 : Nonlinear transition between models

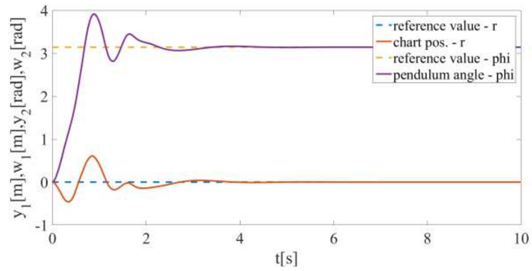

Figure 10 : System output - the nonlinear transition

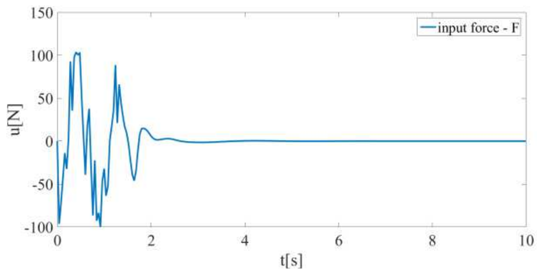

Figure 11 : System input - the nonlinear transition

The table 2 shows the results of the quadratic criterions.

Table 2 :Simulation results

	Edge transition	Linear transition	Nonlinear transition
S_{e1}[m^2]	0.034	0.029	0.017
S_{e2}[rad^2]	0.343	0.378	0.339
S_u[N^2]	361.06	262.40	301.37

CONCLUSION

In this paper, the multimodel approach of the predictive control based on the state-space CARIMA model was presented. The controller was tested on the inverted pendulum system which is an example of the nonlinear single-input two-output system. The goal of this multimodel approach in the predictive controller was the controlled fall of the pendulum from up to down position. The movement of the cart is acting like a disturbance in the system. The sampling period was chosen as T_0 = 40 ms. The mathematical model of the inverted pendulum was made according to the real laboratory model of the inverted pendulum Amira PS600. The aim of this paper is to present a possible approach in control of the nonlinear models using multimodel approach. That means using set of linear models to describe the behaviour of the nonlinear model. This set of models is created by linearization the nonlinear model in multiple operating points. The final linear model used to prediction of the output values is calculated as a combination of two linearized models according to the current state. The transition between linearized models is done using three methods. The first one is the edge transition, where one linearized model is using directly for the output values prediction. The second method is transition between linear models according to the linear function and the third method is transition between linear models according to the nonlinear function. The control signal is calculated by the minimization of the cost function that minimize the differences between the output and the reference signals and the control signal increments. This minimization is achieved by predictor-corrector method. The result section compares the different methods of transition between linear models in multimodel approach. The examined criterions show that the nonlinear transition between models follows the reference signal as the best of the presented methods. The linear transition has the least control signal changes of the presented methods.

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ACKNOWLEDGMENT

This article was created with support of the Ministry of Education of the Czech Republic under grant IGA reg. n. IGA/CebiaTech/2018/002.

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