PARAMETRIC MODEL ORDER REDUCTION OF INDUCTION HEATING SYSTEM

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ABSTRACT

Induction Heating system have been widely used nowadays. Modeling of such system remains a challenge. In this paper, modeling of a axi-symmetrical structured induction heating system is carried out. Few geometrical parameters affect the modeling of the system. Variation of these parameter values leads to multiple simulations, making the design procedure computationally expensive. Original system is produced from finite element method and is reduced to lower order system to make the simulation cost effective. As the size of system matrices are different with different state space models, matrix interpolation is used with two stage model reduction. Simulation results of the models at different parameter values are shown.

Introduction

Induction Heating process has wide application areas from industrial area to household appliances. Induction heating process is a multi-physical system. Uniform current density is applied to the copper coils. It produces a magnetic field in the system. Due to ohmic loss of eddy current induced in the iron body, heat is generated. Experimental design of induction-heated systems is not only tedious but also expensive. Thus, it necessitates the need for a precise mathematical model with help of simulation software[1]-[3].

Most mathematical models of the induction heating system are expressed as distributed parameter models with partial differential equations (PDEs). These PDEs are converted to ordinary differential equations (ODEs) with Finite element (FE) analysis. The generated ODEs are large in number and make the system computationally costly. The original system can be projected to a lower dimensional subspace to produce its lower order approximation using Model Order Reduction (MOR) techniques. There are various methods for model order reduction depending on the need such as Singular value decomposition method, Krylov subspace based method, structure-preserving model order reduction etc[4]-[8].

Sometimes few analytically inexpressible parameters like geometrical parameters in FE model based large systems, play important role in the design process. Changing such parameter values while using standard MOR techniques are computationally costly because of repetitive simulations. In such problems Parametric MOR (pMOR) techniques are used. It tries to preserve the parameter dependency of the original systems in the reduced models so that multiple simulations for different values of the parameters can be carried out in the reduced space. While considering geometric parameters regular pMOR cannot be applied, although the geometric parameters does effect the FE model and solution implicitly. Matrix Interpolation (MI) based pMOR framework [9] has been proposed for such scenarios. In [9], system matrices of reduced order models using MOR techniques, were transformed as the reduced states do not have same physical interpretation. Then weighted interpolation is performed to get a parametric reduced model. In the present induction heating model, it is not directly applicable as the higher order FE models are of different sizes for different parameter values. Hence some modifications are proposed with some results.

Modeling of Induction Heating System

Here we are considering an induction heating system as shown in Fig. 1. The model consists of a iron cylinder and copper wires around it. The

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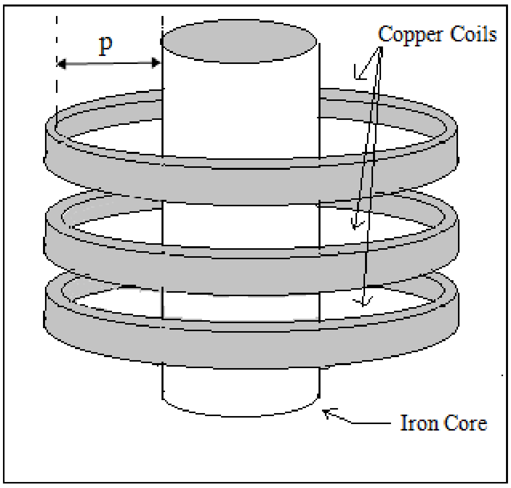

Fig. 1: Three dimensional model of Induction Heating System

copper coils carry alternating current producing electromagnetic effect. Due to ohmic losses of eddy current in the iron cylinder, heat is generated. The distance p, between copper coil and iron core, is considered as a parameter.

The governing equation describing the magnetic vector potential A is given as

$$\frac{1}{\mu}\nabla^2 A - j\nu A + J_s = 0 \quad (1)$$

where J_s is the current density and it varies sinusoidally in time with a single frequency, ω. As it is time harmonic in nature, it is sinusoidal function of time. So it can be represented in terms of vector phasors that depend on the space coordinates but not on time. The permeability is represented with μ and ν is a scalar quantity with $\nu = \omega\sigma$ with σ being electrical conductivity.

The induced heat (Q) can be written as

$$E.J = \sigma A^2 = Q \quad (2)$$

where E and J are electric field intensity and current density respectively. As only magnitude of Magnetic vector potential is considered, Q is a scalar quantity. Due to the ohmic losses of the eddy currents in iron, heat is produced. The time dependent temperature profile $T(r, z, t)$ can be given as

$$c\rho\frac{\partial T(r,z,t)}{\partial t} + \nabla(\kappa\nabla T(r,z,t)) = Q \quad (3)$$

where the model is considered to be thermally insulated and heat is not convected or radiated from the iron body. Here, specific heat and mass density of the materials are represented with c and ρ, whereas the thermal conductivity is denoted by κ.

After discretizing the model, the Finite Element representation of (1) can be written as

$$[\mathbf{K} + j\mathbf{S}]\bar{A} = \bar{F} \quad (4)$$

where $\mathbf{K}$ and $\mathbf{S}$ are the real, sparse, symmetric matrices and $\mathbf{K}$ has full rank.

The heat transfer equation can also be written as

$$\mathbf{C}\frac{d\bar{T}(t)}{dt} + \hat{\mathbf{K}}\bar{T}(t) = \bar{Q} \quad (5)$$

Again, $\mathbf{C}$ and $\hat{\mathbf{K}}$ are sparse, real and symmetric matrices with $\hat{\mathbf{K}}$ having full rank. The input vector is denoted by $\bar{Q}$. Specific heat constant and mass density are consumed in $\mathbf{C}$ and κ in $\hat{\mathbf{K}}$. Hence, (4) & (5) is the coupled model of the induction heating system.

Parametric Model Order Reduction

There are some geometrical parameters which affect the performance of the system. Such a parameter, the distance between the coils and the cylindrical core, is shown in Fig. 1. As the distance increases, the magnetic field or the magnitude of magnetic vector potential in the iron body decreases and heating time increases.

The FE electromagnetic equation where the parameter varies implicitly, can be written as

$$[\mathbf{K}(p) + j\mathbf{S}(p)]\bar{A} = \bar{F}(p) \quad (6)$$

Similarly, the FE heat transfer equation can be written as

$$\mathbf{C}(p)\frac{d\bar{T}(t)}{dt} + \hat{\mathbf{K}}(p)\bar{T}(t) = \bar{Q}(p) \quad (7)$$

where, $p \in \mathbf{R}$.

FE generated ODEs are quite large in number making it complex to handle. These systems can be replaced with reduced order model projecting them to lower dimensional subspaces. The parameters which influences the system modeling, can not be preserved using conventional model order reduction methods. Parametric model order reduction (pMOR) techniques are very useful for preserving the parameter in the reduced models.

All the reduced models from pMOR have equal number of states. But they do not interprete the same physical quantities. This necessitates the need of state transformation. The required state transformation can be achieved employing the matrix interpolation method as described in [9]. Here, all the system matrices are considered to be equal and this leads to equal size of projecting matrices.

While modeling it has been seen that with change in the parameter value, p the size of system matrices $(\mathbf{K}, \mathbf{S}, \mathbf{C}, \hat{\mathbf{K}} \in \mathbf{R}^{\mathbf{n_i} \times \mathbf{n_i}})$ changes. Therefore, the projecting matrices are of different sizes. Usual matrix interpolation method as described in [9] is inapplicable. So, a two stage model order reduction method is suggested. In the first stage reduction, the original systems are projected to a moderately lower dimensional subspace making all the system matrices of same size. In the second stage, these moderately large systems are reduced to get lower order approximations.

In the first stage of order reduction the original system is projected to a lower order $(\mathbf{R}^{\mathbf{r_1}})$ plane using one sided Arnoldi algorithm. The reduced system can be written as

$$[\mathbf{K}_{r_1}(p) + j\mathbf{S}_{r_1}(p)]\bar{A}_{r_1} = \bar{F}_{r_1}(p) \qquad (8)$$

$$\mathbf{C}_{r_1}(p)\frac{d\bar{T}_{r_1}(t)}{dt} + \hat{\mathbf{K}}_{r_1}(p)\bar{T}_{r_1}(t) = \bar{Q}_{r_1}(p) \qquad (9)$$

Here, system matrices are represented as $\mathbf{K}_{r_1} = \mathbf{V}_{r_1}^T\mathbf{K}\mathbf{V}_{r_1}$, $\mathbf{S}_{r_1} = \mathbf{V}_{r_1}^T\mathbf{S}\mathbf{V}_{r_1}$, $\bar{F}_{r_1} = \mathbf{V}_{r_1}^T\bar{F}$, $\mathbf{C}_{r_1} = \mathbf{V}_{r_1}^T\mathbf{C}\mathbf{V}_{r_1}$, $\hat{\mathbf{K}}_{r_1} = \mathbf{V}_{r_1}^T\hat{\mathbf{K}}\mathbf{V}_{r_1}$ and $\bar{Q}_{r_1} = \mathbf{V}_{r_1}^T\bar{Q}$ with $\mathbf{V}_{r_1}$ as the projection matrix.

As described earlier, (8) & (9) are moderately large systems. So, it can be reduced further by projecting it to a lower order subspace $\mathbf{R}^{\mathbf{r_2}}$ with V_{r_2}. The new reduced order model becomes

$$[\mathbf{K}_{r_2}(p) + j\mathbf{S}_{r_2}(p)]\bar{A}_{r_2} = \bar{F}_{r_2}(p) \qquad (10)$$

$$\mathbf{C}_{r_2}(p)\frac{d\bar{T}_{r_2}(t)}{dt} + \hat{\mathbf{K}}_{r_2}(p)\bar{T}_{r_2}(t) = \bar{Q}_{r_2}(p) \qquad (11)$$

For each discrete value of parameter, large scale model and reduced models are obtained employing projection matrices $(\mathbf{V}_{r_2,i})$ with one-sided Arnoldi algorithm.

The transformed reduced systems can be expressed as

$$[\mathbf{M}_i\mathbf{K}_{r_2,i}\mathbf{T}_i^{-1} + j\mathbf{M}_i\mathbf{S}_{r_2.i}\mathbf{T}_i^{-1}]\bar{A}_{r_2} = \mathbf{M}_i\bar{F}_{r_2,i} \qquad (12)$$

$$\mathbf{M}_i\mathbf{C}_{r_2,i}\mathbf{T}_i^{-1}\frac{d\bar{T}_{r_2}(t)}{dt} + \mathbf{M}_i\hat{\mathbf{K}}_{r_2,i}\mathbf{T}_i^{-1}\bar{T}_{r_2}(t) = \mathbf{M}_i\bar{Q}_{r_2,i} \qquad (13)$$

Now, following the standard MI procedure, the transformation matrices can be found from

$$\mathbf{T}_i = \mathbf{R}^T\mathbf{V}_{r_2,i} \text{ and } \mathbf{M}_i = (\mathbf{V}_{r_2,i}^T\mathbf{R})^{-1}$$

For obtaining $\mathbf{R}$, whose q columns span the universal subspace of the MOR strategy, the following has been suggested. The projection matrices $\mathbf{V}_{r_2,i}$s form a pool of independent directions and thus are accumulated in

$$\mathbf{V}_{all} = [\mathbf{V}_{r_2,1}\ \mathbf{V}_{r_2,2}\ \cdots \mathbf{V}_{r_2,k}]$$

followed by its SVD, given by $\mathbf{V}_{all} = \mathbf{U}\mathbf{\Sigma}\mathbf{N}^T$. The first q columns of $\mathbf{U}$ span q different directions and serve as $\mathbf{R}$.

Results and Simulations

The actual model of Induction Heating system is three dimensional. But due to its axi-symmetrical structure, two dimensional model is considered as shown in Fig. 2. The dimensions

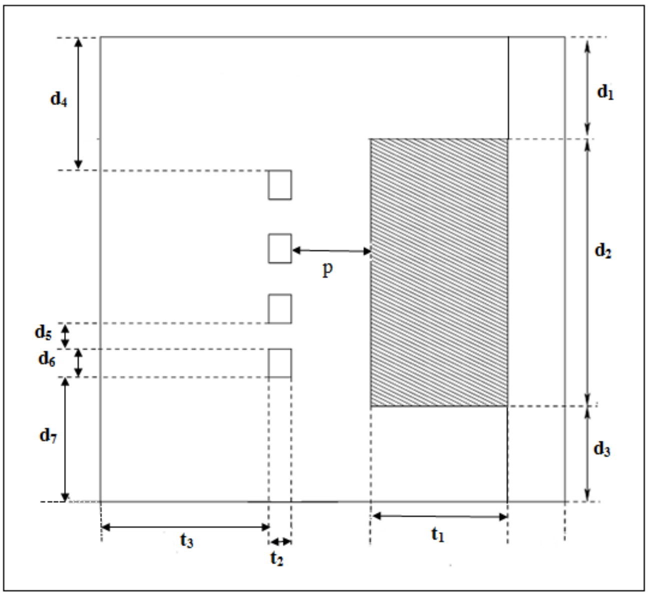

Fig. 2: Parameter affecting the Induction Heating System

of the model considered is given in Table 1. The

TABLE I: Geometric characteristics of the problem domain

p_1, p_2, p_3	$5mm, 10mm, 15mm$
d_1	$70mm$
d_2	$60mm$
d_3	$70mm$
d_4	$71mm$
d_5	$6mm$
d_6	$10mm$
d_7	$71mm$
t_1	$25mm$
t_2	$5mm$
t_3	$55mm - 65mm$

geometric parameter considered here is the distance between coil and the iron core. The parameter value varies from 5 mm to 15 mm. The material properties considered here are given in Table 2. Uniform current density of $8A/m^2$ is applied to the copper coils. Here zero boundary conditions are assumed for (4). As the boundaries are far away from the model, they do not have any effect of the magnetic field. Produced magnetic vector potential in the system is shown in Fig 3. The temperature profile in iron core is shown in Fig 4. The thermal field is simulated for 40 sec. It is clear from Fig. 4 that the heating time varies with the change in the distance between coil and core. As the FE generated models are of different sizes, in first stage all the models are projected to lower dimensional subspace of order 1000. In the second stage all the models are reduced to an order of 16. With the help of matrix interpolation, interpolated reduced system is generated. Fig. 5 shows the bode plots of the original system, first stage reduced system, second stage reduced system and matrix interpolated reduced system. From the plots it can be suggested that the reduced models and the matrix interpolated reduced model can replace the original system for lower frequencies.

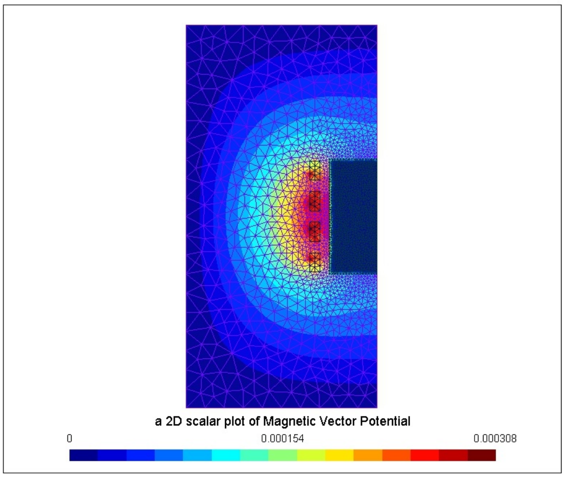

Fig. 3: Magnetic field of Induction Heating System

TABLE II: Material Properties

Parameters	Iron	Copper	Air
Relative Permeability (N/A^2)	1500	1	1
Electrical Conductivity (S/m)	10^7	5.96×10^6	3×10^{-15}
Mass Density (kg/m^3)	7874	8940	1.225
Specific Heat ($J/kg-K$)	450	385	1000
Thermal Conductivity ($W/m-K$)	83.5	401.0	0.001

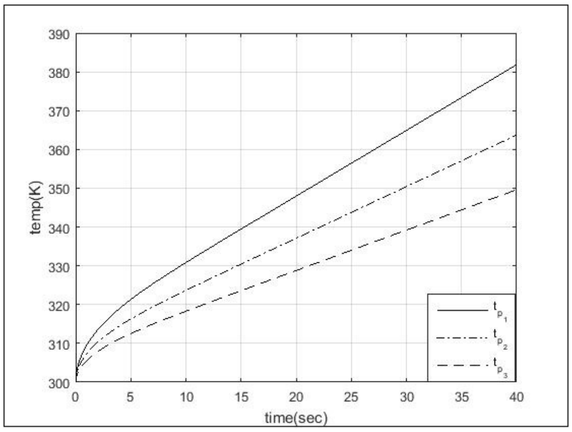

Fig. 4: Temperature plot in Iron core

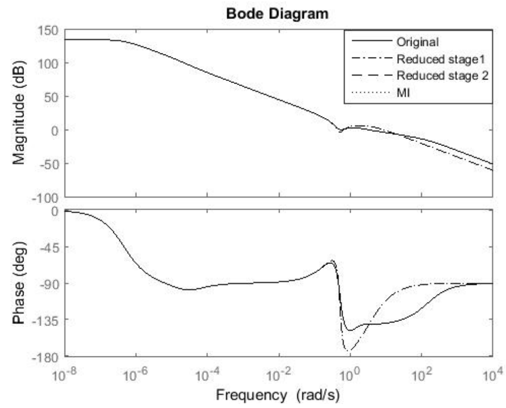

Fig. 5: Bode Plot of Induction Heating System (Thermal Model)

CONCLUSIONS

In this article, finite element modeling of induction heating system is described. In the design process, variation of the parameter value leads to repetitive simulations which is computationally costly. So, the original system is reduced employing parametric model order reduction. The distance between copper coil and iron core is the parameter. As the size of system matrices changes with the variation of parameter values, standard matrix interpolation method cannot be applied directly. Hence, the original finite element model is reduced to a lower order approximation in two stages. Simulation results show the efficacy of the proposed modification.

References

[1] C. Chaboudez, S. Clain, R. Glardon, D. Mari,J. Rappaz, M. Swierkosz, 'Numerical modeling in induction heating for axisymmetric geometries', IEEE Transactions on Magnetics 33.1 (1997): 739-745

[2] M. H. Tavakoli, H. Karbaschi, F. Samavat,'Computational modeling of induction heating process', Progress in Electromagnetics research letters, 11, 93-102

[3] O. Klein, P. Philip,'Correct voltage distribution for axisymmetric sinusoidal modeling of induction heating with prescribed current, voltage, or power', IEEE transactions on Magnetics, 38(3), 1519-1523.

[4] J. N. Reddy, 'An introduction to the finite element method', Vol. 2. No. 2.2. New York: McGraw-Hill, 1993.

[5] Klaus-Jurgen Bathe, E. L. Wilson, 'Numerical methods in finite element analysis' (1976): 6-12.

[6] B. Salimbahrami, B. Lohmann, 'Order reduction of large scale second-order systems using Krylov subspace methods', Linear Algebra and its Applications 415 (2006) 385 - 405.

[7] Zu-Qing Qu, 'Model Order Reduction Techniques with Applications in Finite Element Analysis', Springer, 2013.

[8] A. C. Antoulas, D. C. Sorensen, 'Approximation Of Large-Scale Dynamical Systems: An Overview', Int. J. Appl. Math. Comput. Sci., 2001,Vol.11,No.5,1093-1121.

[9] H. Panzer, J. Mohring, R. Eid, B. Lohmann, 'Parametric Model Order Reduction by Matrix Interpolation', Automatisierungstechnik Methoden und Anwendungen der Steuerungs- Regelungs- und Informationstechnik, Volume 58, Issue 8, Pages 475 - 484, ISSN (Print) 0178-2312, DOI: 10.1524/auto.2010.0863, August 2010.

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