

# Modeling dependencies in complex system dependability

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## ABSTRACT

Technological objects (as well as natural and biological beings) age in time reducing the ability to perform their functions until, eventually, a final catastrophic breakdown occurs. We adopt the term dependability to identify the ability of a system to deliver service that can justifiably be trusted [1]. Dependability is an integrating concept that encompasses various attributes like reliability and availability.

The behavior of these attributes in time cannot be described by deterministic laws, and then, we are led to resort to stochastic models based on the laws of probability [2]. Many stochastic formalisms have been proposed since the beginning of reliability engineering that can be broadly classified based on whether component behaviors are considered statistically independent or statistically dependent.

The *modeling power* of a formalism refers to its ability to correctly incorporate details of the real system, and the *decision power* refers to the ability to analytically compute characterizing measures. The success of any formalism depends on the balance between its modeling power and its decision power.

The formalisms that adopt the simplifying assumptions that the component failures and repairs are statistically independent events (Reliability Block Diagrams, Network Reliability and Fault Trees), exploit the product rule of probabilities and have low modeling power but high analytical tractability.

Statistical independence is often unrealistic: The most common types of phenomena that induce dependencies in the failure process are load dependencies, functional dependencies, common cause failures and coincident faults. In the repair process,

usual causes of dependencies are preventive or deferred maintenance and shared repair capabilities. When considering dependencies, more sophisticated formalisms are necessary, able to incorporate the possible conditional dependence of each component state on the state of the other components, thus resorting to the system state space. Homogeneous Continuous Time Markov Chains (HCTMC) are the basic state space model; they have a very compact close form solution but have the restriction that the random time variables must be exponentially distributed, i.e. with constant failure and repair rates. To remove the exponential assumption and, at the same time, modelling dependencies, various extensions are possible. If the transition rates are dependent on global time, then the system behavior can be represented as a Non Homogeneous Continuous Time Markov Chain (NHCTMC) that still satisfies the Markov property. A semi-Markov process (SMP) permits the random time distributions to be non-exponential, but the epoch of entry into each state must be regeneration points; and then the transition rates depend on the sojourn time in each state. A Markov Regenerative Process (MRGP) is a generalization of the above stochastic processes [3,4]. In a CTMC (homogeneous or non-homogeneous) any time epoch satisfies the Markov property, in an SMP the Markov property is satisfied at all the time instances at which the process undergoes a state transition, in an MRGP the Markov property holds only when the process enters a subset of specific states called regeneration states.

The formalisms considered up to now have a closed form analytical expression even if the numerical solution may be quite cumbersome. However, they require to be defined over the global state space of the system, thus incurring into the well-known state-space explosion problem.

The state-space description appears often over specified with respect to the real modeling needs since it is very unusual to encounter real applications in which each component changes its stochastic behavior according to the state of all the other components of the system. To mitigate the state explosion problem dependencies could be confined locally. In a hierarchical approach, the system model is partitioned into independent submodels (possibly of different model types) that are combined by exchanging quantities across submodels [5].

To avoid the generation of the global state-space and with the aim of providing a flexible and scalable framework for modeling various kinds of local and conditional dependencies among interacting objects, we have recently defined an analytical model, called Markovian Agent Model (MAM) [6,7]. MAM is formed by a spatial collection of interacting Markovian Agents (MAs) whose transition kernel is composed by two components a *local transition* matrix and an *induced transition* matrix. The local matrix contains a fixed component that depends on the MA structure and its position in the space while the induced matrix depends on the interaction of a MA with the other MAs.

Several examples will illustrate all the above techniques.

1. Avizienis, A., J.C. Laprie, B. Randell and C. Landwehr (2004). Basic concepts and taxonomy of dependable and secure computing. IEEE Trans Dependable and Secure Computing 1, 11-33.
2. K. Trivedi and A. Bobbio, Reliability and Availability Engineering, Cambridge University Press, 2017.
3. E. Cinlar, Introduction to Stochastic Processes. (Prentice-Hall, Englewood Cliffs, 1975)..
4. V.G. Kulkarni, Modeling and Analysis of Stochastic Systems. (Chapman&Hall, 1995).
5. H. Sukhwani, A. Bobbio and K. Trivedi, "Largeness avoidance in availability modeling using hierarchical and fixed-point iterative techniques," International Journal of Performability Engineering, vol. 11, no. 4, pp. 305–319, 2015.
6. D. Bruneo and M. Scarpa and A. Bobbio and D. Cerotti and M. Gribaudo. An Intelligent Swarm of Markovian Agents, Performance Evaluation, In: Springer Handbook of Computational Intelligence, J. Kacprzyk and W. Pedrycz Eds., Springer, 1345-1359, 2015
7. A. Bobbio and D. Cerotti and M. Gribaudo and M. Iacono and D. Manini. Markovian Agent Models: A Dynamic Population of Interdependent Markovian Agents, In Seminal Contributions to Modelling and Simulation, K. Al-Begain and A. Bargiela (Eds.), Springer, 185-203, 2016