AGGREGATE PLANNING USING MIXED INTEGER PROGRAMING: A FRUIT JUICE CONCENTRATED FACTORY CASE STUDY

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ABSTRACT

The case study is a newly opened fruit juice concentrated factory. Three different products have been launched to serve both domestic and international markets. For its inbound logistics, it receives fresh fruits from local growers. As the fresh fruits are seasonal, its prices and quantities are higly fluctuated. A cold storage is then used to freeze and store the fruits during their peak periods in order to reduce risks of fruit shortages and high prices in off peak periods. Storaging costs of fresh fruits are, however, substantial. To produce the finished products, Clean-in-Place (CIP) is being managed before changing the product lines and it takes 3 days for each cleaning or setup. After the production process, the finished products are stocked in its warehouse no more than 2 months as its clients require the products to be 80% viable of its shelf life. Aggregate planning is then needed to match demand with supply of the case study while minimising total operating costs including purchasing cost, raw material inventory cost, semi-product cost, finished inventory cost, setup cost and labour cost. Mixed integer programing is used to find the optimal decision variables that are purchasing volumes, semi-product volumes, production volumes, and setup decisions.

INTRODUCTION

To manage supply chains, a general planning framework consists of three levels of planning: strategic tactical, and operations. The strategic planning level is to define major aspects such as production capacity levels for the next 3 years. For the tactical level, planning activities cover aggregate production and distribution planning while the operational level involves operations scheduling for 1 – 18 months including distribution resource planning, master production scheduling and transport scheduling (Miller, 2016). Aggregate planning is an essential part of supply chain planning as it balances between demand sides and supply chain sides in order to meet customers' requirements and minize total operational costs (Stevenson, 2018).

A fruit juice contrated factory is newly opened in Thailand. The product is 100% natural extracted juice. There are three types products for domestic and export markets. As the factory is opened in Jan 2018, the sales forecast has been conducted based on judgement. The supply chain activities are to fulfill the demand forecast. The supply chain activities start from receving fresh fruits from growers or middleman. Freezed fruits can be obtained from the company's cold storage as the company would buy fresh fruits during their high seasons and keep them in stocks in order to minimize supply and price risks. Fresh fruits and/or freezed fruits are then extracted as concentrated juice. The concentrated juice can be called as a semi-product that can be kept in 200 litre drums. The semi-product or *WIP* is filled into two different bottle sizes that are A1 and A2 products. For product B, *WIP* will be mixed with other ingredients and filled in another bottle size. For export markets, the products will be sent to a controlled temperature room before exporting. On the other hand, a distribution center is used to distribute the products thoughout Thailand. The material flow of the case study is exhibited in figure 1.

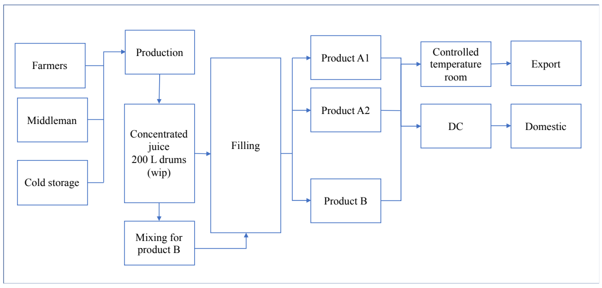

Figure 1: The material flow of the case study

To match demand with supply for the case study is rather complex as many factors are unable to control such as price, quality and quantity of fruits and demand volumes. In addition to these, safety standards such as Clean-In-Place and product shelf life are major issues to be managed.

Aggregate planning model can be conducted by linear programming, mixed integer linear programming, goal programming and heuristics such as genetic algorithm and tabu search. Linear programming could be applied when a objective function and constraints are both linear. While mixed integer liner programming (MILP) occurs when some variables in the model are real variables and some of them are integer variables or

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binary integer variables. Heuristics approaches can be applied on aggregate planning problems when an optimal solution is impossible or impractical as they can quickly identify a satisfactory solution, but not guaranteed to be an optimal solution (Silver et al., 1998). A MILP was applied in a case study of the food industry with having two subproblems. The fist problem is to conduct an annual planning or aggregate planning and the second problem is to conduct short term scheduling or operational planning (Tadei et al., 1995). In addition, Karin et al. (2016) studied a practical problem to define a production plan for a perishable product with a long production lead time. An MILP was applied to identify production plans on non-stationary demand patterns with the objective of minizing the expected total costs using existing solvers. The MILP can give the results and shorter solver time and it is appropriate for use in practice.

This paper is then to propose MILP to determine an aggregate plan that can balance between supply and demand in order to minimize total costs that include purchasing fruits cost, raw material inventory cost, semi-product inventory cost, machine set up cost, finished product inventory cost and labour cost.

PROBLEM DESCRIPTION

The factory started its first production run in March 2018. Demand forecast conducted by sales department was anticipated without any algorithm used, but sales forces driven. Demand forecast is shown in table 1. For supply side, fruits are seasonal. High season of fruits is from June to September with very low price at 5 Baht per kilogram and the maximum of available quantity is 10,000 tons per month. Low season of fruits is from October to May and the price of fruit is 11 – 13 Baht per kilogram and the maximum of available quantity is 260 tons per month. Prices and purchasing capability are exhibited in table 2. To reduce supply uncertainties, the factory has rented a cold storage to keep fruits at high season and the maximum capacity of the cold storage is 1500 tons. The holding cost of fruits at the cold storage is 1 Baht per kilogram per month. Moreover, quality of fruits is also an important issue. It was found that loss of fruits or raw materials is about 3%. Therefore, sources of uncertainty in sourcing include price, quantity and quality.

The production process consists of three processes that are production process, mixing process and filling process. For product A1 and A2, the filling process can immediately start after the production process or using concentrated juice kept in 200 litre drums. The mixing process is only needed for producing product B by using concentrated juice or *WIP* to mix with other ingredients and followed by the filling process. During the test run period in March and May, the production process can receive fruits about 20 tons per day and yields would be about 5% of fruits. The normal period starts from June, the production process can receive fruits about 20 tons per day and yields can be about 8% of fruits.

Table 1: Demand forecast

Month	Demand (bottle)		
	A1	A2	B
Mar	5,000	5,000	26,000
Apr	6,500	6,500	136,852
May	50,000	20,000	211,852
Jun	80,570	34,530	327,566
Jul	46,230	19,810	797,955
Aug	80,000	37,000	997,955
Sep	40,450	17,335	937,955
Oct	40,450	17,335	937,955
Nov	86,680	37,150	718,977
Dec	46,230	19,810	718,977

Table 2 : Maximum purchasing quantity and price

Month	Maximum purchasing quantity (kg)	Price (Baht/kg)
Mar	260,000	12
Apr	260,000	13
May	260,000	13
Jun	10,000,000	5
Jul	10,000,000	5
Aug	10,000,000	5
Sep	260,000	11
Oct	260,000	11
Nov	260,000	10
Dec	260,000	10

The filling process capacity for product A1 and A2 was 1,150 bottles per hour from January to April and it has been increased to 1800 bottles per hour from May. For product B, the fillind process capacity was 63 bottles per hour, but it has been increased to 1,800 bottles per hour from May 2018. The usage of concentrated juice per bottle is different as shown in table 3. Moreover, Clean-In-Plance (CIP) is needed for cleaning and setting up machines for each production batch. It takes 3 days to complete CIP and it costs about 15,000 Baht per set up.

Table 3 : The usage of concentrated juice

Product	Concentrated juice (kg/bottle)
A1	0.1414
A2	0.36764
B	0.011

Work in process is kept in aseptic bags in 200 litre drums with the maximum capacity at 374,500 kilograms or 1440 drums. The holding cost of *WIP* is 250 Baht per drum per month. In addition, loss of the production process is 10% of yields. Finished product inventory policy is needed to follow customers' requirement about the remaining shelf life of products delivered. The remaining shelf life of the finished products delivered must be no less than 80% of their shelf lives. Consequently, the finished products cannot be kept more than 2 months in the warehouse or distribution center.

MATHEMATICAL FORMULATION

The mixed integer linear programing is constructed to obtain anaggregate plan for the case study and the aggregate plan covers 10 months from March to December 2018 with 3 types of products including product A1, product A2 and product B. The objective of the MILP is to minimize total cost including purchasing cost, inventory cost, labor cost and set up cost as mentioned in the previous part. The parameters are shown as follows:

Indices

i = products : $i = 1,2,3,\dots, I$
t = production period : $t = 1,2,\dots, T$

Parameters

D_{it}	finished product demand (kg)
DIn_t	fruit demand (kg)
$DWip_{it}$	*WIP* demand (kg)
$InvIn_t$	fruit inventory (kg)
$InvInCost_t$	fruit holding cost (Baht/kg)
$InvInCap_t$	fruit storage capacity (kg)
$InvWip_t$	*WIP*(kg)
$InvWipCap_t$	*WIP* storage capacity (kg)
$InvWipCost_t$	*WIP* holding cost (Baht/kg)
$InvFg_{it}$	finished product quantity (bottle)
$InvFgCost_{it}$	finished product holding cost (Baht/bottle)
$LossIn_t$	loss of frui ts in production (kg)
$LossWip_t$	loss in *WIP* mixing process(kg)
$\%LossIn_t$	% of fruit loss in production
$\%LossWip_t$	% of loss in *WIP* mixing process
$MinSS_{it}$	minimum stock level (bottle)
$PriceIn_t$	fruit price (Baht)
$ProdRate_i$	production rate (bottle/hr)
$ProdWipRate$	mixing *WIP* rate (kg/hr)
$Recipe_i$	% of fruit used for each product
Sup_i	machine set up time (hr)
$SupCost_{it}$	machine set up cost (Baht)
$SupWip$	set up time for mixing process (hr)
$SupWipCost_t$	mixing machine set up cost (Baht)
$WipTime_t$	mixing time for *WIP*(hr)
$WorkTime_{it}$	production time (hr)
$YieldProd_{it}$	% of product yield

Decision variables

$BSup_{it}$	= 1 if product i is assigned to produce at time t; and 0 otherwise
$BSupWip_t$	= 1 if *WIP* i is assigned to mix at time t; and 0 otherwise
$Prod_{it}$	finished product quantity (bottle)
$ProdWip_t$	*WIP* quantity (kg)
$PurIn_t$	fruit purchasing quantity (kg)

Objective function

$$MinZ = \sum(PurIn_t * Price_t) + \sum(InvIn_t * InvInCost_t) + \sum(InvWip_t * InvWipCost_t) + \sum(InvFg_{it} * InvFgCost_{it}) + \sum(SupCost_{it} * BSup_{it}) + \sum(SupWiCost_t * BSupWip_t) \quad (1)$$

The objective function is to minimize total cost including fruit purchasing cost, fruit holding cost, *WIP* holding cost, finished product holding cost, production set up cost and mixing process for *WIP* set up cost.

Constraints

$$\sum InvFg_{it} \geq \sum D_{it} + MinSS_{it} \quad \forall i,t \quad (2)$$

$$MinSS_{i,t} = D_{i,t}\ x\ 5\% \quad \forall i,t \quad (3)$$

Equation 2 and 3 are to obtain finished products inventory level to meet customer demand for each month while maintaining safety stock at 5% of monthly demand.

$$InvWip_t \geq DWip_t \quad \forall t \quad (4)$$
$$DWip_t = (Recipe_i * Prod_{it}) + LossWip_t \ \forall i,t \quad (5)$$
$$LossWip_t = (Recipe_i * Prod_{it})\ x\ \%LossWip_t \ \forall i,t \quad (6)$$

WIP must be equal or great than demand of products and loss of *WIP*. Loss of *WIP* can be calculated by receipe multiplied by product volumes and loss of *WIP*.

$$InvIn_t + PurIn_t \geq DIn_t \quad \forall t \quad (7)$$
$$DIn_t = (YieldProd_t * ProdWip_t) + LossIn_t \ \forall t \quad (8)$$
$$LossIn_t = (YieldProd_t * ProdWip_t) * \%LossIn_t \ \forall t \quad (9)$$

Equation 7 is to force fruit inventory and fruit purchasing quantity to be equal or greather than fruit demand for each period while equation 8 is to identify fruit demand for each period based on *WIP* quantity, yield of products and loss of fruits in production. Loss of fruit in production can be determined by yields of product multiplied by *WIP* quantity and % loss of fruit.

$$PurIn_t \leq PurInCap_t \quad \forall t \quad (10)$$

Purchasing fruit quantity cannot be greater than the maximum purchasing quantity for each month affected by fruit seasonal.

$$InvIn_t \leq InvInCap_t \quad \forall t \quad (11)$$

Not only purchasing quantity is affected by fruit seasonal, the storage of fruit is also limited according to the storage's space as shown in eqution 11.

$$InvIn_t = InvIn_{t-1} + (PurIn_t - DIn_t) \quad \forall t \quad (12)$$

Equation 12 is to balance inventory level of fruit for each month.

$$\frac{ProdWip_t}{ProdWipRate_t} + (SupWip \cdot BSupWip_t) \leq WipTime_t \forall t \quad (13)$$

Working time to produce *WIP* is determined by *WIP* quantity divided by *WIP* production rate and setup time.

$$\frac{Prod_{it}}{ProdRate_i} + (Sup_i \cdot BSup_i) \leq WorkTime_t \ \forall i,t \quad (14)$$

Working time to produce finished product is calculated by time to produce each product and setup time.

$$InvWip_t \leq InvWipCap_t \qquad \forall t \quad (15)$$

WIP cannot be stored more than *WIP* storage's space.

$$InvWip_t = InvWip_{t-1} + ProdWip_t - DWip_t \ \forall t \quad (16)$$

WIP is calculated by *WIP* of the previous period, *WIP* quantity of the current period and *WIP* demand of the current period.

$$IngFg_{it} = InvFg_{it-1} + Prod_{it} - D_{it} \qquad \forall i,t \quad (17)$$
$$Prod_{it} \leq D_{it} + D_{i,t+1} + D_{i,t+2} \qquad \forall i,t \quad (18)$$
$$InvFg_{it} \leq D_{i,t} + D_{i,t+1} \qquad \forall i,t \quad (19)$$

Equation 17 is to balance finished goods inventory level. Finished goods cannot be produced more than three month demands including the current month demand as shown in equation 18. Finished products can be kept in the warehouse no more than 2 months as its clients require the products to be 80% viable of its shelf life. Equation 19 is then constructed.

$$PurIn_t, InvIn_t, InvWip_t, IngFg_{it} \geq 0 \qquad \forall i,t \quad (20)$$
$$ProdWip_t, Prod_{it} \geq 0 \qquad \forall i,t \quad (21)$$

RESULTS

Total cost minimized by the MILP is 24,277,592 Baht including fruit purchasing cost, fruit inventory cost, *WIP* inventory cost, finished product inventory cost and machine setup cost. The highest cost is purchasing cost. The decisions of production plan, *WIP* production plan, and fruit purchasing plan are identified in table 4 to 6. The fruit cost is very high. It then leads to high purchasing cost in August as it is last month of high season of fruits. It is also needed to produce the highest *WIP* in August in order to have concentrated fruit juice to fill in bottles from Sep to Dec as it can provide lower cost comparing to produce *WIP* in Sep to Dec with using freezed fruits. Each product type can provide enough products according to customer demand at 80% viable of its shelf life. The stock records of each product are shown in table 7 to 9.

Table 4 : Production planning

Month	Finished products (bottle)		
	A1	A2	B
Mar	5,250	5,250	27,300
Apr	6,575	6,575	142,397
May	52,175	20,675	215,600
Jun	82,099	35,257	333,352
Jul	44,513	19,074	835,829
Aug	81,688	37,859	993,600
Sep	38,473	16,352	934,955
Oct	40,450	17,335	937,955
Nov	88,991	38,141	708,028
Dec	44,208	18,943	718,977

Table 5 : *WIP* demand and planning

Month	*WIP* demand (kg)	*WIP* production planning (kg)
Mar	3,270	12,621
Apr	5,405	2,783
May	26,924	20,195
Jun	42,719	42,721
Jul	31,082	31,081
Aug	50,946	190,775
Sep	29,415	0
Oct	30,404	0
Nov	50,488	0
Dec	29,524	0

Table 6 : Fruit demand and purchasing quantity

Month	Fruit demand (kg)	Purchasing quantity (kg)
Mar	259,993	259,993
Apr	57,330	57,351
May	260,011	260,000
Jun	550,033	550,028
Jul	400,168	400,165
Aug	2,456,228	2,456,227
Sep	0	0
Oct	0	0
Nov	0	0
Dec	0	0

Table 7 : Product A1 inventory (bottle)

Month	Produce	Demand	End period Inventory	aging > 2 months
Mar	5,250	5,000	250	0
Apr	6,575	6,500	325	0
May	52,175	50,000	2,500	0
Jun	82,099	80,570	4,029	0
Jul	44,513	46,230	2,312	0
Aug	81,688	80,000	4,000	0
Sep	38,473	40,450	2,023	0
Oct	40,450	40,450	2,023	0
Nov	88,991	86,680	4,334	0
Dec	44,208	46,230	2,312	0

Table 8 : Product A2 inventory (bottle)

Month	Produce	Demand	End period Inventory	aging > 2 months
Mar	5,250	5,000	250	0
Apr	6,575	6,500	325	0
May	20,675	20,000	1,000	0
Jun	35,257	34,530	1,727	0
Jul	19,074	19,810	991	0
Aug	37,859	37,000	1,850	0
Sep	16,352	17,335	867	0
Oct	17,335	17,335	867	0
Nov	38,141	37,150	1,858	0
Dec	18,943	19,810	991	0

Table 9 : Product B inventory (bottle)

Month	Produce	Demand	End period Inventory	aging > 2 months
Mar	27,300	26,000	1,300	0
Apr	142,397	136,852	6,845	0
May	215,600	211,852	10,593	0
Jun	333,352	327,566	16,379	0
Jul	835,829	797,955	54,253	0
Aug	993,600	997,955	49,898	0
Sep	934,955	937,955	46,898	0
Oct	937,955	937,955	46,898	0
Nov	708,028	718,977	35,949	0
Dec	718,977	718,977	35,949	0

CONCLUSION

Aggregate planning is conducted at a tactical level with the main objective to balance between supply and demand with regarding to satisfy customers' requirements while minizing total operational costs. The supply chain planning of the case study is facing with various aspects that can affect the capability of the supply chain in terms of cost, quality and time. Raw material or fruit is seasonality that impacts on quality, quantity and price of fruit. The production capacity, cold storage and WIP storage are constraints needed to be determined to minizing related costs. For demand sides, both the quantity and aging of the finished products must be controlled according to customers' requirement. The MILP was then constructed with specific characteristics of the aggregate planning problem including setup time, setup cost, capacity restrictions, perishable product shelf life, and perishable supply restrictions.

From the results, the MILP can provide an aggregate plan that minimize costs including purchasing cost, inventory holding costs of fruit, *WIP*, finished goods and set up cost while maintain product aging at 80% viable of its shelf life. Purchasing plan, production plan for both *WIP* and finished products can be obtained. However, the effect of fruit season leads to unbalance of workload from September to December and the highest *WIP* in August in order to reduce wip holding cost.

To enhance the MILP developed to suit with the case study, more constraints in order to levelling the production workload or part-time workers can be added. Although the MILP can provide the optimal solutions for the case study, heuristic approaches such as Tabu search, Ant colony and so on can be applied. It should be then noted that advantages of heuristic approaches can quickly find a satisfactory solution on the problem that is impossible to obtain an optimal solution (Silver et al., 1998).

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