ESSAY ON THE STUDY OF THE SELF-OSCILLATING REGIME IN THE CONTROL SYSTEM

Anna V. Korolkova
Department of Applied Probability and Informatics,
Peoples' Friendship University of Russia
(RUDN University),
6 Miklukho-Maklaya St, Moscow, 117198,
Russian Federation
Email: korolkova-av@rudn.ru

Dmitry S. Kulyabov
Department of Applied Probability and Informatics,
Peoples' Friendship University of Russia
(RUDN University),
6 Miklukho-Maklaya St, Moscow, 117198,
Russian Federation
and Laboratory of Information Technologies
Joint Institute for Nuclear Research
6 Joliot-Curie, Dubna, Moscow region, 141980,
Russian Federation
Email: kulyabov-ds@rudn.ru

Tatyana R. Velieva
Department of Applied Probability and Informatics,
Peoples' Friendship University of Russia
(RUDN University),
6 Miklukho-Maklaya St, Moscow, 117198,
Russian Federation
Email: velieva-tr@rudn.ru

Ivan S. Zaryadov
Department of Applied Probability and Informatics,
Peoples' Friendship University of Russia
(RUDN University),
6 Miklukho-Maklaya St, Moscow, 117198,
Russian Federation
and Institute of Informatics Problems,
FRC CSC RAS, IPI FRC CSC RAS,
44-2 Vavilova str., Moscow, 119333, Russian Federation
Email: zaryadov-is@rudn.ru

KEYWORDS

Active queue management, simulation, self-oscillation, RED

ABSTRACT

The authors for a long time have investigated the phenomenon of global synchronization. Specifically, the occurrence of a self-oscillatory regime in a control system was investigated. The model was built for the interaction of the TCP protocol and the active queue management algorithm RED. As a result of the research, the authors identified a set of methods for constructing the model, studying the model structure, calculating parameters of self-oscillations and verifying theoretical results based on a computer simulation. The paper summarizes the main results, obtained by the authors, and gives the description of their approaches.

INTRODUCTION

In data networks with TCP such a parasitic phenomenon as a global synchronization can take place. The TCP packets are sent by sources synchronously, and also sources may synchronously stop transmitting the TCP packets. This phenomenon is extremely negative for the network parameters (such as bandwidth, data transfer delays and etc) manifests itself as the occurrence of a self-oscillatory regime in the system. The use of AQM (Active Queue Management) algorithms, such as RED,for traffic shaping may reduce the likelihood of global synchronization, but does not completely eliminate it.

Quite a lot of work devoted to the modeling of the process of functioning of TCP with a random drop packets. Simplified models are presented in the works Mathis et al. (1997); Padhye et al. (1998); Floyd and Fall (1999). Later and more accurate (in terms of assumptions about the packet loss process), the TCP operation models in the stationary mode are presented, for example, in Misra and Ott (1999); Misra et al. (1999*a*, 2000*a*); Firoiu and Borden (2000); Altman et al. (2000, 2005). In many studies, the construction and analysis of models takes into account the interaction of the data transmission process using the TCP protocol and process flow control state when congestion occurs. The RED algorithm Floyd and Jacobson (1993) or its modifications is often considered as a regulatory

Communications of the ECMS, Volume 33, Issue 1,
Proceedings, ©ECMS Mauro Iacono, Francesco Palmieri,
Marco Gribaudo, Massimo Ficco (Editors)
ISBN: 978-3-937436-65-4/978-3-937436-66-1(CD) ISSN 2522-2414

process.

Making various changes to the classic RED and, as a result, the emergence of a large number of modifications is associated with the problem of selecting the algorithm parameters (threshold values, maximum reset parameter, etc.) at which the system would function stably and efficiently. The analysis of the effectiveness of the RED algorithm and the attempt to solve the problem of selecting its parameters are devoted to the work Floyd (1997); Lin et al. (1997); Floyd et al. (1998); May et al. (1999); Floyd (2000); Atsumi et al. (1998); Pazos et al. (1999); Firoiu and Borden (2000); Ziegler et al. (2001); Iannaccone et al. (2001); Feng et al. (1999).

The authors have studied the problem of occurrence of self-oscillation mode in the interaction of TCP and RED algorithm for a long time and several approaches have been applied. In this paper the authors give an overview of the main approaches to the study of the problem of the emergence of self-oscillatory modes in TCP networks.

MODEL CONSTRUCTION

Our research was based on the model of the TCP-source and RED algorithm interactions based on stochastic differential equations with Poisson's process (see Misra et al. (1999*b*, 2000*b*)). However, we were not satisfied with some artificiality of this constructions. Our team has developed stochastization approach of one-step processes that allows us to obtain models from the first principles. In this case, the obtained model models have immanent stochastic properties (see Hnatič et al. (2016); Gevorkyan et al. (2018)).

TCP-like traffic transmission model with dynamic flow rate, controlled by RED algorithm, is constructed from the following assumptions:

- the model consists of two elements: the source which generates TCP packets, and the recipient (the queue) which acts as a router that processes received packets according to the control algorithm and inform the source device about delivered packages;
- the interaction between the source and the recipient occurs through intermediate node according to the control algorithm, i.e. the value of average queue length affects the source settings, in particular, the TCP-window size;
- from the existing phases of the TCP Reno Protocol we take into account only the slow start phase and the congestion avoidance phase; the loss of packages is taken into account only in the case of triple ACK redundancy.

Thus, we assume to obtain a nonlinear dynamic model of TCP Reno interactions with RED, which uses two variables condition: the size of the congestion window and the average queue size. The change of the size of the TCP Reno window will reflect the dynamics of congestion control of TCP protocol, while the average queue size will reflect the dynamics of the queue in a router (or gateway) with RED control module.

Let consider $W(t)$ to be the function of TCP-window size, $Q(t)$ is the queue size, $T := T(Q(t)) = T_p + Q(t)/C$ is the RTT with delays of packets processing in hardware, T_p is the RTT excluding delays of packets processing in hardware, t is the time, C is the intensity of packets service in the queue, δ is the time spent by a single package in queues, $\hat{Q}(t)$ is the exponentially weighted moving-average (EWMA) queue size (see Floyd and Jacobson (1993)):

$$\hat{Q}(t) = (1 - w_q)\hat{Q}(t) + w_q Q(t), \tag{1}$$

where w_q, $0 < w_q < 1$ is a weight factor.

First, we will write the equation responsible for the size of the TCP Reno window changing.

According to RFC-5681 (see Allman et al. (2009)), the window size increases when the source receives the ACK message, which corresponds to the slow start phase of TCP Reno:

$$W\left(t_n^{ACK} + \Delta t^{ACK}\right) = W\left(t_n^{ACK}\right) + 1.$$

Suppose that during the RTT all confirmations for the sent window segment will come:

$$W(t_n + \Delta t) = W(t_n) + 1 \cdot W(t_n),$$
$$\frac{W(t_n + \Delta t) - W(t_n)}{\Delta t} = \frac{W(t_n)}{\Delta t}.$$

If we consider that $\Delta t = T$, then

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = \frac{W(t)}{T}. \tag{2}$$

By solving (2), we will obtain

$$\mathrm{d}\ln W(t) = \frac{\mathrm{d}t}{T}, \quad \ln W(t) = \frac{1}{T}; \quad W(t) = \exp\left\{\frac{1}{T}\right\},$$

that corresponds to the exponential growth of TCP Reno window size in the slow start phase.

Similarly, we may write the changes of the window size in the phase of overloads prevention:

$$W\left(t_n^{ACK} + \Delta t^{ACK}\right) = W(t)\left(t_n^{ACK}\right) + \frac{1}{W\left(t_n^{ACK}\right)}.$$

Suppose that during the RTT all confirmations for the sent window segment will come:

$$W(t_n + \Delta t) = W(t_n) + \frac{1}{W(t_n)} W(t_n);$$
$$\frac{W(t_n + \Delta t) - W(t_n)}{\Delta t} = \frac{1}{\Delta t}.$$

If $\Delta t = T$ we obtain

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = \frac{1}{T}. \tag{3}$$

By solving (3) the following result is derived

$$\mathrm{d}W(t) = \frac{\mathrm{d}t}{T}, \quad W(t) = \frac{1}{T},$$

which corresponds to the linear growth of TCP Reno window size in the phase of overloads prevention.

Considering that all the packets arrive with intensity $W(t)/T$, and served in queue with intensity C, we may write the equation of change the average queue length:

$$\frac{\mathrm{d}Q(t)}{\mathrm{d}t} = \frac{W(t)}{T} - C.$$

Based on (1), the following statement is valid:

$$\hat{Q}(t_n + \Delta t) = (1 - w_q)\hat{Q}(t_n) + w_q Q(t_n),$$
$$\hat{Q}(t_n + \Delta t) - \hat{Q}(t_n) = -w_q\hat{Q}(t_n) + w_q Q(t_n),$$
$$\frac{\hat{Q}(t_n + \Delta t) - \hat{Q}(t_n)}{\Delta t} = \frac{w_q}{\Delta t}(Q(t_n) - \hat{Q}(t_n)).$$

Assuming that $\delta = \Delta t$ and $\delta = \frac{1}{C}$, we obtain

$$\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} = \frac{w_q}{\delta}(Q(t) - \hat{Q}(t)) = w_q C(Q(t) - \hat{Q}(t)), \qquad (4)$$

which corresponds to the behavior of the exponentially weighted moving average.

Next, we will apply the method of stochastization of one-step processes to the resulting equations and write the corresponding Fokker–Planck and Langevin equations for the random processes $W(t)$ and $Q(t)$.

For the random process $W(t)$ we will denote by k_1^1 the arrival intensity of packets, and by k_2^1 the service rate of packets in the system.

From the equation (3) we obtain:

$$k_1^1 = \frac{1}{W(t)}. \qquad (5)$$

Consideration of packages loss only upon receiving a triple duplicate ACK gives that

$$k_2^1 = \frac{1}{2}\frac{\mathrm{d}N(t)}{\mathrm{d}t}, \qquad (6)$$

where $\mathrm{d}N(t)$ is the Poisson process similar to the introduced in Misra et al. (2000*b*).

The corresponding kinetic equation for the congestion window of TCP Reno will have the form:

$$\begin{cases} 0 \xrightarrow{k_1^1} W(t), \\ W(t) \xrightarrow{k_2^1} 0. \end{cases} \qquad (7)$$

From (7) we can write the operators of the state. In particular, for the number of packages before the interaction $N_{\underline{i}}^{\alpha}$ [1] we get:

$$N_{\underline{i}}^{\alpha} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad (8)$$

and for the number of packets after interaction $M_{\underline{i}}^{\alpha}$ we derive the following expression:

$$M_{\underline{i}}^{\alpha} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

The transition operator $r_i^{\alpha} = M_i^{\alpha} - N_i^{\alpha}$ will look like:

$$r_{\underline{i}}^1 = \begin{pmatrix} 1 \end{pmatrix}, \quad r_{\underline{i}}^2 = \begin{pmatrix} -1 \end{pmatrix}. \qquad (9)$$

[1] We will use the abstract indices notation (see Penrose and Rindler (1987)). In this notation tensor as a complete object is denoted merely by an index (e.g., x^i). Its components are designated by underlined indices (e.g., $x^{\underline{i}}$).

From the general formula for the specific probability of transition in the forward direction:

$$S_{\alpha}^{+} = k_{\alpha} \prod_{\underline{i}} \frac{x_{\underline{i}}!}{(x_{\underline{i}} - N_{\underline{i}}^{\alpha})!}$$

and (8) we get:

$$S_1^{+} = k_1^1, \quad S_2^{+} = k_2^1 W(t). \qquad (10)$$

The Fokker–Planck equation coefficients we get from (10) and (9):

$$A^i = S_{\alpha}^{+} r^{\alpha i} = k_1^1 - k_2^1 W(t),$$
$$B^{ij} = S_{\alpha}^{+} r^{\alpha i} r^{\alpha j} = k_1^1 + k_2^1 W(t),$$

where $a^i := A^i(x^k, t)$ is the drift vector, $B^{ij} := B^{ij}(x^k, t)$ is the diffusion vector for some n-dimensional random process $x^k := x^k(t) \in \mathbb{R}^n$. In this case $x^k := W(t)$.

Taking into account (5) and (6), we may write the Fokker–Planck equation:

$$\frac{\partial w(t)}{\partial t} = -\frac{\partial}{\partial W(t)}\left[\left(\frac{1}{W(t)} - \frac{W(t)}{2}\frac{\mathrm{d}N(t)}{\mathrm{d}t}\right) w(t)\right] + \\ + \frac{1}{2}\frac{\partial^2}{\partial W^2(t)}\left[\left(\frac{1}{W(t)} + \frac{W(t)}{2}\frac{\mathrm{d}N(t)}{\mathrm{d}t}\right) w(t)\right], \qquad (11)$$

where $w(t)$ is the distribution density of the random process $W(t)$.

The form of the corresponding (11) Langevin equation is:

$$\mathrm{d}W(t) = \frac{1}{W(t)}\mathrm{d}t - \frac{W(t)}{2}\mathrm{d}N(t) + \\ + \sqrt{\frac{1}{W(t)} + \frac{W(t)}{2}\frac{\mathrm{d}N(t)}{\mathrm{d}t}}\,\mathrm{d}V^1(t), \qquad (12)$$

where $\mathrm{d}V^1(t)$ is the Wiener process corresponding to the random process $W(t)$.

Similarly, for the random process $Q(t)$, we may denote the packets arrival rate in the queue as k_1^2:

$$k_1^2 = \frac{W(t)}{T}, \qquad (13)$$

and by k_2^2 we will denote the intensity of packets service in the queue:

$$k_2^2 = -C. \qquad (14)$$

The corresponding kinetic equations for the average queue length will have the form:

$$\begin{cases} 0 \xrightarrow{k_1^2} Q(t), \\ 0 \xrightarrow{k_2^2} Q(t). \end{cases} \qquad (15)$$

From (15) state operators are derived:

$$N_{\underline{i}}^{\alpha} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad M_{\underline{i}}^{\alpha} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

as well as the transition operator $r_i^{\alpha} = M_i^{\alpha} - N_i^{\alpha}$:

$$r_{\underline{i}}^1 = (1), \quad r_{\underline{i}}^2 = (1). \qquad (16)$$

The specific probability of transition in the forward direction:

$$S_1^+ = k_1^2, \quad S_2^+ = k_2^2. \tag{17}$$

The coefficients of the Fokker–Planck equation based on expressions (17) and (16) take the form:

$$A^i = S_\alpha^+ r^{\alpha i} = k_1^2 + k_2^2,$$
$$B^{ij} = S_\alpha^+ r^{\alpha i} r^{\alpha j} = k_1^2 + k_2^2.$$

From (13) and (14) the Fokker–Planck equation is derived:

$$\frac{\partial q(t)}{\partial t} = -\frac{\partial}{\partial Q(t)}\left[\left(\frac{W(t)}{T} - C\right) q(t)\right] + \\ + \frac{1}{2}\frac{\partial^2}{\partial Q^2(t)}\left[\left(\frac{W(t)}{T} - C\right) q(t)\right], \tag{18}$$

where $q(t)$ is the distribution density of the stochastic process $Q(t)$.

The corresponding (18) Langevin equation has form:

$$\mathrm{d}Q(t) = \left(\frac{W(t)}{T} - C\right)\mathrm{d}t + \sqrt{\frac{W(t)}{T} - C}\mathrm{d}V^2(t), \tag{19}$$

where $\mathrm{d}V^2(t)$ is the Wiener process, corresponding to $Q(t)$.

From (12), (19) and (4) the resulting system of equations is obtained

$$\begin{cases} \mathrm{d}W(t) = \dfrac{1}{T}\mathrm{d}t - \dfrac{W(t)}{2}\mathrm{d}N(t) + \\ \quad + \sqrt{\dfrac{1}{T} + \dfrac{W(t)}{2}\dfrac{\mathrm{d}N(t)}{\mathrm{d}t}}\mathrm{d}V^1(t), \\ \mathrm{d}Q(t) = \left(\dfrac{W(t)}{T} - C\right)\mathrm{d}Q(t) + \sqrt{\dfrac{W(t)}{T} - C}\mathrm{d}V^2(t), \\ \dfrac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} = w_q C(Q(t) - \hat{Q}(t)), \end{cases} \tag{20}$$

Wiener processes included in the corresponding equations can be interpreted as a random deviation of the packet size from the mean size.

For simplicity, we may write the system (20) in moments:

$$\begin{cases} \dot{W}(t) = \dfrac{1}{T(Q(t),t)} - \\ \quad - \dfrac{W(t)W(t - T(Q(t),t))}{2T(t - T(Q(t),t))}p(t - T(Q(t),t)); \\ \dot{Q}(t) = \dfrac{W(t)}{T(Q(t),t)}N(t) - C; \\ \dot{\hat{Q}}(t) = -w_q C\hat{Q}(t) + w_q CQ(t), \end{cases} \tag{21}$$

This model corresponds to the model described in work Misra et al. (2000*b*).

The classical RED (see Floyd and Jacobson (1993)) algorithm works with weighted average of queue length, marking incoming packets for deletion (or deleting them) with some probability, if the calculated value reaches the threshold:

$$p(\hat{Q}(t)) = \begin{cases} 0, & 0 < \hat{Q}(t) \leqslant Q_{\min}, \\ \dfrac{\hat{Q}(t) - Q_{\min}}{Q_{\max} - Q_{\min}}p_{\max}, & Q_{\min} < \hat{Q}(t) \leqslant Q_{\max}, \\ 1, & \hat{Q}(t) > Q_{\max}. \end{cases} \tag{22}$$

This algorithm is a classic and is the most common RED-like algorithm.

NUMERICAL EXPERIMENT

Our initial approach was to investigate the obtained systems: (20) and (21) numerically. In particular, our goal was to obtain phase and parametric portraits of the system.

A family of phase trajectories for given parameters values forms the phase portrait of the system, giving a complete qualitative representation of its possible behavior. The qualitative picture of the phase portrait is determined by equilibrium positions (special points) and special trajectories, for example, the mutual arrangement of limit cycles, equilibrium points.

The limit cycle corresponds to the regime of periodic oscillations and it is the closed trajectory on the phase plane. The equilibrium point, corresponding to the stationary regime, is the point $\hat{Q}_s$ on phase plane in which the right-hand sides of differential equations turn to zero.

Special points and cycles are stationary solutions. If the initial conditions coincide with the special point or are located on the limit cycle, then the system will always remain in that point. In practice, this will be in the case of sustainability, when under initial conditions from a small area of solutions the system returns to the original stationary mode. The phase portrait gives a visual representation of the possible steady solutions, and also the areas of their attraction, if there are several solutions.

The relevant task is to study the phase portraits changes, depending on the parameters or its deformations. Bifurcation situations may occur during the deformation process, when the structure of the phase portrait changes nearby of the bifurcation carrier (the equilibrium or limit position) cycle) in the phase space. For example, the phase portrait qualitatively changes.

The set of bifurcation values of the parameters, which breaks the parameter space into regions of different types of phase portraits, is the parametric portrait of the system. The ultimate goal of qualitative research is to obtain the parametric portrait of the system, i.e. the space partitioning parameters on the areas of different behavior corresponding to topologically different phase portraits.

Since the reset $p(\hat{Q})$ is a piecewise function, then the system (21) can be written as a set of several systems of nonlinear equations, each for one of the intervals of values of the function $p(\hat{Q})$. This approach is called the hybrid modeling (see Korolkova et al. (2016)).

The solution can be either sustainable or represent the self-oscillating process (see Fig. 1). For our system the appearance

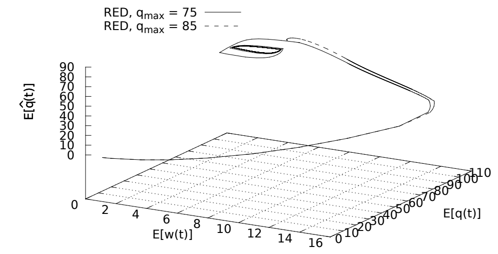

Fig. 1. The phase portrait for the system with the RED, $Q_{\min} = 20$

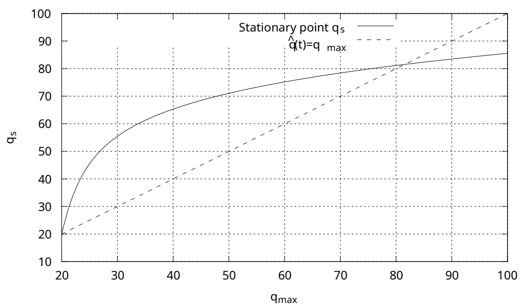

Fig. 2. The parametric portrait of the system with RED, $Q_{\min} = 20$

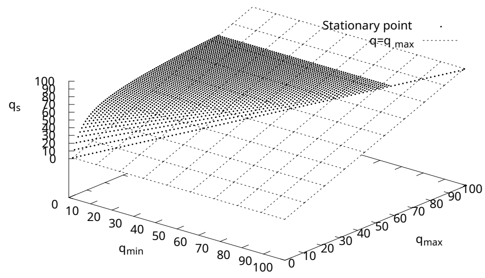

Fig. 3. The parametric portrait of the system with RED

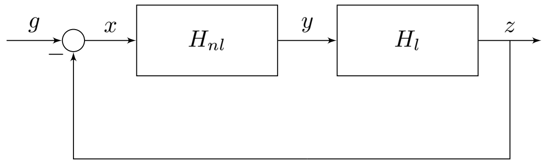

Fig. 4. The block structure of the system for the harmonic linearization method

1

of self-oscillations can be caused by the nature of the discontinuities of the function $p(\hat{Q}(t))$, since there is a gap of the 1st kind. the self-oscillating mode occurs when the fixed point value $\hat{Q}_s$, calculated in the workspace (the region of the slow movements), falls in the interval of the unconditional reset ($p(\hat{Q}(t)) = 1$, fast motion region).

The parametric portrait of the system is the plot which shows the behavior of the stationary point $\hat{Q}_s$ under the assumption that there is no area of unconditional reset. Note that if you leave free n parameters, the result is a surface of the dimension n.

The boundary surface of the dimension $n-1$ will divide the regions of stable and unstable behavior of the system. Self-oscillations will occur in areas of instability of the system lying above the boundary surface. As an example, you can consider parametric portraits with one free parameter (Fig. 2) and with two free parameters (Fig. 3).

The study was conducted by Runge–Kutta method (see Gevorkyan et al. (2016); Eferina et al. (2014)), the programs were written in FORTRAN language.

STRUCTURAL STUDY

With all the convenience and simplicity of the computational experiment it has its drawbacks. The main drawback is that the model is studied as a single monolith. This does not allow to identify the causes of self-oscillations. For atomic studies of the causes of self-oscillations we should use structural approach. As a structural approach we have chosen Krylov–Bogolyubov method (see Kryloff and Bogoliuboff (1934)), and its development — the harmonic linearization method (see Nyquist (1932)).

The harmonic linearization method allows to study approximately stability and accuracy of nonlinear systems, applying methods usually used for analysis of linear systems. The method makes it possible to determine the presence of self-oscillations, and to find their parameters: the frequency and the amplitude. The difference of the harmonic linearization from the usual linearization is the replacement of the nonlinear characteristic with a straight line whose steepness depends on the amplitude of the input signal nonlinear element.

System splits to "linear" H_l and "nonlinear" parts of H_{nl} (see Fig. 4). These names are enough conditional. As the nonlinear part, we present the assumed source of self-oscillations. The linear part serves as a low pass filter. Thus, the linear part allows to restrict the decomposition in a row only to the first member of the row.

For our model the block representation of its linearized model is reduced to the corresponding form fig. 4. In this case, as a nonlinear function the linearized reset function P_{RED} (23), obtained from reset functions (22), will be used:

$$P_{\text{RED}} := \delta p(\hat{Q}, t) = \\ = \begin{cases} 0, & 0 < \hat{Q}(t) \leqslant Q_{\min}, \\ \dfrac{p_{\max}}{Q_{\max} - Q_{\min}} \, \delta\hat{Q}(t)\,, & Q_{\min} < \hat{Q}(t) \leqslant Q_{\max}, \\ 0, & \hat{Q}(t) > Q_{\max}. \end{cases} \quad (23)$$

The structural linearized system has the following form:

$$x_0 + H_l(\omega)\Big|_{\omega=0} \varkappa_0(A, \omega, x_0) = 0,$$
$$1 + H_l(\varkappa(A, \omega, x_0) + \mathrm{i}\varkappa'(A, \omega, x_0)) = 0.$$

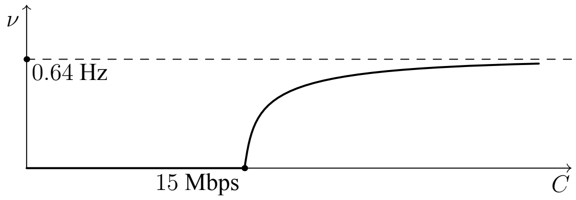

Fig. 5. The parametric portrait that represent self-oscillation frequency dependence from service intensity

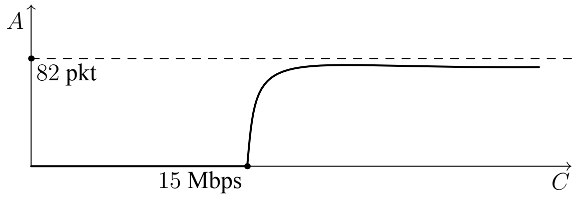

Fig. 6. The parametric portrait that represent self-oscillation amplitude dependence from service intensity

Here a is the oscillation amplitude, ω is the cyclic frequency fluctuations, x_0 is the shift of the oscillations relative to the Ox axis. The functions $\varkappa_0$, $\varkappa$ and $\varkappa'$ define the "nonlinear" part of the system.

As a free parameter, we may set the service intensity C. Let's set the following RED algorithm parameters: number of sessions $N = 60$, RTT without accounting packet delays in hardware $T_p = 0.5$ s, thresholds $Q_{\min} = 75$ packages and $Q_{\max} = 150$ packages, maximum reset level $p_{\max} = 0.1$, the weighting factor of the EWMA $w_q = 0.002$.

The results of the study are the following parametric portraits (see Fig. 5, 6, 7). As a free parameter we use the intensity of services $C_a = 15$ Mbps. That is, judging by the plots, for $C \geqslant C_a$ the system will be in self-oscillating mode.

The structural approach allows to identify and study the sources of self-oscillations occurrence. However, in computational terms, it is much more labor-intensive than the direct numerical simulation.

VERIFICATION

The obtained theoretical studies should be verified by full-scale or simulation experiment. Real network hardware is not always available for experiments. The virtual network has high requirements for computer hardware (see Velieva et al. (2015)) and experiment may take lot of time. In order to save resources and time simulation tools are usually used.

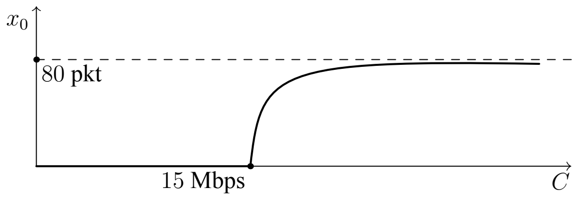

Fig. 7. The parametric portrait that represent self-oscillation shift dependence from service intensity

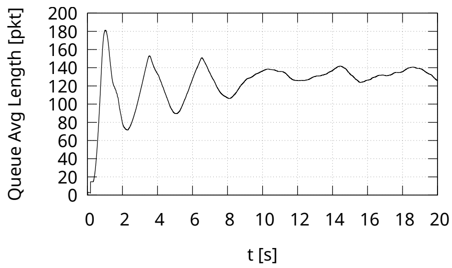

Fig. 8. The behavior of the average queue length when the service intensity is $C = 5$ Mbit/s

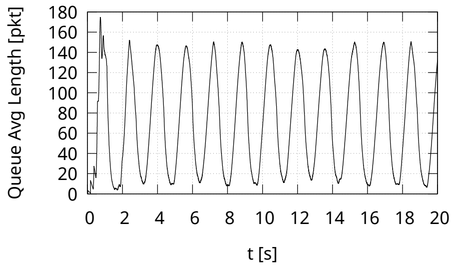

Fig. 9. The behavior of the average queue length when the service intensity is $C = 20$ Mbit/s

As a reference simulation tool for network simulation and replacing the full-scale experiment, researchers are still using ns-2 (see Issariyakul and Hossain (2012); Altman and Jiménez (2012)). Third version of this software has been developing since 2006 to the present time. Ns-3 architecture is more consistent with models of the protocol stacks than ns-2, but ns-3 has not yet been widely used as a tool verification of research results in this area.

The main programming language of ns-2 projects is TCL language (see Welch and Jones (2003); Nadkarni (2017)).

The result of the simulation experiment is saved to large trace-file that contains a description of all simulated events. This data can be visualized (see Fig. 8 and 9). For obtaining the parameters of self-oscillations we found their spectrum using a fast Furies transform (see Rao et al. (2010)) (see Fig. 10). The programs are written in Julia language (see Joshi and Lakhanpal (2017)).

The results of the computer simulation coincides with high accuracy with theoretical results of both the computational experiment and the structural study.

CONCLUSION

We have developed a stochastic model of interaction between the TCP protocol and the RED algorithm based on the one-step process stochastization method. The proposed model (the model in the moments) in general coincides with the models proposed by other authors. The study was conducted in two directions. Structural study allowed to identify the cause of the self-oscillatory regime. The computational experiment made it possible to effectively build parametric portraits of the

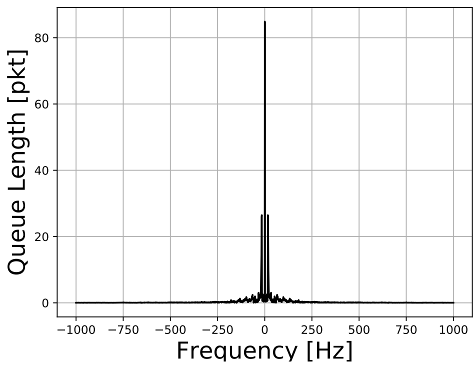

Fig. 10. The spectrum of self-oscillations of the instantaneous queue length when the intensity of the service is $C = 20$ Mbit/s

system and to obtain the characteristics of the zone of self-oscillation and the parameters of self-oscillations themselves. In general, we believe that we successfully solved the problem of investigating the emergence of the self-oscillatory mode in the TCP–RED system.

ACKNOWLEDGMENT

The publication has been prepared with the support of the "RUDN University Program 5-100" and funded by Russian Foundation for Basic Research (RFBR) according to the research project No 19-01-00645.

REFERENCES

Allman, M., Paxson, V. and Blanton, E. (2009), TCP Congestion Control, Technical report.

Altman, E., Avrachenkov, K. and Barakat, Ch. (2000), A stochastic model of tcp/ip with stationary random losses, *SIGCOMM Comput. Commun. Rev.* 30(4), 231–242.

Altman, E., Avrachenkov, K. and Barakat, Ch. (2005), A stochastic model of tcp/ip with stationary random losses, *IEEE/ACM Trans. Netw.* 13(2), 356–369.

Altman, E. and Jiménez, T. (2012), NS Simulator for Beginners, *Synthesis Lectures on Communication Networks* 5(1), 1–184.

Atsumi, Y., Kondoh, E., Altintas, O. and Yoshida, T. (1998), Improving fairness and stability in best effort service: a new congestion control algorithm for sack-tcp, *IEICE Trans. Commun.* E81-B, 2023–2033.

Eferina, E. G., Korolkova, A. V., Gevorkyan, M. N., Kulyabov, D. S. and Sevastyanov, L. A. (2014), One-Step Stochastic Processes Simulation Software Package, *Bulletin of Peoples' Friendship University of Russia. Series "Mathematics. Information Sciences. Physics"* (3), 46–59.

Feng, W.-C., Kandlur, D., Saha, D. and Shin, K. (1999), A self-configuring red gateway, Infocom.

Firoiu, V. and Borden, M. (2000), A study of active queue management for congestion control, INFOCOM 2000. Nineteenth Annual Joint Conference of the IEEE Computer and Communications Societies, Vol. 3, Tel Aviv, Israel, pp. 1435–1444.

Floyd, S. (1997), Red: Discussions of setting parameters.

Floyd, S. (2000), Recommandations on using the gentle variant of red.

Floyd, S. and Fall, K. (1999), Promoting the use of end-to-end congestion control in the internet, *IEEE/ACM Transactions on Networking* 7(4), 458–472.

Floyd, S., Fall, K. and Tieu, K. (1998), Estimating arrival rates from the red packet drop history.

Floyd, S. and Jacobson, V. (1993), Random Early Detection Gateways for Congestion Avoidance, *IEEE/ACM Transactions on Networking* 1(4), 397–413.

Gevorkyan, M. N., Demidova, A. V., Velieva, T. R., Korol'kova, A. V., Kulyabov, D. S. and Sevast'yanov, L. A. (2018), Implementing a Method for Stochastization of One-Step Processes in a Computer Algebra System, *Programming and Computer Software* 44(2), 86–93.

Gevorkyan, M. N., Velieva, T. R., Korolkova, A. V., Kulyabov, D. S. and Sevastyanov, L. A. (2016), Stochastic Runge–Kutta Software Package for Stochastic Differential Equations, Dependability Engineering and Complex Systems, Vol. 470, Springer International Publishing, pp. 169–179.

Hnatič, M., Eferina, E. G., Korolkova, A. V., Kulyabov, D. S. and Sevastyanov, L. A. (2016), Operator Approach to the Master Equation for the One-Step Process, *EPJ Web of Conferences* 108, 58–59.

Iannaccone, G., May, M. and Diot, C. (2001), Aggregate traffic performance with active queue management and drop from tail, *SIGCOMM Comput. Commun. Rev.* 31(3), 4–13.

Issariyakul, T. and Hossain, E. (2012), *Introduction to Network Simulator NS2*, Springer US, Boston, MA.

Joshi, A. and Lakhanpal, R. (2017), *Learning Julia*, Packt Publishing.

Korolkova, A. V., Velieva, T. R., Abaev, P. A., Sevastianov, L. A. and Kulyabov, D. S. (2016), Hybrid Simulation Of Active Traffic Management, *Proceedings 30th European Conference on Modelling and Simulation* pp. 685–691.

Kryloff, N. and Bogoliuboff, N. (1934), Les méthodes symboliques de la Mécanique non Linéaire dans leur application à l'étude de la résonance dans l'oscillateur, *Bulletin de l'Académie des Sciences de l'URSS. Classe des sciences mathématiques* (1), 7–34.

Lin, D., Lin, D., Morris, R. and Morris, R. (1997), Dynamics of random early detection, Proceedings of ACM SIGCOMM, Sophia Antipolis, France, pp. 127–137.

Mathis, M., Semke, J., Mahdavi, J. and Ott, T. (1997), The macroscopic behavior of the tcp congestion avoidance algorithm, *SIGCOMM Comput. Commun. Rev.* 27(3), 67–82.

May, M., Bolot, J., Diot, C. and Lyles, B. (1999), Reasons not to deploy red, Proceedings of IWQoS'99, pp. 260–262.

Misra, A. and Ott, T. J. (1999), The window distribution of idealized tcp congestion avoidance with variable packet loss, INFOCOM, pp. 1564–1572.

Misra, V., Gong, W.-B. and Towsley, D. (1999*a*), Stochastic differential equation modeling and analysis of tcp-windowsize behavior.

Misra, V., Gong, W.-B. and Towsley, D. (1999*b*), Stochastic Differential Equation Modeling and Analysis of TCP-Windowsize Behavior, *Proceedings of PERFORMANCE* 99.

Misra, V., Gong, W.-B. and Towsley, D. (2000*a*), Fluid-based analysis of a network of aqm routers supporting tcp flows with an application to red, *ACM SIGCOMM Computer Communication Review* 30, 151–160.

Misra, V., Gong, W.-B. and Towsley, D. (2000*b*), Fluid-Based Analysis of a Network of AQM Routers Supporting TCP Flows with an Application to RED, *ACM SIGCOMM Computer Communication Review* 30(4), 151–160.

Nadkarni, A. P. (2017), *The Tcl Programming Language: A Comprehensive Guide*, CreateSpace Independent Publishing Platform.

Nyquist, H. (1932), Regeneration Theory, *Bell System Technical Journal* 11(1), 126–147.

Padhye, J., Firoiu, V., Towsley, D. and Kurose, J. (1998), Modeling tcp throughput: A simple model and its empirical validation, Technical report, Amherst, MA, USA.

Pazos, C. M., Juan, S. A. and Gerla, M. (1999), Using back pressure to improve tcp performance with many flows, Proc. INFOCOM99, New York, pp. 431–438.

Penrose, R. and Rindler, W. (1987), *Spinors and Space-Time: Volume 1, Two-Spinor Calculus and Relativistic Fields*, Vol. 1, Cambridge University Press.

Rao, K. R., Kim, D. N. and Hwang, J. J. (2010), *Fast Fourier Transform - Algorithms and Applications*, Signals and Communication Technology, Springer.

Velieva, T. R., Korolkova, A. V. and Kulyabov, D. S. (2015), Designing Installations for Verification of the Model of Active Queue Management Discipline RED in the GNS3, 6th International Congress on Ultra Modern Telecommunications and Control Systems and Workshops (ICUMT), IEEE Computer Society, pp. 570–577.

Welch, B. and Jones, K. (2003), *Practical Programming in Tcl and Tk*, 4th edn, Prentice Hall.

Ziegler, T., Fdida, S. and Brandauer, C. (2001), A quantitative model of red with tcp traffic, IEEE/ACM IWQoS, Karlsruhe.

AUTHOR BIOGRAPHIES

ANNA V. KOROLKOVA received her PhD in Mathematics in 2010. Since then, she has worked as associate professor in Peoples' Friendship University of Russia (RUDN University). Her current research activity focuses on mathematical modeling. Her email address is korolkova-av@rudn.ru.

DMITRY S. KULYABOV received his DSc in Physics and Mathematics in 2017. Since then, he has worked as full professor in Peoples' Friendship University of Russia (RUDN University). His current research activity focuses on mathematical modeling. His email address is kulyabov-ds@rudn.ru.

TATYANA R. VELIEVA PhD student in Peoples' Friendship University of Russia (RUDN University). Her current research activity focuses on mathematical modeling. Her email address is velieva-tr@rudn.ru.

IVAN S. ZARYADOV received his PhD in Mathematics in 2010. Since then, he has worked as associate professor in Peoples' Friendship University of Russia (RUDN University). His current research activity focuses on on probability theory. His email address is zaryadov-is@rudn.ru.