SCENARIO-BASED SIMULTANEOUS INVESTMENT, FINANCING AND OPERATIONAL PLANNING

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KEYWORDS

Simultaneous investment, financing and operational planning, decisions under uncertainty, scenario based linear integer programming

ABSTRACT

Investment, financial as well as operational decisions influence each other and may lead to suboptimal decisions if decided separately. Nevertheless, they are often dealt with separately, both in literature and in practice, or they are considered under rather unrealistic assumptions. This paper describes a new approach for simultaneous investment, financial and operating planning. It is an approach under uncertainty considering different scenarios with given probabilities taking into account individual risk preferences on the basis of suitable criteria like the von Morgenstern-Neumann expected utility. It contains the choice of different investment alternatives and their disinvestment, different financing alternatives as well as the determination of the sales and production quantities on the basis of a mixed-integer linear programme with the aim of maximising the net present value (NPV) of periodical project dividends.

INTRODUCTION

Investment and financial decisions as well as the operational strategies belong to the strategic planning processes. Each of these sub-plans influences the other depending decisions. Separating these planning parts may lead to sub-optimal decisions. The problem includes a set of investment alternatives which can be realised multiple times. There should also be an opportunity to disinvest these investments within their lifetime. These investments build the environment for the operations which deal with the sales and production quantities as well the stock of finished goods. The investments and the operations have to be financed using different financial sources. The results of such long-term oriented decisions are not certain. Therefore, the problem has to be solved under the considerations of a set of scenarios with given probabilities including an individual risk-preference of the decision maker.

This paper proposes a new approach for simultaneous investment, financial and operating planning. A linear, mixed-integer model will be introduced which maximises the NPV of the periodical cash-flow surpluses available as dividend payouts for the shareholders subject to several investment, financial and operations constraints.

The decision is based on scenarios with given probabilities as well as the risk-preference of a decision maker and is determined in a three-stage process. In the first step, the mixed-integer linear model is solved for all scenarios. These scenario-optimal solutions can lead to completely different results if another scenario occurs. In this context, the horizon of all periods to be planned is to be divided into a fixed and a flexible horizon. The frozen horizon contains the first periods of the entire planning horizon. The decisions made for one particular scenario for the frozen horizon cannot be changed if another scenario occurs. Therefore, the mixed-integer linear model is solved again for all other scenarios on the basis of the solution of the fixed horizon of the given scenario to find the solutions for the following flexible horizon.

The scenario-optimal solutions for the frozen horizon can be understood as alternatives and the corresponding NPVs of the project dividends for all scenarios as random variables in a discrete decision model under uncertainty with given probabilities. The last step of the suggested approach is to solve this problem considering specific risk preferences by applying suitable criteria like the von Morgenstern-Neumann expected utility (Neumann and Morgenstern, 1953).

This paper starts with a literature review which is the basis for the proposed new approach described in the following section. The paper ends with conclusions.

LITERATURE REVIEW

Several aspects of simultaneous investment, financial and operating planning have been discussed for more then 50 years.

Approaches of simultaneous *investment and financing planning* want to select several investment and financing alternatives simultaneously often in order to maximise the NPV, the terminal wealth or the dividend payouts of all chosen investment and financing alternatives (Blohm et al., 2012, p. 269f). One of the first models was presented by Dean (1969) in which the investment and financing alternatives are selected by comparing the internal rates of returns of the investment alternatives and the interest rates of the financing alternatives (Götze et al., 2015, p. 216). Beside this rather simple model, there are a couple of proposals using linear programming techniques. A good overview of such approaches at that time are given in Bernhard (1969). E.g., Hax (1964) and Weingartner (1963) introduced similar dynamic models with more realistic assumptions than Dean (1969) in which the investments and financing alternatives are chosen simultaneously using a linear programming model in order to maximise the terminal wealth. These models are extended by Park (2008) by repetitive multiphase decisions sequences. Albach (1962) developed a linear programming model which maximises the sum of the net present values of the selected investment and financial alternatives. Bernhard (1969) introduced a model with assump-

Communications of the ECMS, Volume 34, Issue 1,
Proceedings, ©ECMS Mike Steglich, Christian Mueller,
Gaby Neumann, Mathias Walther (Editors)
ISBN: 978-3-937436-68-5/978-3-937436-69-2(CD) ISSN 2522-2414

tions and constraints similar to Hax (1964) and Weingartner (1963) with an objective function that maximises the terminal wealth and the stream of dividend payments. Byrne et al. (1967) introduced a chance-constraint approach to capital budgeting including liquidity constraints. Majumdar and Chattopadhyay (1999) proposed an application orientated model for an integrated planning of capacity expansions and financial planning to maximise the net worth for electricity generation. They consider different financial sources (e.g. debt, cash flow from projects, equity) but a choice between different external financial sources is not designated.

One of the first models for integrated *investment and operational planning* was proposed by (Förstner and Henn, 1957, p. 119ff). They extended a production programme model by flexible constraints influenced by investments in order to maximise the terminal wealth. Another approach was introduced by Jacob (1964) with more realistic assumptions regarding the production system. The capacities of several types of machines measured in time units are flexible due to the opportunity to invest and also to disinvest in these machines depending on financial constraints. The objective function maximises the total profit including the sales revenues, the variable and fixed costs, the capital expenditures and the proceeds of liquidation (Jacob, 1964, p. 34ff). The author extended the objective function by interest earnings of the gross profits of the previous period in a second version of his model (Jacob, 1964, p.62f). More recently, Bradley and Arntzen (1999) developed a model for simultaneous capacity and inventory investment planning as well as production schedule in order to maximise the return on assets. Rajagopalan and Swaminathan (2001) proposed an mixed integer nonlinear production planning model with capacity expansion and inventory management which minimises the discounted costs of capacity expansion, inventories and production. Another nonlinear mixed integer programming model was introduced by Hsu and Li (2009) for high-tech manufacturers to determine the optimal supply chain network design including economy of scale effects. The model minimises the total average product costs per unit. In a wider range, models which deal with strategic design decisions in supply chain in combination with operational planning are to be mentioned. E.g., an integrated multi-objective supply chain model was introduced by Sabri and Beamon (2000) which considers the design of a supply chain system simultaneously with the selling and production decisions.

There are also simultaneous *operational and financial planning* models. For instance, Charnes et al. (1959) proposed an approach dealing with warehouse operations of goods to be bought and sold subject to financial constraints. Another recently published example is Guillén et al. (2006). They expanded a supply chain model maximising the profit of all products by cash balance constraints including the opportunity to borrow external capital. Different types of external financial sources are not considered.

All of these approaches consider different aspects of simultaneous investment, financial and operating planning, but none integrates the sub-plans into one approach. It can also be criticised that most of these models assume unrealistically only one relevant scenario and therefore certain results. Additionally, some of the approaches use an exogenous weighted average cost of capital (*WACC*) and do not consider the impact of the different chosen financial sources on the *WACC* (Blumentrath, 1969, p. 271ff).

Despite applicable sub-aspects of these approaches, only models for simultaneous *investment, financial and operating planning* are suitable for the described problem. Blumentrath (1969) was one of the first authors who developed simultaneous investment, financial and operating planning models which maximise the terminal wealth for single- and multi-stage production systems. The models include the opportunity of investment and disinvestment of different investment alternatives and also different financial sources (Blumentrath, 1969, p. 334ff and 412ff). The model by Grundmann (1973) extends the second version of the model by Jacob (1964) by multiple financial sources and corresponding constraints as well as tax aspects (Grundmann, 1973, p. 59-66). Other models introduced by Jääskeläinen (1966) and Haberstock (1971) use rather simple assumptions for the operations (only one product, one type of machine and one raw material) and therefore they are less interesting for further discussion. All of these models consider only one scenario and assume therefore unrealistically all results as certain.

The most relevant model was introduced by Hahn and Kuhn (2012) which extends an own earlier approach (Hahn and Kuhn, 2011). It is a robust optimisation approach which combines investment, operations, and financial planning simultaneously in supply chains. The objective function takes into account both value-based performance and risk aspects by maximising the expected value of the economic value added (EVA) minus the downside risk of EVA. It is to be noted that this objective function does not allow risk-seeking preferences. Only risk preferences between risk-neutral and risk-averse can be chosen by selecting a risk preference parameter (Hahn and Kuhn, 2012, p. 562). Another aspect to be criticised is the handling of external financial sources. The model invokes different internal and external financial sources. Regarding the external financial sources, only the amount of a long-term debt can be chosen, but different types of external financial sources are not considered (Hahn and Kuhn, 2012, p. 565). Additionally, the variable structure and amount of the used financial sources should have an impact on the WACC used for the EVA in the objective function, which was not taken into account in the model.

CONCEPT OF THE SCENARIO-BASED SIMULTANEOUS INVESTMENT, FINANCING AND OPERATIONAL PLANNING APPROACH

Conceptual overview

This section is intended to describe a new approach for simultaneous investment, financial and operational planning.

There is a set of investment alternatives which are not only single machines but rather investment packages which can contain for instance machines, new facilities, licences, investment in markets, etc. (Blumentrath, 1969, p. 341). The investment decisions have to be made at the beginning of each period whereby each investment package can be realised multiple times with the opportunity to disinvest it during its lifetime.

The investments and the operations have to be financed. This can be done by using a set of financial alternatives. These long-term orientated financial resources can be equity, loans, venture capital, etc., for which the periodical cash-flows, containing the raising and repayment of the capital as well as the capital costs, are known at the beginning of the first planning period. The decision about the utilised financial alternatives has to be made once at the at the beginning of the first planning period. There is also the opportunity of short-term loans which can be borrowed at the beginning of each of the periods and have to be paid back including the interests in the following period. Other financial sources are the sales revenues and the proceeds of liquidation of disinvested investment packages.

The operational decisions deal with the sales and production quantities in a rolling planning approach based on the resources built by realised investment alternatives and financed by the mentioned financial sources.

All these decisions have to be made simultaneously under the perspective of the shareholders. Therefore, a mixed-integer linear model is introduced which maximises the NPV of the periodical cash-flow surpluses available as project dividends for the shareholders subject to several investment, financial and operations constraints.

Since such long-term oriented decisions are usually problems under uncertainty, the new approach covers also a set of scenarios with given probabilities. The final decision is based on the scenarios and the individual risk-preference of a decision maker. It is determined in a three-stage process.

In the first step, the mentioned mixed-integer linear model is solved for all scenarios. However, such an optimal decision can lead to different results in the other scenarios, whereby the horizon of all periods to be planned is to be divided into two horizons. There is a frozen horizon at the beginning of the entire planning horizon in which the decision made for one scenario cannot be changed if an other scenario happens. That means that the values of the variables found for a scenario-specific optimal solution are fixed for the frozen horizon in the other scenarios for which in the second step of the proposed approach the mixed-integer linear model is solved again. These are the scenario-based solutions of the flexible horizon which follows the fixed horizon.

The different solutions of the scenarios lead to specific opportunities or risks. If it is for example assumed that the demands of customers are uncertain then the decisions made for a low-demand scenario could lead to a loss of sales revenues (and of the NPV for the cash-flow surpluses) in a high-demand scenario due to the restricted capacities determined for the low-demand scenario. If in the low-demand scenario the maximum customer demands are the restrictive factor then higher sales quantities in a high-demand scenario might occur. Additionally, the capacities determined for a high-demand scenario could be underutilised in a low-demand scenario.

The scenario-optimal solutions for the frozen horizon can be understood as alternatives and the corresponding NPVs of the project dividends for all scenarios as random variables in a discrete decision model under uncertainty with given probabilities. The last step of the suggested approach is to solve this problem considering specific risk preferences by applying suitable criteria like the von Morgenstern-Neumann expected utility.

Optimisation model

This subsection describes the mixed-integer linear model for simultaneous investment, financial and operational planning using the following indices, sets, parameters, and variables.

Indices and sets

$i \in O$	investment alternatives
$j \in F$	financing alternatives
$n \in P$	products
$(n,i) \in PO$	valid combinations of products and investment alternatives
$t \in Y$	periods $Y = \{0,1,\ldots,T\}$
$u \in LY_i$	life time of investment i
$k \in R$	investment-independent production factors
$m \in RM_i$	production factors depending on investment i

Parameters

is	interest rate for the shareholders
ps_{nt}	unit selling price for product n in period t
ch_{nt}	unit holding cost for for product n in period t
c_{nit}	variable unit cost for product-investment combination (n,i) in period t
cs_{nit}	set-up cost for product-investment combination (n,i) in period t
co_{iu}	operating costs of one realisation of investment alternative i at the age of u periods
cre_{mit}	unit costs for the extension of investment-dependent production factor m of investment i in period t
re^u_{mit}	upper bound for the extension of investment-dependent production factor m of investment i in period t
cpx_i	capital expenditures for one realisation of investment i
bv_{iu}	book or market value of one realisation of investment alternative i at the age of u periods
yo^u_i	maximum investments in investment alternative i in a period
ydo^u_i	maximum disinvestments of investment alternative i in a period
xo^u_i	maximum number of investment packages of investment alternative i
xs^l_{nt}, xs^u_{nt}	lower and upper bound for the selling quantity of product n in period t
q^l_{nt}, q^u_{nt}	minimum and maximum inventory of product n at the end period t
cff_{jt}	cash-flow of one utilisation of financing alternative j in period t
yf^l_j, yf^u_j	lower and upper bound of utilisations of financing alternative j
ifs_t	interest rate for the short-term loan in period t
cfs^u_t	upper bound of short-term loan in period t
a_{knt}	use or consumption of investment-independent production factor k per unit of product n in period t

b_{kt}	upper bound of investment-independent production factor k in period t
ai_{mnit}	use or consumption of investment-dependent production factor m for investment alternative i per unit of product n in period t
bi_{miu}	upper bound of investment-dependent production factor m for investment alternative i at the age of u periods
fc_t	cash-relevant fixed costs in period t

Variables

cs_t	cash-flow surplus in period t
yo_{it}	number of investments in investment alternative i in period t
ydo_{iut}	number of disinvestments of investment alternative i at the age of u periods in period t
yf_j	number of utilisations of financing alternative j
cfs_t	short term loan borrowed in period t
cf_t	cash-flow in period t
xs_{nt}	sales quantity of product n in period t
x_{nt}	production quantity of product n in period t
xo_{iut}	number of available packages of investment alternative i at the age of u periods in period t
cap_{mit}	available amount of investment-dependent production factor m for investment alternative i in period t
xi_{nit}	production quantity of product-investment combination (n,i) in period t
yi_{nit}	lot realisation variable of product-investment combination (n,i) in period t
q_{nt}	inventory of product n at the end of period t
qa_{nt}	average inventory of product n in period t
re_{mit}	resource extension of investment-dependent production factor m of investment i in period t

In this model, it is assumed that the investments and the operations are fully financed by using the sales revenues, the financial alternatives, the proceeds of liquidation of disinvestments and if necessary also by short-term loans. If there is a positive cash-flow surplus $cs_t; t \in Y$ after all cash consumptions then it can be used to satisfy the shareholder's wish of a risk-adequate return. The objective of the model is is therefore to maximise the NPV of the non-negative periodical cash-flow surpluses. These cash-flow surpluses can be interpreted as dividend payouts of the entire investment, financial and operational programme.

$$z = \sum_{t \in Y} cs_t \cdot (1+is)^{-t} \rightarrow max \tag{1}$$

The values of the periodical cash-flow surpluses result from the following cash-flow balance constraint (expressions (2-9) which contain all relevant cash generators and consumers. This constraint has to be formulated for all periods and starts in expression (2) with the sum of the sales revenues of the goods sold of all products and the total holding costs for the average stock of finished goods of the products. The total cash-relevant variable costs of the quantities produced and also the corresponding total set-up costs of all valid combinations of products and investments $(n,i) \in PO$ have to subtracted as well as the cash-relevant fixed costs fc_t as shown in (3). The first term in expression (4) describes the cash-relevant operating costs of the existing investment objects in the different ages. These investment objects provide resources that can be extended. The corresponding extension costs represent the second part in (4). The capital expenditures of the investments alternatives and the proceeds of liquidation of disinvested investment packages are described in (5). The cash-flow impacts of the utilisation of the long-term investment alternatives are shown in (6) and for the short-term loans in (7). The last part of the cash-flow balance constraint (8) contains the non-negative cash-flows of this and the previous year as well as the cash-flow surplus. That means that the previous cash-flow is used in the current year if it is necessary. Only if all cash requirements are satisfied then the cash-flow surplus is positive and available for the shareholders.

$$\sum_{n \in P} ps_{nt} \cdot xs_{nt} - \sum_{n \in P} ch_{nt} \cdot qa_{nt} \tag{2}$$

$$- \sum_{(n,i) \in PO} c_{nit} \cdot xi_{nit} - \sum_{(n,i) \in PO} cs_{nit} \cdot yi_{nit} - fc_t \tag{3}$$

$$- \sum_{i \in O} \sum_{\substack{u \in LY_i \\ u \le t}} co_{iu} \cdot xo_{iut} - \sum_{i \in O} \sum_{m \in RM_i} cre_{mit} \cdot re_{mit} \tag{4}$$

$$- \sum_{i \in O} cpx_i \cdot yo_{it} + \sum_{i \in O} \sum_{\substack{u \in LY_i \\ 0 < u \le t}} bv_{iu} \cdot ydo_{iut} \tag{5}$$

$$+ \sum_{j \in F} cff_{jt} \cdot yf_j \tag{6}$$

$$+ cfs_t - cfs_{t-1} \cdot (1 + ifs_{t-1}) \tag{7}$$

$$+ cf_{t-1} - cs_t = cf_t \tag{8}$$

$$; t \in Y, cf_{-1} = 0, cfs_{-1} = 0, cfs_T = 0 \tag{9}$$

This constraint can be extended easily by cash-flow impacts of changes of the working capital if necessary. But this paper is focused on the main cash-flow impacts as shown above.

The following constraint (10) describes the use or consumption of all investment-independent production factors $k \in R$ by the quantities to be produced for all products $i \in P$ and the corresponding upper bounds in all periods.

$$\sum_{n \in P} a_{knt} \cdot x_{nt} \le b_{kt} \quad ; t \in Y, k \in R \tag{10}$$

The relationship between the production quantities of the products $i \in P$ and the quantities produced using the several investment alternatives $(n,i) \in PO$ are given in expression (11). These quantities are required for the calculation of the total variable production costs in (3).

$$\sum_{\{i \mid (n,i) \in PO\}} xi_{nit} = x_{nt} \quad ; n \in P, t \in Y \tag{11}$$

The balance between the products produced, the sales quantities and the stock of finished goods is defined in (12) (Billington et al., 1983). The sales quantities are the basis for the sales revenues in expression (2). The average stock of finished goods equals the average of the beginning and ending inventories of finished goods as in (13). These average stocks are needed to calculate the holding costs in (2).

$$x_{nt} - q_{nt-1} - q_{nt} = xs_{nt} \quad ;n \in P, t \in Y \tag{12}$$

$$0.5 \cdot q_{nt-1} + 0.5 \cdot q_{nt} = qa_{nt} \quad ;n \in P, t \in Y, q_{n0} = 0, q_{nT} = 0 \tag{13}$$

The relationship between the product variables and the lot variables in (14) for all valid product-investment combinations $(n,i) \in PO$ is required to calculate the set-up costs in the cash-balance constraint (3).

$$xi_{nit} \leq M \cdot yi_{nit} \quad ;(n,i) \in PO, t \in Y \tag{14}$$

The constraints (15) and (16) are intended to determine the number of investment packages at the different ages for all investment alternatives and periods. As shown in (15), the number of investment packages at the age of zero depends on the number of investments in such packages in a given period. The amount of investment objects at the ages $u \in LY_i$ bases on the amount of these investment packages in the year before and the disinvestments as of (16). Similar formulations can be found in Hahn and Kuhn (2012).

$$xo_{i0t} = yo_{it} \quad i \in O, t \in Y \tag{15}$$

$$xo_{iut} = xo_{iu-1t-1} - ydo_{iut} \quad i \in O, t \in Y, t > 0, u \in LY_i, 0 < u \leq t \tag{16}$$

There is a set of investment-dependent production factors RM_i for which the available amounts per period are to be determined. The upper bounds of these production factors depend on the age of the corresponding investment objects. Therefore, the amounts of the available investment objects in the different ages xo_{iut} have to be multiplied by the age-dependent upper bounds bi_{miu} of the investment-dependent production factors to determine in sum the available amount cap_{mit} of an investment-dependent production factor $m \in RM_i$ for an investment alternative $i \in O$ in a period $t \in Y$ as shown in (17).

$$cap_{mit} = \sum_{\substack{u \in LY_i \\ u \leq t}} bi_{miu} \cdot xo_{iut} \quad ;m \in RM_i, i \in O, t \in Y \tag{17}$$

The investment-dependent production factors are used or consumed by the investment-dependent production quantities xi_{nit} as shown in the constraints (18). The available amounts cap_{mit} cannot be exceeded by this consumption or usages, but if necessary extended in a certain interval as of (32) which lead to additional extension costs shown in (4).

$$\sum_{\{n|(n,i) \in PO\}} ai_{mnit} \cdot xi_{nit} - re_{mit} \leq cap_{mit} \quad ;t \in Y, i \in O, m \in RM_i \tag{18}$$

The ranges of all variables are defined as follows.

$$cs_t \geq 0 \quad ;t \in Y \tag{19}$$

$$xs_{nt} = \{xs^l_{it}, xs^l_{it}+1, \ldots, xs^u_{it}\} \quad ;n \in P, t \in Y \tag{20}$$

$$x_{nt} = \{0, 1, \ldots\} \quad ;n \in P, t \in Y \tag{21}$$

$$xi_{nit} = \{0, 1, \ldots\} \quad ;(n,i) \in PO, t \in Y \tag{22}$$

$$yi_{nit} \in \{0, 1\} \quad ;(n,i) \in PO, t \in Y \tag{23}$$

$$cf_t \geq 0 \quad ;t \in Y \tag{24}$$

$$0 \leq cfs_t \leq cfs^u_t \quad ;t \in Y \tag{25}$$

$$q_{nt} = \{q^l_{nt}, q^l_{nt}+1, \ldots, q^u_{nt}\} \quad ;n \in P, t \in Y \tag{26}$$

$$qa_{nt} \geq 0 \quad ;n \in P, t \in Y \tag{27}$$

$$xo_{iut} \in \{0, 1, \ldots, xo^u_i\} \quad ;i \in O \tag{28}$$

$$yo_i \in \{0, 1, \ldots, yo^u_i\} \quad ;i \in O \tag{29}$$

$$ydo_i \in \{0, 1, \ldots, ydo^u_i\} \quad ;i \in O \tag{30}$$

$$yf_j \in \{yf^l_j yf^l_j+1, \ldots, yf^u_j\} \quad ;j \in F \tag{31}$$

$$0 \leq re_{mit} \leq re^u_{mit} \quad ;m \in RM_i, i \in O, t \in Y \tag{32}$$

Solving the discrete decision problem under uncertainty

The proposed simultaneous investment, financing and operational decision model leads to a discrete problem under uncertainty for which a set of scenarios $s \in S$ with given probabilities $p_s : \sum_{s \in S} p_s = 1$ exists. All parameters of the model described in the previous section can be uncertain. The optimal investment, financing and operating decision is based on the scenarios and the risk-preference of a decision maker and to be determined in a three-stage process. To carry out the three working steps, the horizon of all periods to be planned $Y = \{0, 1, \ldots, T\}$ is to be divided in a fixed planning horizon $\{0, \ldots, \tau\}$ and a flexible planning horizon $\{\tau+1, \ldots, T\}$.

Step 1

As already described, all decisions have to be made at the beginning of the periods, with the financing decision being made only once at the beginning of the planning horizon. It is therefore not possible to predict which of the scenarios will occur before the decisions are made. In the first step, the optimisation model described in the previous section is therefore solved for all of the scenarios. The solutions of all variables found for a scenario $w \in S$ are divided into the set of the solutions of all variables for the frozen horizon A_w and the set of the solutions of all variables for the flexible horizon Ω_w which determine in total the objective function value $Z(A_w, \Omega_w)$ of this scenario.

Step 2

However, such an optimal solution for a scenario $w \in S$ can lead to completely different results in the other scenarios $s \in S \backslash \{w\}$. The decision made for one particular scenario cannot be changed if another scenario occurs. That means that the set of the optimal values A_w of all variables found for scenario w in the first working step are also fixed for the frozen horizon in the other scenarios.

$$A_s = A_w \quad ;w \in S, s \in S \backslash \{w\} \tag{33}$$

In the second working step, the optimisation model described in the previous section has to be solved again for all of these scenario combinations $w \in S, s \in S \backslash \{w\}$, whereby the values of the variables of the frozen horizon $A_s = A_w$ are fixed as shown in (33). The values of the variables of the flexible horizon Ω_s have to be determined optimally in order to maximise the objective function (1) resulting in the optimal objective function value $Z(A_w, \Omega_s)$.

Step 3

The outcome of the two previous working steps leads to a discrete decision model under uncertainty with given probabilities. The optimal solutions of the variables A_w of the frozen horizon of the scenarios $w \in S$ can be understood as alternatives with a distribution of the resulting objective function values in all scenarios $Z(A_w, \Omega_s)$ $s \in S$. The last step of the suggested approach is to solve this problem considering specific risk preferences by applying suitable criteria.

The simplest approach to solve this decision problem is the expected value principle which is applicable for *risk-neutral* decision makers (Klein, 2009, p. 6-12ff). For all of the alternatives $w \in S$ the expected value

$$\mu_w = \sum_{s \in S} p_s \cdot Z(A_w, \Omega_s) \qquad ; w \in S \tag{34}$$

is to be calculated and the alternative with the maximum expected value has to be chosen (Drury, 2018, p. 288ff). If additionally a *risk-averse* or *risk-seeking* preference (Klein, 2009, p. 6-12ff) has to be invoked into this decision then a measure of the opportunities and threats has to be determined. This can be done by the standard deviation of the stochastic results of the alternatives (Drury, 2018, p. 288ff).

$$\sigma_w = \sqrt[2]{\sum_{s \in S} p_s \cdot (Z(A_w, \Omega_s) - \mu_w)^2} \qquad ; w \in S \tag{35}$$

The utility of an alternative w results from the addition of the expected value with the standard deviation weighted by the parameter alpha (Drury, 2018, p. 288ff).

$$U(\mu_w, \sigma_w) = \mu_w + \alpha \cdot \sigma_w, \quad 0 \le \alpha \le 1 \qquad ; w \in S \tag{36}$$

The parameter α determines the risk-preference of the decision-maker. If α is negative the decision maker evaluates the standard deviation as a threat for results less than the expected value. In contrast to this *risk-averse* attitude, a *risk-seeking* decision-maker uses a positive α because the standard deviation promises in his or her perception higher results than the expected value. An $\alpha = 0$ leads to the expected value principle used for *risk-neutral* decision-makers (Mulvey et al., 1995, p. 126f). The optimal alternative is the alternative with the maximum value of these utilities.

Another criterion to include specific risk-preferences into this decision is the von Morgenstern-Neumann expected utility (vNM) (Neumann and Morgenstern, 1953). For each alternative and all scenarios, the utility of the results $u(Z(A_w, \Omega_s))$ are calculated. The expected utility per alternative is the sum over all scenarios of these utilities multiplied by the probabilities (Winston, 2004, p. 744).

$$vNM_w = \sum_{s \in S} p_s \cdot u(Z(A_w, \Omega_s)) \qquad ; w \in S \tag{37}$$

The specific risk-preferences depend on the utility functions. A concave utility function implies a *risk-averse* behaviour because it is assumed that such a decision maker has a decreasing marginal utility (Winston, 2004, p. 750f). A linear function is used for a *risk-neutral* preference and a convex utility function is interpreted as the utility of a *risk-seeking* decision maker (Klein, 2009, p. 6-12ff). As with the other criteria, the alternative with the maximum von Neumann-Morgenstern expected utility is to be chosen as the optimal alternative.

CONCLUSIONS AND OUTLOOK

Investment and financial decisions as well as the operational plans influence each other and have to be solved simultaneously to avoid suboptimal decisions. Unfortunately, these depending problems are often solved (partially) separately or considered under rather unrealistic assumptions.

This paper describes a new approach for simultaneous investment, financial and operating planning. A mixed-integer linear model is introduced which maximises the NPV of the periodical project dividend payouts subject to several investment, financial and operational constraints. This model is used in a three-stage process. In the first step, the mixed-integer linear model is solved for all scenarios. These scenario-optimal solutions can lead to completely different results if another scenario occurs, for which the optimisation model is solved again under considerations of the results and consequences of the original solution. To do this, the horizon of all periods is to be divided into a frozen and a flexible horizon. The decisions made for a frozen horizon at the beginning of the entire planning horizon cannot be changed if another scenario occurs. Therefore, the mixed-integer linear model is solved again for all other scenarios on the basis of the solution of the fixed horizon to find the solutions for the following flexible horizon. This is the second step of the proposed approach. The set of the alternatives with their solutions for the different scenarios can be understood as a discrete decision model under uncertainty with given probabilities. The last step of the suggested approach is to solve this problem considering specific risk preferences by applying suitable criteria.

This approach tries to fully address the problem described and to avoid the problems of other approaches. It solves investment, financial and operating decisions simultaneously based on a reasonable production-mix model including the opportunity to expand the available amount of the related production factors. It offers several internal and external financing sources including the multiple long-term orientated financial alternatives. Since the objective function is only related to the shareholders, the interest rate for the NPV of the project dividends depends only on their request of a risk-adequate return. This interest rate is not affected by the chosen financial sources in contrast to some of the other published approaches which use an exogenous given WACC without considering the impact of the chosen financial sources. The new approach involves several scenarios, whereby every decision criteria applicable for such problems can be used and therefore every kind of risk-preference can be considered.

A problem of the proposed approach is the effort involved in formulating and solving all of the required scenario combinations which should not be carried out by hand. An opportunity are mathematical programming languages like AMPL (Fourer et al., 2003) or (py)CMPL (Steglich and Schleiff, 2018) which enable a user to formulate mathematical models, to manage the parameters and to obtain the solutions of the models. This can be done iteratively in that a model is solved and its solution is used to specify the parameters of a depending model. Another problem is restricted hardware resources to solve a series of the proposed optimisation model for realistic problem sizes in a reasonable time. To avoid such problems, a distributed or grid optimisation approach can be applied with which large models can be solved remotely on a single optimisation server or in a grid of optimisation servers installed on high performance systems (Steglich, 2016).

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