ANALYSIS OF TIP RELIEF PROFILES FOR INVOLUTE SPUR GEARS

Jakab Molnár*
Attila Csobán
Péter T. Zwierczyk
Department of Machine and Product Design
Faculty of Mechanical Engineering
Budapest University of Technology and Economics
1111, Műegyetem rkp. 3, Budapest, Hungary
E-mail: molnar.jakab@gt3.bme.hu
*Corresponding author

KEYWORDS
involute spur gears, profile modification, tip relief, tooth flank stress, finite element method

ABSTRACT

This research's main goal was to study the influence of involute spur gear tip relief on the contact stress at the engagement meshing point (the beginning point of the line of contact A), as Figure 1. shows.

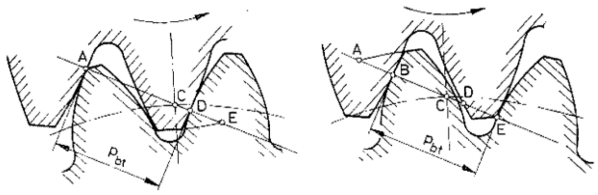


Figure 1. The starting point of single tooth connection (Erney 1983)

Different predefined involute spur gears and modification parameters (amount and length of modification) were already available from previous studies (Schmidt 2019), as Figure 2. shows.

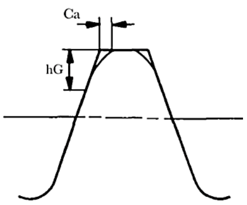


Figure 2. Interpretation of amount of modification and modification length (Schmidt 2019)

In this study, both the drive and the driven gear have tip relief. The modification of the gear profile was achieved through the modification of the gear rack cutter's profile. This way, the gear profiles' profile modification is generated during the gear generation (gear planning) process. The gears have been nitrided, so the heat treatment did not deform the modified gear profile after the gear manufacturing process. The gear modifications were generated in a CAD system, and the calculations were made with FEM. The results show that the tip relief influences the magnitude of the gear contact stress at the first connection point. With the use of tip relief modification, the contact stress of the meshing gears can be reduced at the beginning of the meshing line.

Communications of the ECMS, Volume 35, Issue 1,
Proceedings, ©ECMS Khalid Al-Begain, Mauro Iacono,
Lelio Campanile, Andrzej Bargiela (Editors)
ISBN: 978-3-937436-72-2 / 78-3-937436-73-9(CD) ISSN 2522-2414

INTRODUCTION

Spur gear drives and transmissions are widely used in mechanical engineering. It is necessary to improve the design life, load capacity of the spur gears, and this way, the transmission's stability. Because of the elastic deformation of gear teeth under heavy load, the base pitch of the drive and driven gear differ from each other. This phenomenon leads to contact shock at the beginning of the meshing, significant fluctuation of the transmission ratio, generation of vibrations and noises, reducing the design life, and the transmission accuracy of the gear drive. With tooth profile modifications, the original true involute profile of the spur gear's teeth was modified by removing material from the potential deformation region of gear teeth.

The tooth profile modification of the involute spur gears can correct the deformation of the gear teeth, thus decreasing the noises and vibrations in the gear drive and the fluctuation of the gear ratio. In this study, we limited our focus just to tip relief modification. Tip relief modification is defined as the material that was removed along the tooth flank with reference to the nominal involute profile at the tip circle. In this study, we used tip relief on both the gear and pinion.
The generating, machining process of the gears was gear planning with MAAG rack type gear-cutter tool (DIN 3972). Profile modification can be achieved through the change of the default gear cutter machine parameters or through the modification of the MAAG gear-cutter tool profile.

METHOD

At the start of our study, both the modification parameters (amount of modification and modification length) and the CAD models of the analysed gear pairs were obtained from previous studies (Schmidt 2019). Only spur gears were analysed with the module of 1 [mm]. The gear pairs have zero backlash and addendum modifications. For the tip relief modifications, Inventor 2018 CAD system, and for the preprocessing and FE studies, ANSYS Workbench 18.2 was used.
The main parameters of the analysed gear pairs can be seen in the following table on the next page (Table 1.).

Table 1. Calculated values of
the main geometrical parameters of gear pairs

z_1 [-]	i [-]	z_2 [-]	d_{w1} [mm]	d_{w2} [mm]	a [mm]	b [mm]
17	1	17	17	17	17	10.2
17	4	68	17	68	42.5	10.2
17	6	102	17	102	59.5	10.2
30	1	30	30	30	30	18
30	4	120	30	120	75	18
30	6	180	30	180	105	18
40	1	40	40	40	40	24
40	4	160	40	160	100	24
40	6	240	40	240	140	24

The first goal was to modify the gear profiles with the given modification parameters. As previously mentioned, in this study, we modified the original gear cutter tool profile to achieve tip relief on the gears. The basic idea comes from the paper (Gonzalez et al. 2015) and from the study of the standard DIN ISO 21771. The tool paths of the original rack-cutter tool can be seen in Figure 3.

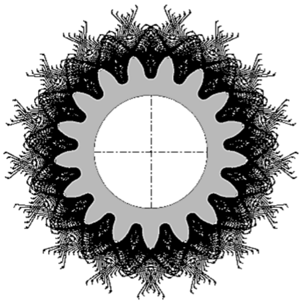


Figure 3. Tool paths of the original gear-cutter tool

The main goal was to make the modification of the rack cutter-tool profile as simple as possible. On the original DIN 3972 rack type gear-cutter profile, the starting point of the gear-cutter tool modification (u) was measured from the tool centerline. From this starting point, the original profile angle (α) was increased on average with 2-3 [°] (α'). This change on the tool profile means that only the required amount of material will be removed at the tip circle. The modification of the tool profile is performed with Electric Discharge Machine (EDM) machine, and because of that, we specified a rounding with the value of 0.3 [mm] at the joining point of the two tool edges with different profile angles. The modified gear cutter tool profile can be seen in Figure 4.

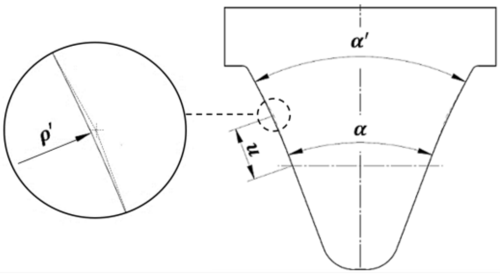


Figure 4. Modified rack-cutter tool profile

Gear profile modification with CAD

Having modified tool profiles, we were able to modify the original gear profiles with CAD. In order to generate a true involute profile in the modification process, we generated tool positions every 0.1 [°]. Where the tool edges crossed, a sketch point was placed in the intersection point.
The intersection points of the tool edges can be seen in Figure 5.

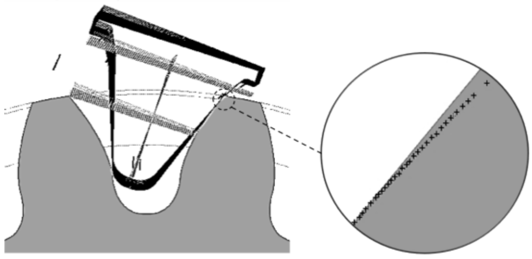


Figure 5. Intersection points of the tool edges

Multiple interpolation splines were placed on the intersection points resulting from the true modified involute profile. The area created by the splines was subtracted from each of the CAD models of the gears. The interpolation splines of the modified involute profile can be seen in Figure 6.

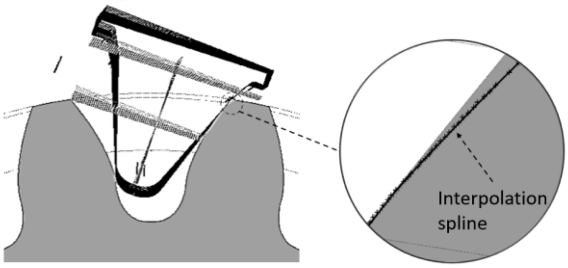


Figure 6. Interpolation splines of the modified profile

As mentioned above, only nitrided gears were analysed, thus the heat treatment process takes place after the machining process of the gears, which already had a modified profile.

Finite Element Model

The goal of the FEM analyses was to determine the changes in the distribution of stress on the contact point of entry's environment on the driving gear while changes were made on driven gears tooth profile, namely the tip relief modification. For the purpose of the analysis, static 2D plane stress was assumed. During the preprocessing part of the analysis, only the midplane surfaces were kept with the original gear thickness. During the studies, the connection of one gear pair was thoroughly analysed, the rest of the gear teeths were kept on each gear to take the stiffening effect of the neighboring teeth into account. In the geometric preparation, each individual tooth was separated from each other with splitting, so multibody parts were created. Because of the need for mesh refinement, the connection region of the analysed gear pairs was separated from the part.

The preprocessing of the CAD models can be seen in figure 7.

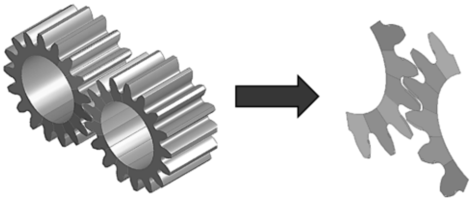


Figure 7. Preprocessing of the CAD models

The applied material for the analysed gears was 16MnCr5 with linear, isotropic, elastic parameters. Thus the linear-elastic study of the stiffness-increasing effect of the heat treatment process was omitted. The material properties can be seen in the following table (Table 2.).

Table 2. Linear, elastic material properties for 16MnCr5

Material property	Value of
Young's Modulus [MPa]	210 000
Poisson's ratio [-]	0.3

Since the friction coefficient is specific to any given gear-pair, a frictionless contact was used between the connecting tooth surfaces in order to generalize the problem definition. The contact between the analysed gear pair was calculated using the Augmented Lagrange method. As previously mentioned, the contact region of the surfaces of the analysed gears was separated, the connection between the teeth body and the meshed region was defined as bonded contact with MPC calculation.
The mesh was constructed using primarily second-order quadrilateral elements, with less than 2 [%] of the mesh consisting of second-order triangular elements. Based on previous studies (Schmidt 2019), the global element size for the gear geometries was selected to 0.1 [mm]. In order to determine the gear root stresses precisely, half of the global element size was set to 0.05 [mm] for the gear roots. In the meshing region of the analysed gear pairs, mesh refinement was used according to the results of evaluated mesh independence studies. The contact region's element size was selected to be 0.006 [mm], two orders of magnitude smaller than the global mesh size to precisely determine the Hertzian contact pressure.
The main parameters of the used FEM mesh are shown in the following table (Table 3.).

Table 3. Main parameters of the FEM mesh

Property	Value
Global element size [mm]	0.1
Element size at the gear root [mm]	0.05
Element size at meshing region [mm]	0.006
Number of nodes [-]	150-300 000
Number of elements[-]	50-100 000
Maximum Aspect ratio [-]	4.3

An example of the structure of the used FE mesh can be seen in Figure 8.

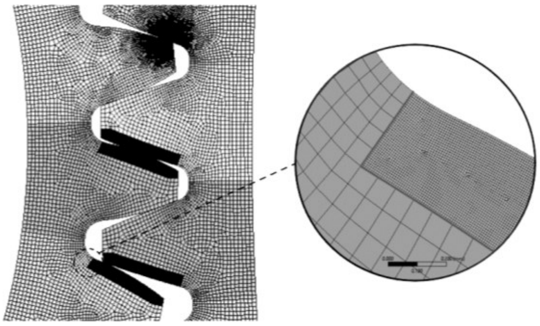


Figure 8. FE mesh

The body of the spur gears was fixed to the z-axis of their coordinate systems as remote points. Only free rotation in Z-axis was allowed. The load of the driving gear was applied to the inner surface of the spur gear. The magnitude of the torque was calculated from the allowable bending stress of the spur gear according to the following equation (Erney 1983):

$$T_{max} = \frac{m \cdot a \cdot b_1 \cdot \sigma_{Flim}}{2 \cdot K_A \cdot 2.7 \cdot 10^6 \cdot (u+1)} \qquad (1)$$

, where:
T_{max} [Nm] is the maximum allowable torque
m [mm] is the module
a [mm] is the center distance
b_1 is the width of the driving gear
σ_{Flim} is the allowable bending stress of the spur gear
K_A [-] is the operating factor
u [mm] is the gear ratio

The boundary conditions and the applied load of the gear pairs can be seen in Figure 9.

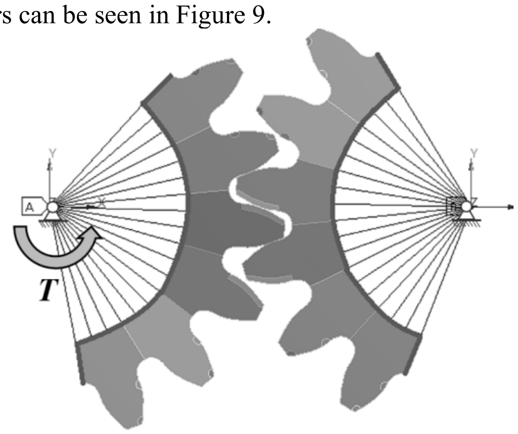


Figure 9. Boundary conditions and applied load

RESULTS

The total displacements showed the expected results, namely, the maximum displacement was located at the tip of the gear tooth where the maximum value was 2 [mm]. The resulted equivalent surface stress field of gear pairs was expected from photoelastic studies. Minimum principal stress was calculated, and the maximum value of the stress was the same as analytically calculated Hertzian pressure.

In Figures 10. and 11., an example for the surface stress field can be seen.

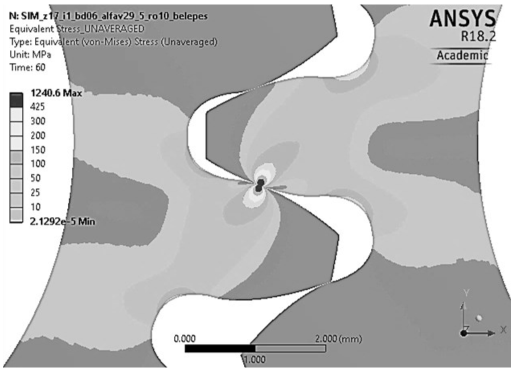


Figure 10. Equivalent stress field of the gear pair

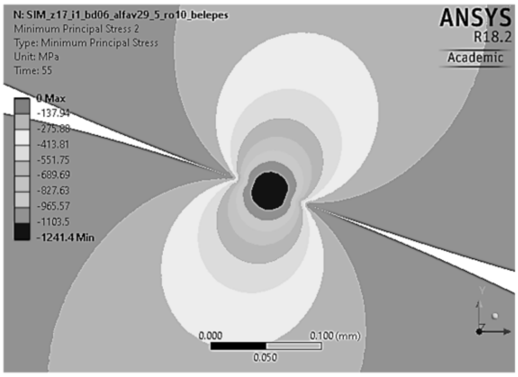


Figure 11. Minimum principal stress field of the gears

Validation

The magnitude of the torque was calculated from the allowable bending stress of the spur gear according to the following equation (Erney 1983):

$$\sigma_{H\_B} = \sqrt{\frac{1}{\pi} \cdot w_n \cdot \left(\frac{1}{\rho_{1,B}} + \frac{1}{\rho_{2,B}}\right) \cdot E_e} \qquad (2)$$

, where:
σ_{H_B} [MPa] is the contact surface stress at the beginning of the single teeth mesh point (point B)
w_n [N/mm] is the line of pressure
E_e [MPa] is the equivalent of Young's modulus
$\rho_{1,B}$ [mm] is the equivalent radius of the driving gear curvature
$\rho_{2,B}$ [mm] is the equivalent radius of the driven gear curvature

In the validation process, the analytically calculated surface contact stress at the beginning of the single teeth meshing (point B, see eq. 2.) were compared to the numerically calculated minimal principal stress (FEM simulations) at the same meshing point. The results show that the difference between the analytically and numerically calculated surface stress at the meshing point B differ by a maximum of 7 [%] from each other. This concludes that the maximum contact stress at the beginning of the gear contact can be validated.

Table 4. Analytically and numerically calculated surface contact stresses

z_1 $[-]$	i $[-]$	z_2 $[-]$	σ_{H-B} $[MPa]$	$\sigma_{H-B}{}^{FEM}$ $[MPa]$	$\sigma_{H-A}{}^{FEM}$ $[MPa]$
17	1	17	1350	1264	1863
17	4	68	1157	1087	2506
17	6	102	1129	1063	1815
30	1	30	990	933	1892
30	4	120	810	769	1345
30	6	180	786	745	943
40	1	40	855	803	1257
40	4	160	692	654	1366
40	6	240	670	630	1716

Summary

Based on the above-mentioned results, the conclusion was reached that the distribution of load, the noise of the gear system, and the life expectancy of the component are greatly influenced by the modification of the teeth profile. In order to reduce the amount of stress present on the teeth surface at the moment of connection, tip relief is recommended. Of all the different types of tip reliefs that are available, the tip relief modification made by the adjustments of the rack-cutter profile had the most promising results, while the linear profile modification only delayed the emergence of the maximum stress point. The teeth modification made by the linear profile modification does not reduce the stress present on the gear teeth' surface but delays its emergence. With the use of rack-cutter modification, the maximum surface stress (at the gear entry point location *A*) could be reduced up to 50[%], compared to the original shape or tip relief made with chamfer. Increasing the gear ratio reduces the stress at the instantaneous point of contact. By increasing the number of teeth on the driven gear, the teeth profile has a straighter form which causes the growth of the gear root that reduces the amount of stress on the teeth surface.

The comparison of the original gear and chamfer and tool profile modification tip relief can be seen in the following figure (Figure 12.).

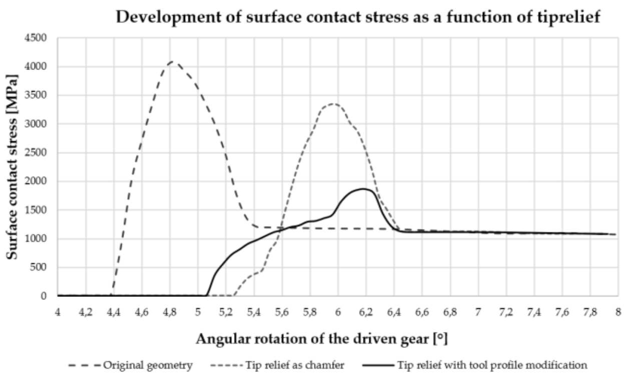


Figure 12. Development of surface contact stress as a function of tip relief

DISCUSSION

The profiles of the tip reliefs examined in our study are partially accurate because of the CAD program. In some cases, a few hundredths of a millimeter difference occurred between the gear profiles' exact positions. The tip relief's spline profile could have been better refined during the analyses, which caused a minimal amount of difference between them. Our examinations would be more precise with further refining of the modification of the tip edge in the simulation program. The amount of time required to run a simulation could be reduced by making submodels that have fewer calculations because of the fewer mesh points. In this study, only connections without backlash were analysed, but it would be recommended to analyze the effect of the gear pair backlash on the stress formation. Thus the stiffness-increasing effect of the heat treatment process and different backlash types should be taken into account.

Due to the short time available for research, only the base problems could be studied. In the future, we would like to continue our research in this field to take a deeper look at the gear optimization possibilities.

ACKNOWLEDGMENT

Hereby, the authors would like to express my thanks to Bence Schmidt, a mechanical engineer and a former student, who contributed hugely to this research by his master's thesis and work.

The research reported in this paper and carried out at BME has been supported by the NRDI Fund (TKP2020 NC, Grant No. BME-NCS) based on the charter of bolster issued by the NRDI Office under the auspices of the Ministry for Innovation and Technology

REFERENCES

György Erney, 1983. , “Fogaskerekek.”, *Műszaki Könyvkiadó*, Budapest, 1983

Li, X., Wang, N., & Lv, Y. 2016. "Tooth Profile Modification and Simulation Analysis of Involute Spur Gear". *International Journal of Simulation Modelling*, vol. 15, pp. 649-662.

Beghini, M., Presicce, F., & Santus, C., 2005. "Proposal for Tip Relief Modification to reduce Noise in Spur Gears and sensitivity to meshing conditions". *Gear Technology,* vol. 23.

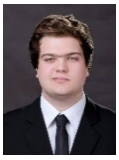
Gonzalez-Perez, I., Roda-Casanova, V. & Fuentes, A., 2015. "Modified geometry of spur gear drives for compensation of shaft deflections". *Meccanica 50*, pp. 1855–1867

DIN ISO 21771:2014-08, Zahnräder - Zylinderräder und Zylinderradpaare mit Evolventenverzahnung - Begriffe und Geometrie (ISO 21771:2007)

DIN 3972:1952-02, Bezugsprofile von Verzahnwerkzeugen für Evolventen-Verzahnungen nach DIN 867

Bence Schmidt, 2019. "Influence of the bearing stiffness on the load distribution of spur gears." *Master's thesis*. (supervisor: Attila, Csobán PhD.)

AUTHOR BIOGRAPHIES



JAKAB MOLNÁR was born in Győr, Hungary, and went to the Budapest University of Technology and Economics, where he studied mechanical engineering and machine design and obtained his bachelor's degree in 2019. He continues his studies at Budapest University of Technology and Economics as a mechanical engineer and machine design master's student. His e-mail address is: molnar.jakab@gt3.bme.hu and his webpage can be found at: http://gt3.bme.hu

ATTILA CSOBÁN Assistant professor at Budapest University of Technology and Economics. Member of the Association of Hungarian Inventors since 2000. Member of the Entrepreneurship Council of the Hungarian Research Student Association since 2006.Member of the public body of the Hungarian Academy of Sciences (MTA) since 2012. Gold level member of the European Who is Who Association since 2013. Research field: gear drives, gearboxes, planetary gear drives, cycloidal drives. His email address is: csoban.attila@gt3.bme.hu, and his webpage can be found at: http://gt3.bme.hu

PÉTER T. ZWIERCZYK is an assistant professor at Budapest University of Technology and Economics Department of Machine and Product Design, where he received his M.Sc. degree and then completed his Ph.D. in mechanical engineering. His main research field is the railway wheel-rail connection. He is a member of the finite element modeling (FEM) research group. His email address is: z.peter@gt3.bme.hu, and his webpage can be found at: http://gt3.bme.hu