FROM STEADY-STATE AND DYNAMIC ANALYSIS TO ADAPTIVE CONTROL OF THE CSTR REACTOR

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ABSTRACT

Simulation is the technical discipline which shows the behavior and reactions of any system on its model. Most processes in nature have nonlinear properties and simulation is one way to examine behaviour of these systems. The behaviour is then obtained by steady-state and dynamic analysis of the model which is usually represented by a set of differential equations. The next step after dynamic analysis is choosing a suitable control strategy and finally designing a controller. The paper presents the progress from modeling and simulation of a nonlinear process represented by a continuous stirred tank reactor (CSTR) to adaptive control of the system based on polynomial synthesis with pole-placement method.

INTRODUCTION

Computer simulation is very often used at present as it has advantages over an experiment on a real system, which is sometimes not feasible and can be dangerous, or time and money demanding. A large amount of processes in chemical industry is nonlinear and computer simulation can help us to give us an insight into the system behaviour. The first step is usually introduction of a mathematical model (Ingham et al. 2000, Luyben 1989). Mathematical model can be described by a set of linear, nonlinear or differential equations.
A Continuous Stirred Tank Reactor (CSTR) is widely used for control because input flow of the reactant or cooling liquid can be easily controlled. From the system engineering point of view, CSTR belongs to the class of nonlinear systems with lumped parameters. Mathematical models of these reactors are described by a set of nonlinear ordinary differential equations (ODEs). In this paper the simple differential method (Luyben 1989) and the standard Runge-Kutta's method are used to solve the set of ODEs.
Simulation results are then used for control, in our case adaptive control (Åström 1989). The first step here is to find an appropriate control method. The polynomial approach (Kucera 1993) has satisfied control requirements and moreover, it could be used for systems with negative properties such as non-minimum phase behaviour and processes with time delays. This method connected with pole-placement fulfills stability, asymptotic tracking of the reference signal and compensation of disturbances.
Two control system configurations (Grimble 1994) wre used – the first with one degree-of-freedom (1DOF) which has regulator only in feedback part and the secont with two degrees-of-freedom (2DOF) with feedback and feedforward parts. Adaptivity was ensured by estimating model parameters with recursive least-squares method with directional forgetting (Kulhavy and Karny 1984). Since the model is nonlinear, an external linear continuous-time model (Wahlberg 1990) was used as an input-output description of the controlled system.
The goal of this work is to describe the progress from simulation of the behaviour to control of a nonlinear process represented by the CSTR reactor taken from (Chen *at al.* 1995).

THEORETICAL BACKGROUND

As it is written above, the goal of this paper is to describe the process from modeling and simulation to control of a dynamic process. This can be expressed for example as in Figure 1. There are four main steps connected mutually together.

1. Define goals

It is necessary to define goals of simulation study including collection of all the available knowledge and practical experience with the investigated type of the plant. Unfortunately, a major group of systems in nature are nonlinear or number of variables is very high. This complexity leads to introduction of some simplifications.

2. Modeling

Modeling is mostly the hardest step. A process is described by its characteristic variables (temperature, concentration, flow rate, pressure etc.) usually called *state variables* and mathematical relations between

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these quantities then form a *mathematical model*. The mathematical model comes from balances inside the plant and could be expressed by a set of linear or nonlinear equations, ordinary or partial differential equations etc.

3. Simulation and validity of the model

Simulation experiment results are good ways how to test suitability of the obtained mathematical model. They usually consist of two parts – static and dynamic analysis. The static analysis results in appropriate working point. On the other hand, the dynamic analysis provides step, frequency responses etc. which display dynamic behaviour of the system and they are a base for choosing an external linear model. Comparison between outputs from real system and simulated outputs demonstrate validity of mathematical model. If the difference between the real system and the model is unacceptable, it is necessary to jump back to modeling and cancel some simplifications. The goal is to find the simplest model with satisfactory description of the real process.

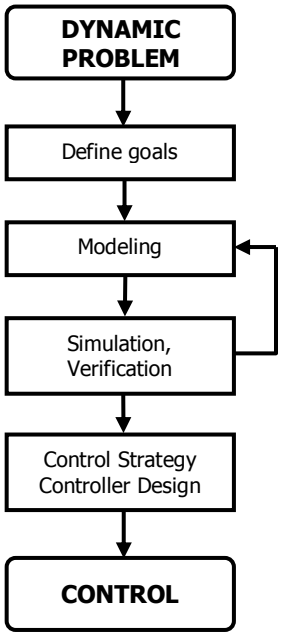

Figure 1: Steps from simulation to control of a dynamic system

4. Control strategy and controller design

The next step after simulation is to look on the problem from the control point of view. That means to find a regulator controling our dynamic process in a defined way. The obtained regulator must e.g. fulfill requirements of stability, properness, asymptotic tracking of the reference or disturbance attenuation. There are lot of ways for designing such a controller. Configurations with one degree-of-freedom (1DOF) and two degrees-of-freedom (2DOF) and their combinations with polynomial methods satisfy control conditions mentioned above. As it was shown (Dostal et al. 2002), these methods can by applied on systems with negative control properties, such as instability, nonminimum phase or on systems with transport delays.
Obtained regulators are then subject to simulation experiments and if results are acceptable, they can be used to control dynamic problem.

APPLICATION ON REAL SYSTEM

The procedure which is described in previous chapter was applied on real nonlinear process represented by a Continuous Stirred Tank Reactor (CSTR).

1. Define goals

The first goal of the simulation study is to find the best working-point for control study, where the production is best balanced – i.e. obtain the biggest production with the smallest energy usage. An External Linear Model (ELM) is the second goal and we obtain it from dynamic study. We presume that both reactant and cooling liquid are perfectly mixed and volumes, densities, heat capacities of reactant and cooling liquid are supposed to be constant.

2. Modeling

The examined reactor has real background and graphical diagram of the CSTR reactor is shown in Figure 2.

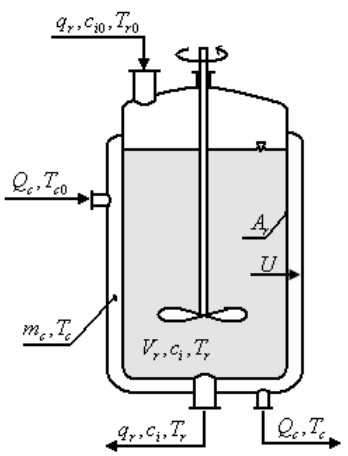

Figure 2: Continuous Stirred Tank Reactor (CSTR)

The reaction inside the reactor is called *van der Vusse reaction.* This reaction can be described by the following scheme:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$
$$2A \xrightarrow{k_3} D$$

The mathematical model of this reactor comes from balances inside the reactor. The first two equations come from material balances of compounds A and B:

$$\frac{dc_A}{dt} = \frac{q_r}{V_r}\left(c_{A0} - c_A\right) - k_1 c_A - k_3 {c_A}^2 \quad (1)$$

$$\frac{dc_B}{dt} = -\frac{q_r}{V_r} c_B + k_1 c_A - k_2 c_B \quad (2)$$

Where $c_A \geq 0$, $c_B \geq 0$. The next balances represent heat balances of the reactive compound and cooling liquid

$$\frac{dT_r}{dt} = \frac{q_r}{V_r}\left(T_{r0} - T_r\right) - \frac{h_r}{\rho_r c_{pr}} + \frac{A_r U}{V_r \rho_r c_{pr}}\left(T_c - T_r\right) \quad (3)$$

$$\frac{dT_c}{dt} = \frac{1}{m_c c_{pc}}\left(Q_c + A_r U\left(T_r - T_c\right)\right) \quad (4)$$

In equations (1) – (4) t is the time, c are concentrations, T represents temperatures, c_p is used for specific heat capacities, q represents volumetric flow rate, Q_c is heat removal, V are volumes, ρ represents densities, A_r is the heat exchange surface and U is the heat transfer coefficient. Indexes $(\cdot)_A$ and $(\cdot)_B$ belong to compounds A and B, $(\cdot)_r$ denotes the reactant mixture, $(\cdot)_c$ cooling liquid and $(\cdot)_0$ are feed (inlet) values.

Table 1: Parameters of the reactor

$k_{01} = 2.145 \cdot 10^{10}$ min^{-1}	$k_{02} = 2.145 \cdot 10^{10}$ min^{-1}
$k_{03} = 1.5072 \cdot 10^{8}$ min^{-1}mol^{-1}	E_1/R = 9758.3 K
E_2/R = 9758.3 K	E_3/R = 8560 K
h_1 =-4200 kJ.kmol^{-1}	h_2 = 11000 kJ.kmol^{-1}
h_3 = 41850 kJ.kmol^{-1}	
V_r = 0.01 m^3	ρ_r = 934.2 kg.m^{-3}
c_{pr} = 3.01 kJ.kg^{-1}.K^{-1}	c_{pc} = 2.0 kJ.kg^{-1}.K^{-1}
U = 67.2 kJ.min^{-1}m^{-2}K^{-1}	A_r = 0.215 m^2
c_{A0} = 5.1 kmol.m^{-3}	c_{B0} = 0 kmol.m^{-3}
T_{r0} = 387.05 K	m_c = 5 kg

The set of ordinary differential equations (ODE) then together with simplifications described in the previous part then mathematically represents examined CSTR reactor. The model of the reactor belongs to the class of *lumped-parameter nonlinear systems*. Nonlinearity can be found in reaction rates (k_j) which are described via Arrhenius law:

$$k_j\left(T_r\right) = k_{0j} \cdot \exp\left(\frac{-E_j}{RT_r}\right), \text{ for } j = 1,2,3$$

where k_0 represent pre-exponential factors and E are activation energies.

The reaction heat (h_r) in equation (3) is expressed as:

$$h_r = h_1 \cdot k_1 \cdot c_A + h_2 \cdot k_2 \cdot c_B + h_3 \cdot k_3 \cdot c_A^2$$

where h_i means reaction enthalpies.

Parameters of the reactor are given in Table 1 (Chen *at al.* 1995).

3. Simulation and validity of the model

Two simulation studies were performed – steady-state analysis and dynamic analysis.

Steady-state analysis

Steady-state analysis for stable systems involves computing values of state variables in time $t \rightarrow \infty$, when changes of these variables are equal to zero. That means, that the set of ODEs (1) – (4) is solved with the condition $\partial(\cdot)/\partial t = 0$. A simple iteration method was used to solve this problem. Steady values of quantities were examined for various rates of the volumetric flow of the reactant, q_r, and heat removal of the cooling liquid, Q_c. The first study for volumetric flow rate of the reactant q_r [m^3.min^{-1}] = <5·10^{-4}; 3·10^{-2}> and heat removal Q_c = -18.56 kJ.min^{-1} displayed in Figure 3 shows strong nonlinear course of steady-state values of both concentrations c_A and c_B. Maximal concentration of the product, c_B = 1.09 kmol.m^{-3}, was obtained for flow rate q_r = 2.365·10^{-3} m^3.min^{-1}. Curves of steady-state temperatures T_r and T_c are displayed in Figure 4.

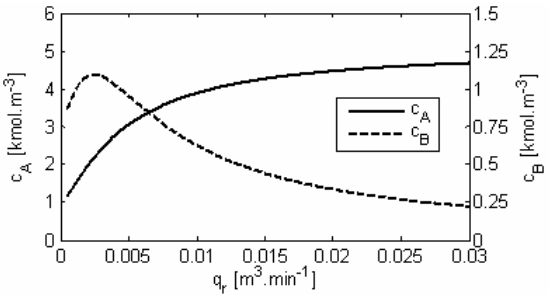

Figure 3: Steady-state values of concentrations c_A and c_B for various flow rate q_r

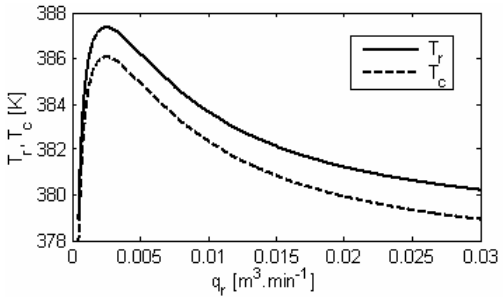

Figure 4: Steady-state values of temperatures T_r and T_c for various flow rate q_r

Simulation for different heat removal of cooling liquid Q_c [kJ.min^{-1}] = <-500; 500> and flow rate q_r = 2.365·10^{-3} m^3.min^{-1} has similar nonlinear behaviour as in the previous study – see Figure 5. This course has again a maximal value of product's concentration, c_B, in this case, the maximal concentration is achieved approximately for heat removal Q_c = -18.56 kJ.min^{-1}. Figure 6 shows steady values of temperatures T_r and T_c.

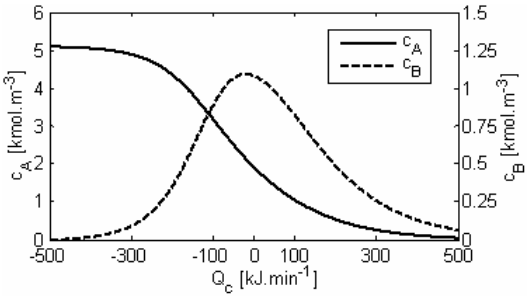

Figure 5: Steady-state values of concentrations c_A and c_B for various heat removal Q_c

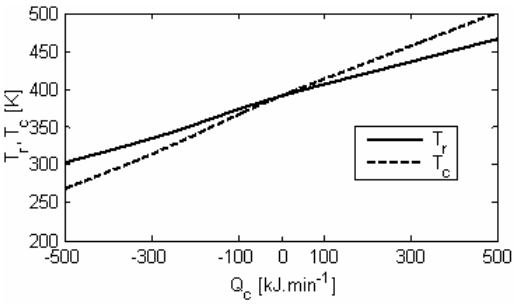

Figure 6: Steady-state values of temperatures T_r and T_c for various heat removal Q_c

The goal of the steady-state study is to find an optimal working point, where the product's concentration is maximal and this point was found for volumetric flow rate q_r^s = $2.365 \cdot 10^{-3}$ $m^3.min^{-1}$ and heat removal Q_c^s = -18.56 $kJ.min^{-1}$. Steady values of the quantities are following

$$c_A^s = 2.1403\ kmol.m^{-3} \quad c_B^s = 1.0903\ kmol.m^{-3}$$
$$T_r^s = 387.3397\ K \quad T_c^s = 386.0551\ K \tag{5}$$

Dynamic analysis

The next step after steady-state study is dynamic study. For this nonlinear lumped-parameter system dynamic analysis involves solving the set of nonlinear ODE. Computed steady-state values of (5) are used as an input for the dynamic study and Runge-Kutta's standard method with a fixed step was used for solving equations (1) to (4). The behaviour of the system was obtained after a step change of the input quantities Δq_r and ΔQ_c. Simulation took 30 minutes and the integration step was set to 0.1 min. Output step responses of the values y_1 and y_2 in next figures illustrate the difference of variables c_B and T_r from their steady state values c_B^s and T_r^s, i.e.

$$y_1 = c_B - c_B^s; \quad y_2 = T_r - T_r^s$$

The first study for four values of step changes of volumetric flow rate Δq_r [$m^3.min^{-1}$] = $-1.18 \cdot 10^{-3}$(-50% of its steady-state value q_r^s), $-0.59 \cdot 10^{-3}$(-25%), $0.59 \cdot 10^{-3}$(25%), $1.18 \cdot 10^{-3}$(50%) shows that both outputs y_1 and y_2 has negative properties from the control point of view – non-minimum phase behaviour and the changing sign of gain. Step responses of these output variables are displayed in Figure 7 and Figure 8.

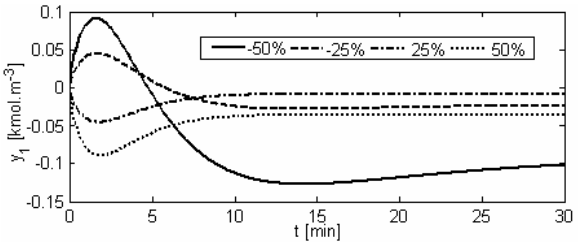

Figure 7: Course of output concentration c_B (y_1) for various step changes of flow rate Δq_r

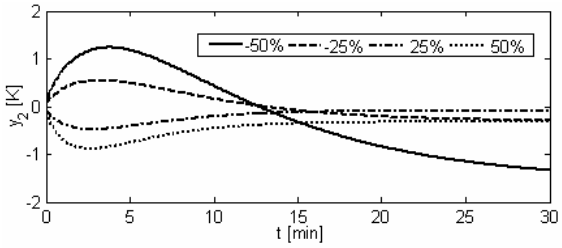

Figure 8: Course of output temperature T_r (y_2) for various step changes of flow rate Δq_r

Figure 9 shows step responses for different step changes of cooling heat removal ΔQ_c [$kJ.min^{-1}$] = 9.28 (-50% of its steady value Q_c^s), 4.64 (-25%), -4.64 (25%) and -9.28 (50%).

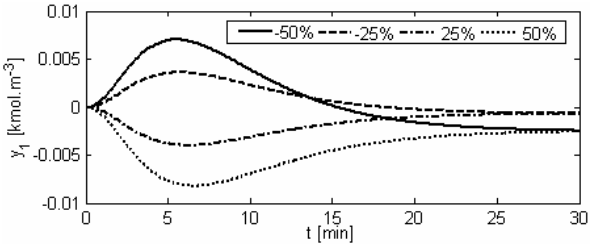

Figure 9: Course of output concentration c_B (y_1) for various step changes of heat removal ΔQ_c

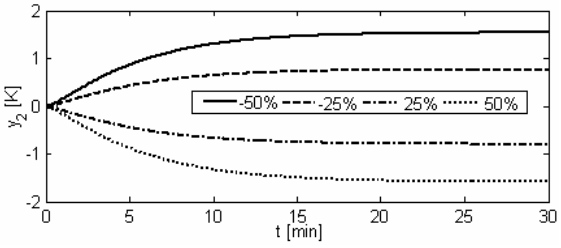

Figure 10: Course of output temperature T_r (y_1) for various step changes of heat removal ΔQ_c

Again, the output product concentration c_B (output y_1) has non-minimum phase behaviour and changes the sign of gain.

On the other hand, the output reactive temperature T_r (output y_2) in Figure 10 could be expressed with a second-order linear model. This output was used as the controlled variable in the control section and a change of the heat removal was considered as a manipulated variable.
We did not have a real model of this plant at our disposal, there was no chance to check the validity of the model and we supposed that the model is satisfactory.

4. Control strategy and controller design

The adaptive control with usage of polynomial synthesis and pole-placement method was used for control of this reactor. This method could be used for systems with negative control properties such as non-minimum phase behaviour or transport delay.

External continuous-time linear model
We used a continuous-time (CT) external linear model (ELM) as a description or the relation between the output and the input signals. The CT model is in time domain generally described by a differential equation

$$a(\sigma)y(t) = b(\sigma)u(t) \tag{6}$$

where $\sigma = d/dt$ is the differentiation operator and if initial conditions are equal to zero it si possible to transform equation (6) to the complex domain using the Laplace transform (L-transform):

$$a(s)Y(s) = b(s)U(s) + o_1(s) \tag{7}$$

with s as a complex variable and $o_1(s)$ denoting initial conditions.
Polynomials $a(s)$ and $b(s)$ have the following form

$$a(s) = \sum_{i=0}^{\deg a} a_i s^i,\ b(s) = \sum_{j=0}^{\deg b} b_j s^j$$

If the initial values are equal to zero, we can introduce a transfer function

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b(s)}{a(s)} \tag{8}$$

This transfer function must be proper, i.e.

$$\deg b \le \deg a \tag{9}$$

Parameter estimation
It is clear that the equation (6) could not be used directly for parameter estimation because of derivatives of the input and output variables. It is necessary to introduce filtered variables u_f and y_f which are computed from differential equations

$$\begin{aligned} c(\sigma)u_f(t) &= u(t) \\ c(\sigma)y_f(t) &= y(t) \end{aligned} \tag{10}$$

where $c(\sigma)$ is a stable polynomial which satisfies condition $\deg c \le \deg a$.
The L-transform of (10) to complex domain yields

$$\begin{aligned} c(s)U_f(s) &= U(s) + o_2(s) \\ c(s)Y_f(s) &= Y(s) + o_3(s) \end{aligned} \tag{11}$$

where o_1, o_2 are polynomials which take into account initial conditions of filtered variables. If we substitute the equation (11) into the equation (7), the result is

$$Y_f(s) = \frac{b(s)}{a(s)}U_f(s) + \Psi(s)$$

Where $\Psi(s)$ is a rational function in s which takes into account initial conditions of filtered and non-filtered variables.
Dynamics of filters (10) must be faster then dynamics of the controlled process. The selection of filter parameters is problematic and it is closely connected with the knowledge about the system behaviour. If we do not have any knowledge about the system it is good to set these parameters a priory small.
Values of filtered variables were acquired in discrete time moments $t_k = k \cdot T_v$ for $k = 0,1,2 \ldots$, where T_v denotes a sampling period and the regression vector for $n = \deg a$ and $m = \deg b$ has the following form

$$\boldsymbol{\varphi}^T(t_k) = \left[-y_f(t_k), -y_f^{(1)}(t_k), \ldots, -y_f^{(n-1)}(t_k), u_f(t_k), u_f^{(1)}(t_k), \ldots, u_f^{(m)}(t_k), 1\right]$$

The vector of parameters is

$$\boldsymbol{\theta}^T(t_k) = \left[a_0, a_1, \ldots, a_{n-1}, b_0, b_1, \ldots, b_m\right]$$

Then, parameters of polynomials a and b can be estimated in discrete time intervals from

$$y_f^{(n)}(t_k) = \boldsymbol{\theta}^T(t_k) \cdot \boldsymbol{\varphi}(t_k) + \Psi(t_k)$$

Adaptivity of the process is fulfilled by recursive parameter estimation during the control. Recursive Least Squares (RLS) method with direct forgetting (Kulhavy and Karny 1984) was used for this parameter estimation. The discrete version of the algorithm could be described by following equations:

$$\varepsilon(k) = y(k) - \boldsymbol{\varphi}^T(k) \cdot \boldsymbol{\theta}(k-1)$$

$$r(k-1) = \boldsymbol{\varphi}^T(k) \cdot \mathbf{P}(k-1) \cdot \boldsymbol{\varphi}(k)$$

$$\boldsymbol{L}(k) = \frac{\mathbf{P}(k-1) \cdot \boldsymbol{\varphi}(k)}{1 + r(k-1)}$$

$$\gamma(k) = \frac{1}{\lambda_1(k-1)/\lambda_2 + \boldsymbol{\varphi}^T(k) \cdot \mathbf{P}(k-1) \cdot \boldsymbol{\varphi}(k)}$$

$$\beta(k-1) = \begin{cases} \lambda_1(k-1) - \dfrac{1-\lambda_1(k-1)}{r(k-1)} & \text{for } r(k-1) > 0 \\ 1 & \text{for } r(k-1) = 0 \end{cases}$$

$$\mathbf{P}(k) = \mathbf{P}(k-1) - \frac{\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}(k)\cdot\boldsymbol{\varphi}^T(k)\cdot\mathbf{P}(k-1)}{\beta(k-1)^{-1} + r(k-1)}$$

where λ_1 is the forgetting factor computed e.g. via

$$\lambda_1(k) = 1 - K\cdot\gamma(k)\cdot\varepsilon^2(k)$$

for a small value of the constant K.

Control system configuration

Two control system configurations were adopted. The first 1DOF (one degree-of-freedom) configuration displayed in Figure 11 has a regulator only in the feedback part. On the other hand, configuration with two degrees-of-freedom (2DOF) has both feedback and feedforward parts – see Figure 12.

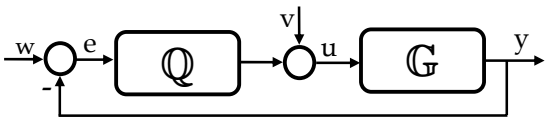

Figure 11: 1DOF control scheme

In both schemes G is an approximate transfer function from (8), Q is feedback and R feedforward part of the controller. The signal w is the reference signal, u is a control variable, e is an error, y represents an output variable and v is disturbance in the input to the system.

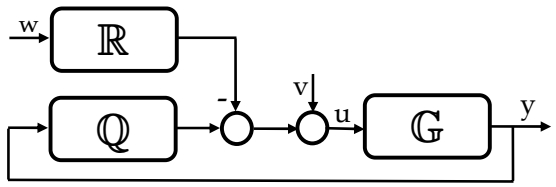

Figure 12: 2DOF control scheme

Both, control variable u and disturbance v are considered from the ring of step functions because of simplicity. For step chanted the transfer functions of the feedback and feedforward parts include integration component $1/s$:

$$Q(s) = \frac{q(s)}{s\cdot p(s)}, R(s) = \frac{r(s)}{s\cdot p(s)} \qquad (12)$$

where q, p and r are polynomials in s. Algebraic methods were used for computation of coefficients of the polynomials in (12). The first diophantic equation

$$a(s)\cdot s\cdot p(s) + b(s)\cdot q(s) = d(s) \qquad (13)$$

is used for computation of coefficients of the polynomials $p(s)$ and $q(s)$ and coefficients of the polynomial $r(s)$ in the feedforward part are obtained from the second diophantic equation

$$t(s)\cdot s + b(s)\cdot r(s) = d(s) \qquad (14)$$

where $t(s)$ is an additive stable polynomial with arbitrary coefficients, because these coefficients are not used for computation of the coefficients of the polynomial $r(s)$. In both equations, $d(s)$ on the right side is a stable polynomial. The feedback regulator $R(s)$ ensures stability, load disturbance attenuation for both configurations and asymptotic tracking for 1DOF configuration. On the other hand, feedforward part $Q(s)$ ensures asymptotic tracking in 2DOF configuration.

A demand for a stable controller is fulfilled if the polynomial $p(s)$ in the denominators of (12) is stable. Inner properness holds if all transfer functions are proper. Transfer functions of the feedforward and feedback parts (12) are proper if

$$\deg q \le \deg p + 1, \deg r \le \deg p + 1 \qquad (15)$$

Degrees of the polynomials p, q and r are computed with respect to conditions (9), (15) and solubility of diophantic equations (13) and (14) as follows

$$\deg q = \deg a, \deg p \ge \deg a - 1, \deg r = 0 \qquad (16)$$

Roots of the polynomial $d(s)$ on the right side of equations (13) and (14) are poles of the closed-loop and the control quality is determined by the placement of these poles. A method, where poles were connected to parameters of the controlled system was used to set poles of the characteristic polynomial. Then, the polynomial $d(s)$ could be rewritten for aperiodical processes to the form

$$d(s) = n(s)\cdot(s+\alpha)^{\deg d - \deg n}$$

for $\alpha > 0$ be an optional coefficient reflecting closed-loop poles and stable polynomial $n(s)$ is obtained from the spectral factorization of the polynomial $a(s)$

$$n^*(s)\cdot n(s) = a^*(s)\cdot a(s)$$

Simulation experiment

Changes of the heat removal were used as manipulated variable and the controlled output was the temperature of the reactant:

$$y(t) = T_r(t) - T_r^s(t)\,[K]; u(t) = 100\cdot\frac{Q_c(t) - Q_c^s(t)}{Q_c^s(t)}\,[\%]$$

Dynamic analysis of the output temperature shows that ELM could be expressed by a second order system with the relative order one described in the continuous-time form by the differential equation

$$y^{(2)}(t) + a_1 y^{(1)}(t) + a_0 y(t) = b_1 u^{(1)}(t) + b_0 u(t)$$

And in the complex s-plane as the transfer function

$$G(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \qquad (17)$$

Filtered variables u_f and y_f were computed from differential equations similar as in (10)

$$y_f^{(2)}(t) + c_1 y_f^{(1)}(t) + c_0 y_f(t) = y(t)$$
$$u_f^{(2)}(t) + c_1 u_f^{(1)}(t) + c_0 u_f(t) = u(t)$$

where parameters c_1 and c_0 were chosen equal to c_1 = 1.4 and c_0 = 0.49. The recursive least squares method with directional forgetting was used for parameter estimation. The regression vector and vector of parameters for ELM (17) have the form

$$\boldsymbol{\varphi}^T(t_k) = \left[-y_f(t_k), -y_f^{(1)}(t_k), u_f(t_k), u_f^{(1)}(t_k)\right]$$
$$\boldsymbol{\theta}^T(t_k) = \left[a_0, a_1, b_0, b_1\right]$$

Starting values for the RLS method with direct forgetting were following: the vector of parameters $\boldsymbol{\theta}^T(k)$ = [0.1,0.1,0.1,0.1]; the covariance matrix $\boldsymbol{P}(0)$ with the dimension 4 x 4 has 10^{-6} on the diagonal; a starting value of the forgetting factor was $\lambda_1(0) = 0.95$ and further, $\varepsilon(0) = 0$, $\gamma(0) = 0$. Degrees of polynomials of the feedforward $R(s)$ and feedback $Q(s)$ parts of the controller were computed from the transfer function (17) via (16): deg q = 2, deg p = 1, deg r = 0 and then, transfer functions were

$$\tilde{Q}(s) = \frac{q_2 s^2 + q_1 s + q_0}{s \cdot (p_1 s + p_0)},\ \tilde{R}(s) = \frac{r_0}{s \cdot (p_1 s + p_0)}$$

Coefficients of polynomials p, q and r were computed from diophantic equations (13) and (14) where polynomial $d(s)$ was of the fourth degree

$$d(s) = n(s) \cdot (s + \alpha)^2$$

With parameter $\alpha > 0$. Coefficients of the polynomial $n(s)$ are computed via

$$n_0 = \sqrt{a_0^2},\ n_1 = \sqrt{a_1^2 + 2n_0 - 2a_0}$$

The simulation took 300 min and three changes of the reference value were made during this time. The reference value was $w(t)$ = 2 K in the time domain $0<t<100$ min, $w(t)$ = -1 K on the interval $100<t<200$ min and $w(t)$ = 1 K in the last interval $200<t<300$ min. A proportional regulator, with gain k_p = -1, was used for first 15 steps because of identification, which does not work well with oscillating values in the beginning.

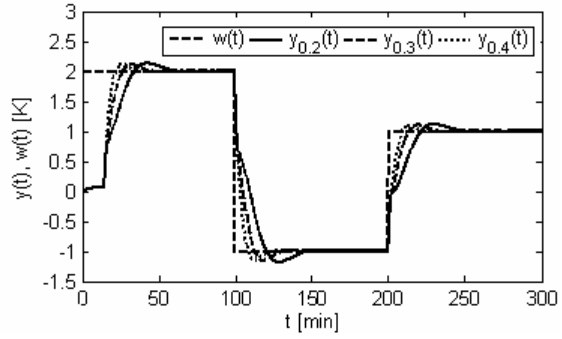

Figure 13: Course of output $y(t)$ for 1DOF set-up and different values of the parameter α = 0.2; 0.3 and 0.4

The sampling period for the identification and the control action value was T_v = 0.5 min. The manipulated variable was limited to the interval $-75\% \le u(t) \le 75\%$.

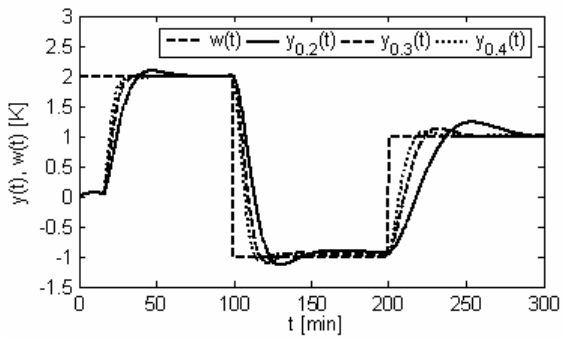

Figure 14: Course of output $y(t)$ for 2DOF set-up and different values of the parameter α = 0.2; 0.3 and 0.4

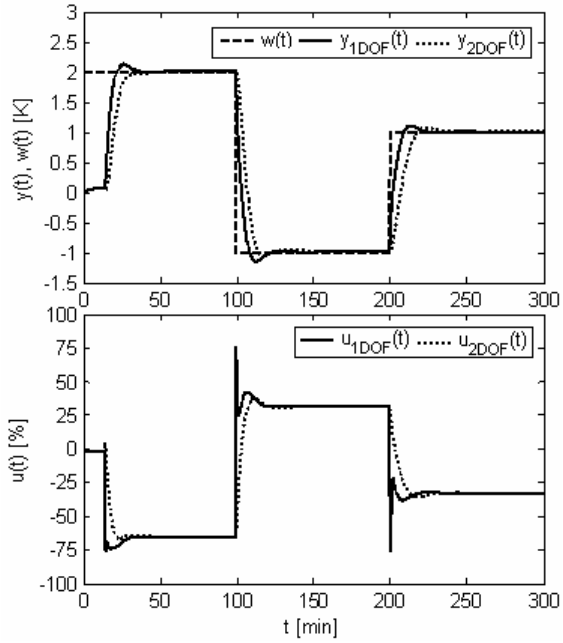

Figure 15: Comparison of outputs $y(t)$ and action values $u(t)$ for 1DOF and 2DOF set-ups with root α = 0.4

Figure 13 and Figure 14 show results for 1DOF and 2DOF configuration and different parameters α. As it can be seen in both figures, this parameter affects mainly the overshoot and speed of the control. With increasing value of parameter α the output response is quicker and the overshoot smaller in this case.

Figure 15 compares results for 1DOF and 2DOF configuration for the same placement of the parameter α = 0.4. The main advantages of 2DOF configuration are lower overshoots and smoother changes of action value. Parameter estimation during the simulation for 1DOF configuration displayed in Figure 16 shows that the chosen RLS method with direct forgetting has a problem only in the beginning of the simulation because of small amount of information.

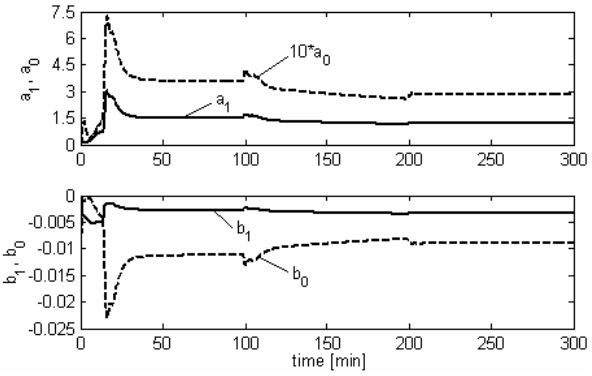

Figure 16: Course of estimated parameters a_0, a_1, b_0 and b_1 for 1DOF configuration and root $\alpha = 0.4$

CONCLUSION

The paper shows methodology from simulation of a nonlinear process represented by the CSTR reactor to adaptive control of this system. Simulation of the steady-state reveals the ideal working point and dynamic analysis shows step responses for more input values. Proposed adaptive control based on polynomial synthesis and the pole-placement method provides good control results although the system has negative control properties such as non-minimum phase behaviour and changes the sign of gain. The control process has quicker slope and overshoots lower with increasing value of the parameter α, but increasing cannot be infinite because of limitation of the action variable. The main advantage of the 2DOF control configuration is represented by smaller overshoots and smoother changes of the action variables which is important from the practical point of view, where action value could be e.g. valve twist. Shock changes of the twist could destroy or damage the valve. The results demonstrate the usability of this control method, which is represented by a good quality and stability of the output response.

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