PORTFOLIO MODELLING USING THE THEORY OF COPULA IN LATVIAN AND AMERICAN EQUITY MARKET

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KEYWORDS

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ABSTRACT

Portfolio building is the most important process in investment management. In the world there are many methods which can help both to estimate the investment characteristics of a financial instrument and to build a portfolio. The overwhelming majority of them are based on the assumption that analyzed data are normally distributed. The theory of copula allows to study non-linear dependences between selected assets and to build unified distribution function based on the distribution functions of each asset. In the theory of copula the set of financial instruments can be considered as one asset. The authors of this paper analyzed the effectiveness of copula implementation in the financial markets represented by the American market and the Latvian equity market. The authors compared the cumulative returns of two portfolios formed by traditional Markowitz's approach and simulating copula. Different data time formats were used in this research.

INTRODUCTION

The problem of portfolio building is divided into two sub-problems:

1) the estimation of the investment characteristics of an asset;

2) forming of portfolio.

Many specialists work out and offer their own methods like digital portfolio theory which consider the asset movements as an electric signal. Other methods are the modifications of traditional Markowitz approach or Value-at-Risk methodology. Anyway, the overwhelming majority of these methods are based on the assumption that analyzed data are normally distributed. In fact, this assumption significantly simplifies calculations and gives accepTable results. In addition, traditional Markowitz's approach takes into account the correlation and linear dependence between the assets.

Modern data analysis techniques and the latest achievements of information technologies allow us to analyze the financial markets from another point of view. Now there are a lot of scientific papers which are the evident of new features of the financial markets. They show that the usage of normal distribution function in data analysis is not correct because the kurtosis and skewness of data distribution is too far from normal. Also the assets can have non-linear dependences between each other or their investment characteristics.

The theory of copula is one of the interesting methods which can facilitate to solve a part of the problems mentioned above. This theory allows us to analyze non-linear dependences between assets and to merge different distribution functions in one unified.

MARKOWITZ'S APPROACH

In 1952 year Harry Markowitz published his famous work named "Portfolio Selection". Markowitz's paper is the first mathematical formalization of the idea of diversification of investments: the financial version of "the whole is greater than the sum of its parts". Through diversification, risk can be reduced (but not generally eliminated) without changing expected portfolio return. Markowitz postulates that an investor should maximize expected portfolio return (μ_p) while minimizing portfolio variance of return (σ_p^2). Later it became a part of modern portfolio theory.

According to traditional approach worked out by Markowitz portfolio return can be calculated as

$$\mu_p = \sum_j w_j \mu_j \tag{1}$$

where the μ_j is expected security's return.

The variance of portfolio return can be calculated as

$$\sigma_p^2 = \sum_j w_j^2 \sigma_j^2 + \sum_j \sum_{k \neq j} w_j w_k \rho_{jk} \sigma_j \sigma_k \tag{2}$$

where the w_j are the portfolio proportions and ρ_{jk} is the pairwise correlation of the returns of securities j and k.

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THEORY OF COPULA

An n-dimensional copula is basically a multivariate cumulative distribution function with uniform distributed margins in [0, 1].

Sklar's Theorem

Let H denote a n-dimensional distribution function with margins $F_1, ..., F_n$. Then there exists a n-copula C such that for all real $(x_1, \ldots, x_n)$

$$H(x_1, ..., x_n)=C(F(x_1), ..., F(x_n)) \quad (3)$$

If all the margins are continuous, then the copula is unique. A copula is thus a function that, when applied to univariate marginals, results in a proper multivariate probability distribution function: since this probability distribution function embodies all the information about the random vector, it contains all the information about the dependence structure of its components. Using copulas in this way splits the distribution of a random vector into individual components (marginals) with a dependence structure (the copula) among them without losing any information. It is important to highlight that this theorem does not require F_1 and F_n to be identical or even to belong to the same distribution family.

Archimedean Copulas

In this paper the authors utilized the Archimedean copulas. The Archimedean copulas provide analytical tractability and a large spectrum of different dependence measure. These copulas can be used in a wide range of applications for the following reasons:

1. The many parametric families of copulas belonging to this class.

2. The great variety of different dependence structures.

3. The ease with which they can be constructed, and the nice properties possessed by the members of this class.

An Archimedean copula can be defined as follows: let us consider a function φ :[0; 1]→[0;1] which is continuous, strictly decreasing $\varphi'(u)<0$, convex $\varphi''(u)>0$, and for which $\varphi(0) = \infty$ and $\varphi(1)=0$. We then define the pseudo inverse of $\varphi^{[-1]}$: [0; ∞] → [0;1] such that:

$$\varphi^{[-1]}(t) = \begin{cases} \varphi^{-1}(t) & 0 \le t \le \varphi(0) \\ 0 & \varphi(0) \le t \le \infty \end{cases} \quad (4)$$

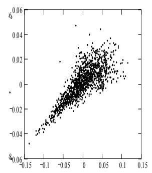
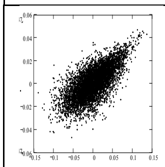
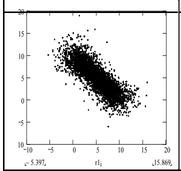
As φ is convex, the function C: $[0; 1]^2$ → [0;1] defined as

$$C(u_1, u_2)=\varphi^{-1}[\varphi(u_1)+\varphi(u_2)] \quad (5)$$

is an Archimedean copula and φ is called the "generator" of the copula.

In this paper the authors used three Archimedean copulas (see Table 1).

Table 1: Used bivariate copulas and generators

	**Clayton copula** $C(u_1,u_2) = \left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta}$ $\varphi(t) = \frac{t^{-\theta} - 1}{\theta}$ with $\theta \in (0, \infty)$
	**Gumbel copula** $C(u_1,u_2) = \exp\left\{-\left[(-\ln u_1)^{\theta} + (-\ln u_2)^{\theta}\right]^{1/\theta}\right\}$ $\varphi(t) = \left(-\ln t\right)^{\theta}$ with $\theta \geq 1$
	**Frank copula** $C(u,v) = -\theta^{-1}\ln(1 + \frac{g(u)\cdot g(v)}{g(1)})$ $\varphi_\theta(t) = -\ln\frac{g(t)}{g(1)}$

As mentioned above the traditional portfolio theory based on multivariate normal distribution assumes that investors can benefit from diversification by investing in assets with lower correlations.

Kendall's Rank Correlation

The pairwise correlation coefficient takes into account the linear dependence between the assets and also it is limited by some specific restrictions. In the theory of copula there is another correlation coefficient called Kendall's Tau. Kendall's rank correlation for the sample can be calculated by

$$\hat{\tau} = \binom{n}{2}^{-1} \sum_{i<j} sign\left[(X_i - X_j)(Y_i - Y_j)\right] \quad (6)$$

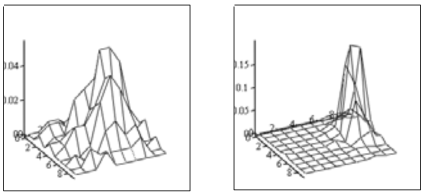
τ(X,Y) can be considered as measure of the degree of monotonic dependence between X and Y.

Table 2: Kendall's Tau for Some Copulas

	Kendall's Tau
Clayton copula	$\tau(\theta) = \frac{\theta}{2+\theta}$
Gumbel copula	$\tau(\theta) = 1 - \frac{1}{\theta}$
Frank copula	$\tau(\theta) = 1 - \frac{4}{\theta} + \frac{4}{\theta^2}\int_0^{\theta}\frac{t}{e^t - 1}dt$

Optimal Copula

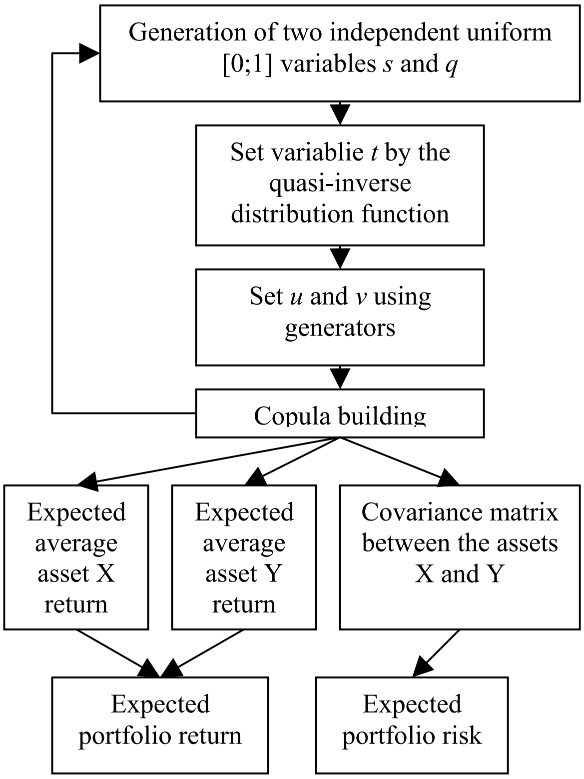
The main problem is how to choose the best copula from the set of estimated ones. The authors used the following method. Two density distributions are built: theoretical and empirical. If maximal distance between two densities is minimal for different copulas then this copula is the best one.



Figures 1: Two Distribution Densities: Theoretical (left) and Empirical (right)

THE PROCESS OF PORTFOLIO BUILDING BACED ON COPULA SIMULATING

Copula implementation in portfolio building is shown in the Figures 2.



Figures 2: Theory of Copula in Portfolio Building

According to this modelling process the assets returns and covariance matrix are calculated separately. In the case of traditional Markowitz's approach the asset return is calculated as simple average (see formula 1) and covariance matrix as product of correlation and standard deviations (see formula 2). In the case of modeled portfolio the asset return is calculated as simple average on x simulations based on the corresponding copula and covariance matrix is calculated as average covariance based on x simulations of non-linear dependences between two assets.

EMPERICAL RESULTS

The authors analyzed three stocks traded in the American stock market (COKE – Coca-Cola Bottling Co Consolidated, F – Ford Motor Co, Microsoft CP) and three more active stocks traded in the Latvian equity market (GZE1R – Latvian Gas, VNF1R – Ventspils Oil, GRD1R - Grindex). D1, W1 and M1 data represent the average daily, weekly and monthly prices, respectively over the corresponding time period. The general information about analyzed periods and test periods is shown in the Table 3.

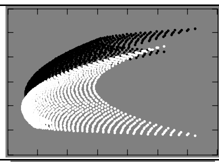
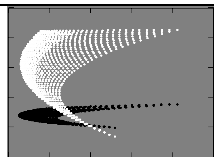
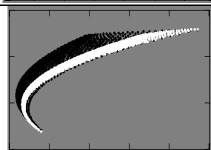
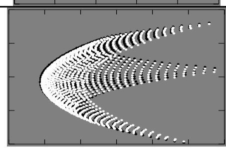
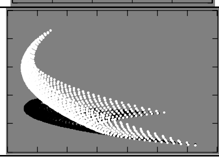
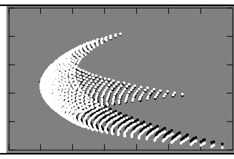
Table 3: Periods for Data Analyzing

Data	Analyzed period	Test period
D1	January 3, 2000 - December 31, 2003	January 3, 2004 – December 31, 2004
W1		
M1	January 3, 1990 - December 31	January 3, 2003 – December 31, 2004

Thus, the data of analyzed period are used for portfolio building. Then the cumulative returns of two portfolios are compared within the test period.

Using the formulas (1) and (2) it is very simple to calculate portfolio risk and return against its structure and then to build a portfolio set. In the same way another portfolio set, based on modelling, is built. In the Table 4 we can see the portfolio sets sorted by data time formats and financial markets.

Table 4: Behaviour of Two Portfolios (White – Modelled Portfolio, Black – Markowitz's Portfolio)

	American market	**Latvian market**
D1		
W1		
M1		

In the Table 6 we can see that modelled portfolio offers

1) less return and risk on D1 data, more risk on W1 data and more return and less risk on M1 data in the American market;

2) more return and risk on D1 data, practically identical return and risk on W1 and M1 data in the Latvian equity market.

To compare the effectiveness of two portfolio management it is necessary to choose an optimal portfolio from the optimal portfolio set. The authors used the tangent to the optimal portfolio set going from the point with zero risk and return. The structures of optimal portfolios are shown in the Table 5.

Table 5: The Structures of Two Portfolios

	COKE	**F**	**MSFT**
D1			
Traditional portfolio	46%	28%	26%
Modelled portfolio	54%	36%	10%
W1			
Traditional portfolio	44%	28%	28%
Modelled portfolio	52%	24%	24%
M1			
Traditional portfolio	70%	22% sell	8%
Modelled portfolio	74%	18% sell	8%
	GZE1R	**GRD1R**	**VNF1R**
D1			
Traditional portfolio	48%	26%	26%
Modelled portfolio	54%	28%	18%
W1			
Traditional portfolio	32%	38%	30%
Modelled portfolio	32%	36%	32%
M1			
Traditional portfolio	30%	48%	22%
Modelled portfolio	30%	48%	22%

As we can see the structures of optimal portfolios against their types differ in the American market but in the Latvian equity market the difference between the structures is significantly decreased on W1 and M1 data. In M1 data the expected return of one stock (F) is negative. It means that this asset must be sold, i.e. an investor should open the short position.

To determine the significance of this difference it is necessary to compare the cumulative returns of two portfolios on test period. The results are shown in the Table 6.

Table 6: Cumulative Portfolio Returns

	Markowitz's portfolio	**Modelled portfolio**
Portfolio of 3 American stocks		
D1	106.13%	106.82%
W1	91.31%	92.88%
M1	86.21%	92.30%
Portfolio of 3 Latvian stocks		
D1	144.73%	145.97%
W1	157.64%	155.77%
M1	167.22%	167.22&

The Table 6 shows some interesting effects, i.e.

1) the quality of the modelled portfolio increases as used data time format increases in the American market;

2) the quality of the modelled portfolio decreases as used data time format increases in the Latvian market.

CONCLUSIONS

Taking into account the given results the authors have came to the conclusions:

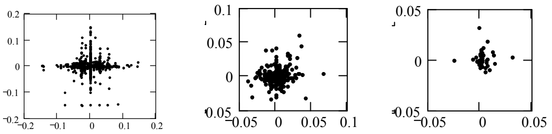
1. In most cases traditional Markowitz's portfolio is more optimistic than a portfolio formed with copula theory in the mature markets but practically does not differ in the emerging markets like the Latvian one.

2. The implementation of copula theory allows an investor to estimate portfolio risk more precisely in comparison with traditional Markowitz's approach in the mature markets. However, portfolio building based on copula theory is not rational in the emerging markets represented by the Latvian equity market because of complexity of computing and minimal deviation in the results (see the Table 5).

3. The difference between the cumulative returns in the portfolios formed utilizing copula theory and traditional Markowitz's approach is increasing function of time format in case of the mature markets and decreasing function in case of the emerging markets. It means that the difference is more evident in monthly data, less evident in weekly data and almost similar in daily data in the mature markets but vice versa in the Latvian equity market.

4. The features of non-linear dependences between Latvian stocks require to use more specific copulas for quality increasing of the corresponding portfolio

(see Figures 3). It means that it is necessary to find such copulas which could describe these unusual non-linear dependences.



Figures 3: Examples of Non-linear Dependences Between Two Latvian stocks (Daily, Weekly and Monthly)

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