

# MODELING AND COMPUTER SIMULATION FOR THE PREDICTION OF FORCES IN HIGH-SPEED MACHINING PROCESSES

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## KEYWORDS

Forces, high-speed machining, modeling, simulation.

## ABSTRACT

This paper reports some results in modeling and simulation of the high-speed cutting process based on cutting force signal. The mathematical model has two fundamental, co-dependent parts. The first is a multiple-input system that defines the model's kinematics, where constants and variables describe the tool geometry, the material type and the cutting parameters. The second addresses the dynamics, represented by integral-differential equations. The results of the simulation for cylindrical mills show the suitability of forces signal in describing the high-speed machining processes.

## INTRODUCTION

One of the basic tasks manufacturing systems have to perform today is machining, especially high-speed machining (HSM) (Haber et al. 2004a). High-speed machining may be defined according to the available literature as the process of machining at the highest possible cutting speeds allowed by existing restrictions set by the workpiece and the material to be machined, the machine tools to be used and the control and operating devices available (e.g., CAD/CAM systems, numerical controls). In this paper, we focus on high-speed milling machines where the workpiece remains fixed and the tool is clamped to a rotating head. Using this high-performance rotating head and working with each of its axes, cutting speeds can be reached that are much higher than those found in conventional machining, leading to what is known nowadays as high-performance machining (HPM).

The conventional machining process may be regarded as a complex electromechanical process in which force and torque signals are representative of the physical processes taking place during cutting (Haber et al. 2004b). Precise knowledge of the dynamic behavior of such signals can be interpreted to evaluate the status of

the cutting process, so force and torque signals are extremely useful in monitoring and control systems. Despite the enormous progress made in HSM, it is unclear what usefulness the mathematical basis already developed for conventional machining may have for modeling and characterizing high-speed machining. In addition, more light needs to be thrown upon the role that variables such as cutting force play in HSM, where they may furnish relevant information on cutting-tool condition, vibration and surface finish.

The main goal of this paper is to derive a mathematical model from the characterization of the physical processes taking place during high-speed machining and to perform simulations. Modeling shall be done following classic patterns used in conventional machining, looking upon cutting force as the output variable.

At the present time, modeling high-speed machining processes, especially high-speed milling, is a very active area of investigation that is peppering the scientific community with challenges. High-speed machining has now been adopted and put into regular use at many companies, and yet certain points of how to monitor cutting-tool condition have been worked out only partially as yet, largely for lack of a mathematical model of the process that can feasibly be used in real-time applications. Even so, HSM today is a cutting technology whose solid bases open the doors to the possibility of machining materials of a hardness of over 50 HRc and narrow walls just 0.2 mm thick.

If we confine ourselves to HSM, and more particularly the modeling of HSM, and we analyze the literature available to us, we can see that there are few papers dealing with the relationship of cutting force as an output variable with the constants and input variables that define mill or cutting-tool geometry, the type of material to be machined and the cutting parameters themselves.

## GEOMETRIC MODEL OF THE TOOL

Geometrical modeling of the helical cutting edge includes the kinematic and dynamic analysis of the cutting process. Predicting cutting forces requires a system of coordinates, the helix angle and the angular distance of a point along the cutting edge (Yucesan and Altintas 1996). The mathematical expressions that define this geometry in a global coordinate system are presented below in the geometric model, using classic vector notation.

Vector  $\vec{r}(z)$  drawn from point  $O$  to a point  $P$  in cylindrical coordinates is expressed mathematically in equation 1.

$$\vec{r}_j = x_j \vec{i} + y_j \vec{j} + z_j \vec{k} = r(\phi_j)(\sin \phi_j \vec{i} + \cos \phi_j \vec{j}) + z(\phi_j) \vec{k} \quad (1)$$

where  $\phi_j$  is the radial rake angle of a point  $P$  at tooth  $j$ .

Point  $P$  lies at an axial depth of cut  $a_p$  in the direction of axis  $Z$ , at a radial distance  $r(z)$  on the  $XY$  plane, with an axial rake angle  $\kappa(z)$  and a radial lag angle of  $\psi(z)$ .

The geometry of the tool is represented mathematically, considering that the helical cutting edge wraps parametrically around a cylinder. The mathematical model dictated for the cutting edge considers that the edge is divided into small increments, where the cutting coefficients can vary for each location. The initial point of reference to the cutting edge of the tool ( $j = 1$ ) is considered to be the angle of rotation when  $z = 0$  is  $\phi$ . The radial rake angle for the cutting edge  $j$  in a certain axial position  $z$  is expressed as:

$$\phi_j(z) = \phi + \sum_{n=1}^j \phi_p - \psi(z) \quad (2)$$

The lag angle  $\psi(z)$  appears due to the helix angle  $\theta$ . This angle is constant in the case of a cylindrical mill, and it varies for a ball-end mill. In the generalized model for the geometry of a mill with helical teeth, the tool diameter may differ along the length of the tool, depending on the shape of the tool (cylindrical, ball-end, spherical, angular, etc.). An infinitesimal length of this cutting edge may be expressed as

$$\begin{aligned} dS &= |dr| = \sqrt{r^2(\phi) + (r'(\phi))^2 + (z'(\phi))^2} d\phi \\ r'(\phi) &= \frac{dr(\phi)}{d\phi} \\ z' &= \frac{dz(\phi)}{d\phi} \end{aligned} \quad (3)$$

Chip thickness changes as a function of the radial rake ( $\phi$ ) and axial rake ( $\kappa$ ):

$$h_j(\phi_j) = s_{ij} \sin \phi_j \cdot \sin \kappa \quad (4)$$

As mentioned above, this paper looks at cylindrical mills. Analyzing the geometry, for a cylindrical mill, the following conditions are defined for finding the general solution:

$$r(z) = \frac{D}{2} \quad (5)$$

$$\kappa = 90^\circ \quad (6)$$

$$\psi = k_\theta z \quad (7)$$

$$k_\theta = (2 \tan \theta) / D \quad (8)$$

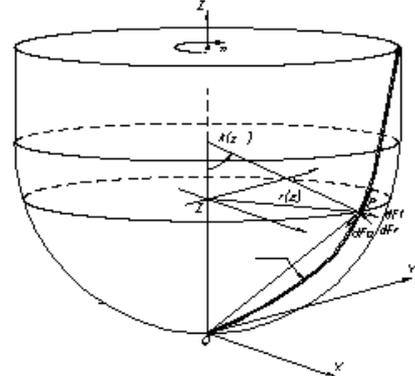


Figure 1: Tool geometry

## MODEL OF CUTTING FORCE

In order to find the cutting forces in high-speed machining, cutting is considered to occur in opposition. The force differentials ( $dF_t$ ), ( $dF_r$ ), ( $dF_a$ ) act on an infinitesimal element of the cutting edge of the tool (Altintas and Lee 1998):

$$\begin{aligned} dF_t &= K_{te} dS + K_{tc} h_j(\phi, \kappa) db \\ dF_r &= K_{re} dS + K_{rc} h_j(\phi, \kappa) db \\ dF_a &= K_{ae} dS + K_{ac} h_j(\phi, \kappa) db \end{aligned} \quad (9)$$

It is also considered that:

$$db = \frac{dz}{\sin \kappa} \quad (10)$$

In order to facilitate finding the mathematical relations inherent in this set-up, very small time increments are used. The positions of the points along the cutting edge are evaluated with the geometrical model presented herein above.

Furthermore, the characteristics of a point on the cutting surface are identified using the properties of kinematic rigidity and the displacements between the tool and the workpiece. The constants or cutting coefficients ( $K_{te}$ ,  $K_{re}$ ,  $K_{ae}$ ,  $K_{tc}$ ,  $K_{rc}$ ,  $K_{ac}$ ) can be found experimentally using cutting forces per tooth averaged for a specific type of tool and material (Fu et al. 1984, Budak et al. 1996). We might point out that these coefficients are highly dependent on the location (axial depth) of the cutting edge. How these coefficients are found shall not be addressed in this paper.

Cutting forces can be evaluated employing a system of Cartesian coordinates:

$$\begin{bmatrix} dF_x \\ dF_y \\ dF_z \end{bmatrix} = \begin{bmatrix} -\sin \phi \sin \kappa & -\cos \phi & -\sin \phi \cos \kappa \\ -\cos \phi \sin \kappa & \sin \kappa & -\cos \phi \cos \kappa \\ -\cos \kappa & 0 & -\sin \kappa \end{bmatrix} \cdot \begin{bmatrix} dF_r \\ dF_t \\ dF_a \end{bmatrix} \quad (11)$$

The total cutting forces as a function of  $\phi$  are found by integrating equation (11) along the axial depth of cut for all the lips of the mill that are in contact with the workpiece:

$$\begin{aligned} F_x(\phi) &= \sum_{j=1}^{Nf} \int_{z_1}^{z_2} [-dF_{rj} \sin \phi_j \sin \kappa_j - dF_{tj} \cos \phi_j - dF_{aj} \sin \phi_j \cos \kappa_j] dz \\ F_y(\phi) &= \sum_{j=1}^{Nf} \int_{z_1}^{z_2} [-dF_{rj} \cos \phi_j \sin \kappa_j + dF_{tj} \sin \phi_j - dF_{aj} \cos \phi_j \cos \kappa_j] dz \\ F_z(\phi) &= \sum_{j=1}^{Nf} \int_{z_1}^{z_2} [-dF_{rj} \cos \kappa_j \quad 0 \quad -dF_{aj} \sin \kappa_j] dz \end{aligned} \quad (12)$$

where  $z_1$  and  $z_2$  are the integration limits of the contact zone at each moment of cutting and can be calculated from the geometrical model described herein above. For the numerical calculation, the axial depth of cut is divided into disks having an infinitesimal height  $dz$ . The differentials of the cutting forces are calculated along the length of the cutting edge in contact, and they are summed to find the resulting forces for each axis  $F_x(\phi)$ ,  $F_y(\phi)$ ,  $F_z(\phi)$  in an angle of rotation defined by

$$\phi = \Omega \cdot dt \quad (13)$$

where  $\Omega$  is the speed of the head in ( $rad/s$ ) and  $dt$  is the time differential in the interval for the integration.

The left end of the tool is the initial point of reference for the radial rake angle ( $\phi$ ), designated by the distance  $a_e$  as the entry point and  $a_s$  as the exit point (figure 2). The points lying between  $a_e$  and  $a_s$  remain at the angles designated by:  $\phi_j(0) = \phi + j\phi_p$ ;  $j = 1, 2, \dots, (Nf)$ , where  $j$  indicates the lip of the tool.

The rake angle for lip  $j$  of the mill, due to the depth of cut, is defined along the  $Z$  axis by:

$$\phi_j(z) = \phi + j\phi_p - k_\theta z \quad (14)$$

in which chip thickness:

$$h_j(\phi, z) = s_{ij} \sin \phi_j(z) \cdot \sin \kappa(z) \quad (15)$$

The cutting constants can be calculated using equation (19) and the transformation of the orthogonal cut into an oblique cut, regarding the helix angle as an oblique cutting angle (i.e.,  $i = \theta$ ) (Engin and Altintas 2001).

$$\begin{aligned} K_{tc} &= \frac{\tau_s}{\sin \phi_n} \frac{\cos(\beta_n - \alpha_n) + \tan i \tan \eta \sin \beta_n}{\sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2 \eta \sin^2 \beta_n}} \\ K_{ac} &= \frac{\tau_s}{\sin \phi_n \cos i} \frac{\sin(\beta_n - \alpha_n)}{\sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2 \eta \sin^2 \beta_n}} \\ K_{rc} &= \frac{\tau_s}{\sin \phi_n} \frac{\cos(\beta_n - \alpha_n) \tan i - \tan \eta \sin \beta_n}{\sqrt{\cos^2(\phi_n + \beta_n - \alpha_n) + \tan^2 \eta \sin^2 \beta_n}} \end{aligned} \quad (16)$$

where  $\tau_s$  is the cutting or shearing force defined as the quotient between the shearing force and the flat cutting area,  $i, \alpha_n$  are the oblique angle and the normal rake angle, respectively,  $\phi_n$  is the normal shear or cutting angle,  $\eta$  is the chip-flow angle and  $\beta_n = \arctan(\tan \beta_A \cos \eta)$  where  $\beta_A$  is the friction angle.

Of course, the cutting coefficients described in (16) are considered constant for the tool/material as a block, and these values can be adjusted either empirically in milling operations or by using the oblique-cut transformation.

The primary forces are calculated in the direction of feed  $X$ , normal direction  $Y$  and axial direction  $Z$ , which are derived from the transformation indicated in equation (17) for the particular case of a cylindrical mill.

$$\begin{aligned} dF_{x,j}(\phi_j(z)) &= -dF_{t,j} \cos \phi_j(z) - dF_{r,j} \sin \phi_j(z) \\ dF_{y,j}(\phi_j(z)) &= +dF_{t,j} \sin \phi_j(z) - dF_{r,j} \cos \phi_j(z) \\ dF_{z,j}(\phi_j(z)) &= +dF_{a,j} \end{aligned} \quad (17)$$

Furthermore, the exact solution for cylindrical mill can be found. By substituting (14) and (15) into (12) and making  $\kappa = 90^\circ$ , we obtain:

$$\begin{aligned} F_{x,j}(\phi_j(z)) &= \left\{ \begin{array}{l} \frac{s_{ij}}{4k_\beta} [-K_{tc} \cos 2\phi_j(z) + K_{rc} [2\phi_j(z) - \sin 2\phi_j(z)]] \\ + \frac{1}{k_\beta} [K_{te} \sin \phi_j(z) - K_{re} \cos \phi_j(z)] \end{array} \right\}_{z_{j,1}(\phi_j(z))}^{z_{j,2}(\phi_j(z))} \\ F_{y,j}(\phi_j(z)) &= \left\{ \begin{array}{l} \frac{-s_{ij}}{4k_\beta} [K_{tc} (2\phi_j(z) - \sin 2\phi_j(z)) + K_{rc} \cos 2\phi_j(z)] \\ + \frac{1}{k_\beta} [K_{te} \cos \phi_j(z) - K_{re} \sin \phi_j(z)] \end{array} \right\}_{z_{j,1}(\phi_j(z))}^{z_{j,2}(\phi_j(z))} \\ F_{z,j}(\phi_j(z)) &= \frac{1}{k_\beta} [K_{ac} s_{ij} \cos \phi_j(z) - K_{ae} \phi_j(z)]_{z_{j,1}(\phi_j(z))}^{z_{j,2}(\phi_j(z))} \end{aligned} \quad (18)$$

where  $z_{j,1}(\phi_j(z))$  and  $z_{j,2}(\phi_j(z))$  are the lower and upper limits, respectively, that establish the axial depth of cut at lip  $j$  of the mill.

## SIMULATION AND MODEL VALIDATION

The algorithms were implemented in MATLAB, drawing upon the mathematical models. MATLAB is the abbreviated name for “MATrix LABoratory.” MATLAB is a software tool for doing numerical computations with matrices and vectors (Matlab 2003). It can also display information graphically and includes many toolboxes for several research and applications areas.

MATLAB was chosen for its simplicity of programming, the possibility it affords of running simulations and applications in real time and the portability of the programs that are developed with its use (i.e., it is possible to generate C/C++ programs from MATLAB files).

Through the simulation based on these models, studies may be conducted to gain an understanding of the influence the variables and parameters have. The main difficulties are choosing the cutting coefficients and the properties of the materials, which were taken from earlier work on the subject (Altintas 2000). As mentioned above, this paper is not an attempt to develop a methodology for finding such coefficients, but an attempt to use the information already available in the literature.

The tool type considered (cylindrical mill) is one of the tools most frequently used in machining molds and equipment in the construction of parts for the aerospace and automobile industry.

The workpiece-material properties that were used for the simulation in the MATLAB environment are the properties of GGG-70 cast iron with nodular graphite. In this study, in the simulation and in the real tests, the cutting condition for high-speed milling operations were regarded:  $V_c=640$  m/min,  $sp=17500$  rpm,  $f=2700$  mm/min,  $a_p = 0.5$  mm,  $a_e = 0$ ,  $a_s = 12$  mm,  $\theta=30^\circ$ ,  $H = 25.0$  mm,  $D = 12.0$  mm.

The constants used in the simulation and in the experimental validation were  $K_{tc} = 2172$  N/mm<sup>2</sup>,  $K_{rc} = 850$  N/mm<sup>2</sup>,  $K_{te}=17.3$  N/mm,  $K_{re} = 7.8$  N/mm,  $K_{ac} = 726$  N/mm<sup>2</sup>,  $K_{ae} = 6.7$  N/mm. These constants or cutting coefficients referring to the material and the tool were drawn from the available literature, due to their similarity to the characteristics of the tool/material set-up used in the study in question.

### Experimental validation

The validation tests were conducted at a KONDIA HS1000 high-speed machining center equipped with a Siemens 840D open CNC. Cutting force was measured using a Kistler 9257 dynamometric platform installed on the testbed. The main technical specifications of the Kistler platform are: a [-5, 5] kN measurement range

along each axis, a natural frequency of  $>4$  kHz, a full-scale linearity of  $\leq 2\%$  and a sensitivity of  $-7.5$  pC/N. Three 5011 load amplifiers were used as well, one each to measure the cutting forces  $F_x$ ,  $F_y$  and  $F_z$ . Measurement was done by means of a DAQBOARD-2005 data-acquisition card at a sampling frequency of 50 kHz.

A Karnasch 30.6472 cylindrical mill 12 mm in diameter was selected to be used in validating the model developed under the procedure described herein. The tool was 12% cobalt micrograin with a 1-3-micra coating of TiAlN-Al, microhardness 3300 (HV 0.05), withstanding a maximum temperature of 800°C. The chosen test piece, measuring 200x185x50 mm, was made of GGG-70 iron and was machined in a spiral pattern. The real cutting conditions chosen were the same as considered above for the simulation. A view of the tool used in the tests (a), the test piece and the configuration of the cut (b) and the laboratory (c) may be seen in figure 2.

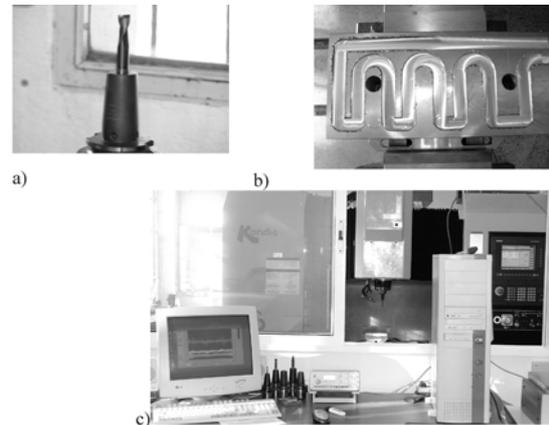


Figure 2: Cutting tool for experiments, b) experimental workpiece, c) partial view of the Laboratory for machine tool research

The objectives focused not only on validating the theoretical model for the cylindrical mill, but also on ascertaining the importance of cutting forces in HSM and their ability to provide relevant information on the condition of the cutting tool during high-speed cutting processes. Figures 3, 4, 5 and 6 show the real behavior of cutting forces  $F_x$ ,  $F_y$  and  $F_z$  and the resulting force  $F_{qT}$  for each of the four cases analyzed. The model's response is shown as a solid line.

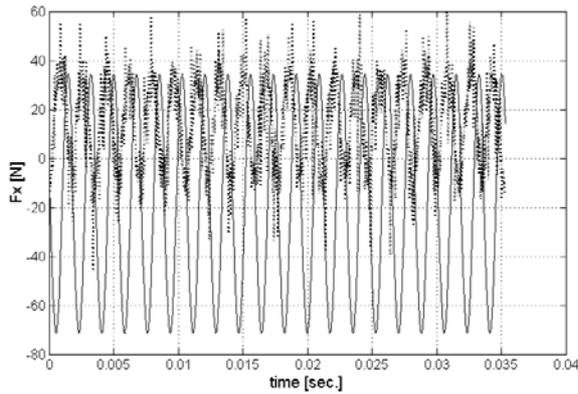


Figure 3: Measured (straight line) and predicted (dashed line) cutting force  $F_x$

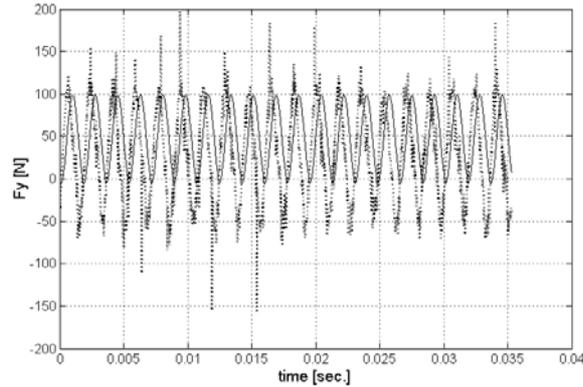


Figure 4: Measured (straight line) and predicted (dashed line) cutting force  $F_y$

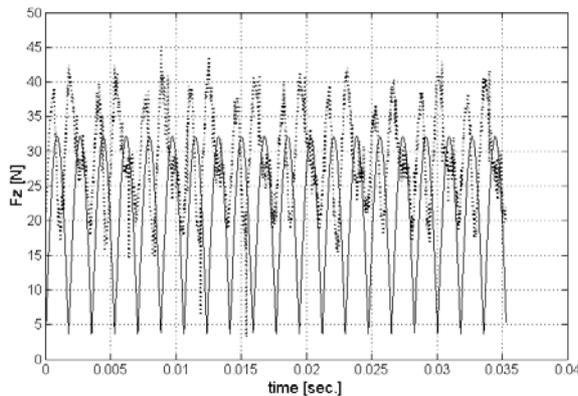


Figure 5: Measured (straight line) and predicted (dashed line) cutting force  $F_z$

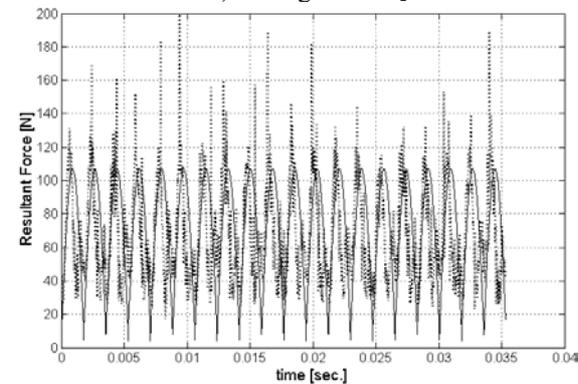


Figure 5: Measured (straight line) and predicted (dashed line) resultant cutting force

The average resulting cutting force  $\bar{F}_e$  estimated by the model is 70.6N and the average resulting cutting force  $\bar{F}_{qT}$  measured in a real high-speed cutting operation is 71.7N. The error criterion  $\bar{E} = \frac{(\bar{F}_{qT} - \bar{F}_e) \cdot 100}{\bar{F}_{qT}}$ , is 7.9%.

## CONCLUSIONS

This paper reports some initial results in modeling the high-speed cutting process and the validation of the model in question. The mathematical model has two fundamental, co-dependent parts. The first is a multiple-input system that defines the model's kinematics, where constants and variables describe the tool geometry, the material type and the cutting parameters. The second addresses the dynamics, represented by integral-differential equations. The exact analytical solution is found, inasmuch as the limits of integration and the boundary conditions along the tool geometry can be pre-established.

The flexibility of the model developed in this paper makes simulation studies valid for both conventional and high-speed machining, maintaining a consistent relationship among cutting parameters such as feed, spindle speed and cutting speed.

Furthermore, the importance of cutting force as a representative variable of high-speed machining processes has been proved. Thus, what cutting conditions will maximize the chip-removal rate or useful lifetime of a specific tool can be evaluated by comparing average cutting-force values. In this way, improvements in machining time, useful tool lifetime and waste-product reduction can be achieved. It is also possible to obtain information necessary for estimating tool wear. Whether cutting-force figures provide enough information about the condition of the cutting tool is a matter that should be analyzed in the future.

With this paper and the implementation conducted using MATLAB, the doors are thrown open onto the use of a model for predicting surface finish and stability lobes in high-speed machining processes.

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