Simulation of Spacecraft Attitude and Orbit Dynamics

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ABSTRACT

In this paper, the simulation model of satellite attitude and orbit dynamics is discussed. The satellite attitude model has been represented in term of a quaternion and a ordinary differential equation is used to describe the satellite orbital motion. The different actuators and sensors have been modeled with suitable faults and failures. The simulation model enables us to consider the satellite motion under different environmental perturbations (for example aerodynamic drag, external celestial body etc.) and failure in actuators and sensors. The simulation model is utilized in the development of attitude and orbit control algorithms or fault detection, isolation and recovery (FDIR) technologies. Simulation results are also given.

INTRODUCTION

During the last decades modeling, simulation, and wider computational science and engineering have become more and more important tools in the research and development projects. The design phase has to be reduced in time and cost when the use of new ideas and tools becomes possible. This is also the trend in space application in which the real tests are not possible or at least they are expensive. New demands on the aerospace and control engineering have become up and they have to be able to answer to requirements.

Spacecraft simulators or simulators in general, are software tools that can be used by researchers, engineers, students or everybody to analyze and assess system operations, behaviors, and to answer to the questions regarding phenomenon or product. The simulations are essential tools in the mission and spacecraft control design. For example, the scientific missions are unique and the instrumentation of a spacecraft is designed only for this specific mission. There are not any ready-to-use platforms that can be used. Hence, it is not possible to verify the operation of control algorithms and strategies in real process but the simulation environments can be used.

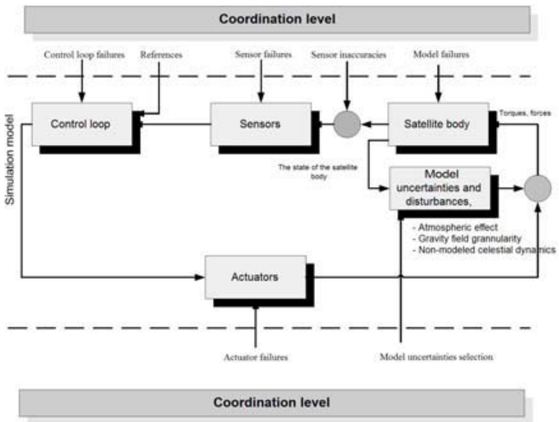
There are plenty of companies that offer their simulation services to the research institutes and space companies.

MODEL STRUCTURE AND MATHEMATICS

Model Structure

The spacecraft simulation model is organized like any actual control loops (See Figures 1.). The interfaces of the components are defined and modeled in such a way that the simulation model would be as modular as possible. Modifications to the simulation model are easy to do and one part of the model can be easily replaced with another.

The model is initialized and controlled from the coordination level. This means that the model parameters and possible faults and failures in the FDIR simulation case are defined.



Figures 1. Spacecraft simulation model structure.

Coordinate Systems

Three different coordinate systems are defined in the simulator:

1. Inertial Coordinate System (ICS),
2. Orbit Coordinate System (OCS), and
3. Body Coordinate System (BCS).

The inertial coordinate system is usually defined such that the center of mass of the Earth (cm) acts as origin and the direction of the axes are fixed to the solar system. This kind of coordinate system is not exactly inertial but it is enough for all engineering purposes (Sidi 1997). The **Z**-axis of the ICS is the rotation axis of the Earth in a positive direction and the **X**-**Y** plane is the equatorial plane of the Earth, which is perpendicular to the Earth's rotation axis. The vernal equinox vector ϒ is selected to be the **X**-axis of the ICS. Finally, the **Y**-axis

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has been chosen in such a way that the ICS is right-handed orthogonal coordinate system.

Orbit coordinate system is also a right-handed orthogonal coordinate system with origin in the center of the satellite mass. The **Z**-axis is pointing towards the center of the Earth; **X**-axis to the direction of satellite perpendicular to the **Z**-axis, and **Y**-axis completes the coordinate system such that it is right-handed and orthogonal. The third coordinate system, which has been fixed to the moving and rotating spacecraft, defines satellite orientation.

Rotation

The attitude transformation in space can be executed by using various different aspects. In the simulation model, the quaternion technique is used. The main feature of quaternions is that they provide a convenient product rule for successive rotations and they have simple form of kinematics (Wertz 1978, Wis´niewski 1996).

The basic definition of the quaternion is a consequence of the property of the direction cosine matrix **A** that it has at least one eigenvalue of unity. This means that there is an eigenvector **e** (Euler axis) that is unchanged in every rotation. The quaternion is defined as a vector (1) where $q_i \in R$, **i**, **j** and **k** satisfy the Hamilton's rule (2) and where the length of the quaternion is unity. (Sidi 1997)

$$\mathbf{q} = q_4 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k} \tag{1}$$

$$\begin{aligned} \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1 \\ \mathbf{ij} = -\mathbf{ji} = \mathbf{k} \\ \mathbf{jk} = -\mathbf{kj} = \mathbf{i} \\ \mathbf{ki} = -\mathbf{ik} = \mathbf{j} \end{aligned} \tag{2}$$

When the Euler axis **e** of the rotation is known the connection between quaternion and the rotation Euler axis is

$$\begin{cases} q_1 = e_1 \sin(\alpha/2) \\ q_2 = e_2 \sin(\alpha/2) \\ q_3 = e_3 \sin(\alpha/2) \\ q_4 = \cos(\alpha/2) \end{cases}$$

where e_i is a component of Euler axis and α is the magnitude of the rotation.

The final combined rotation of two successive rotations can be performed as a matrix-vector multiplication (3) where **q** and **q**´ are the individual rotations.

$$\mathbf{q}'' = \mathbf{q}\mathbf{q}' = \begin{pmatrix} q_4' & q_3' & -q_2' & q_1' \\ -q_3' & q_4' & q_1' & q_2' \\ q_2' & -q_1' & q_4' & q_3' \\ -q_1' & -q_2' & -q_3' & q_4' \end{pmatrix} \cdot \begin{pmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{pmatrix} \tag{3}$$

SIMULATION MODEL

The simulation model is realized in the MATLAB/ SIMULINK-environment.

Orbit Model

The motion of a celestial body is based on the quite elementary principles of celestial mechanics. In the 17th century, J. Kepler provided three basic empirical laws that describe the motion of planet in unperturbed planetary orbit. The orbital dynamics of a satellite is extensively explained in many books, for example (Sidi 1997) and (Wertz 1978).

If we consider a system of two particles P_1 and P_2 of masses m_1 and m_2 and apply Newton's second law and the law of gravity to the two-body system, we can get the fundamental equation (4) of the motion of the two-body system where the symbol $\mu = G(m_1+m_2)$ and G is the universal constant of gravitation. This equation describes the motion of the particle P_1 relative to the second mass P_2.

$$\ddot{\mathbf{r}} + \frac{\mu}{r^3}\mathbf{r} = 0 \tag{4}$$

In general, if a particle P moves in a force field **F**, the momentum of the force **F** about origin O is

$$\mathbf{M}=\mathbf{r}\times\mathbf{F}$$

where **r** is the position vector of the particle P. The angular momentum about origin is

$$\mathbf{h}=m(\mathbf{r}\times\mathbf{v})=\mathbf{r}\times\mathbf{p}$$

where **p** is the linear momentum of the particle. Thus, the time rate of the angular momentum **h** is equal to the moment of the force **F.**

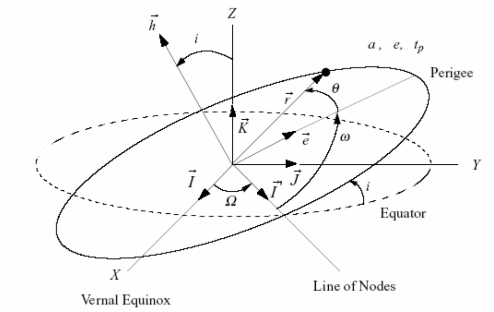
$$\frac{d\mathbf{h}}{dt} = \frac{d}{dt}(\mathbf{r}\times m\mathbf{v}) = 0 + \mathbf{r}\times\mathbf{F} = \mathbf{M} \tag{5}$$

The equation (5) states the fundamental fact that the momentum acting on a particle is equal to the time rate of the change of its angular momentum.

In space science it is common to describe the satellite orbit by five numbers, known as orbital elements or classical orbital elements (COE). A sixth element is added to determine the location of the satellite in its orbit (Wertz 1978 and Sidi 1997). The classical orbital elements have been described in the Table 1. Because these elements are poorly defined if e and/or i is equal to zero, so-called equinoctial orbital elements (EOE) have been defined in terms of the classical orbital elements. The equinoctial orbital elements have been defined in Table 2.

Table 1. The classical orbital elements. (Sidi 1997)

Symbol	
a	the semi major axis
e	the eccentricity
i	the inclination
Ω	the right ascension of the ascending node
ω	the argument of perigee
M	the mean anomaly



Figures 2. The definitions of the elements.

Table 2. The definitions of the equinoctial orbital elements (EOE). (We and Roithmayr, C.M. 2001)

EOE	
a	a
P_1	$e\sin(\Omega+\omega)$
P_2	$e\cos(\Omega+\omega)$
Q_1	$\tan\left(\frac{i}{2}\right)\sin(\Omega)$
Q_2	$\tan\left(\frac{i}{2}\right)\cos(\Omega)$
l	$\Omega+\omega+M$

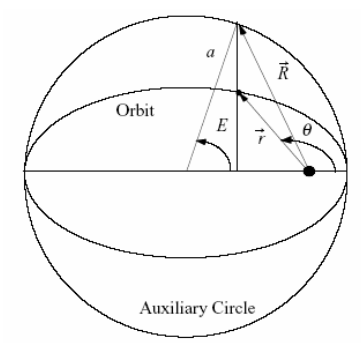
In Keplerian orbit the derivative of the first five orbital elements are equal to zero. If the satellite orbital elements are known the satellite location **r** and the velocity vector **v** can be calculated, and vise versa. Algorithms to do this can be found in any textbook concerning orbital dynamics, for example (Sidi 1997), (Wertz 1978).

In the general case, in which any kind of perturbing force can exist, the equation of orbital motion is

$$\ddot{\mathbf{r}} + \frac{\mu}{r^3}\mathbf{r} = \mathbf{f}_p$$

with initial condition and where $\mathbf{f}_p$ is the perturbing force per unit mass. Due to the perturbing acceleration the orbital elements are not constants. Hence, so-called Gauss form of Lagrange's planetary equations describes the derivative of classical orbital elements when the perturbing force is conservative or non-conservative. Knowing the initial condition of COE the Gauss equations can be integrated to calculate the evolution of the elements. Gauss equation is represented in equation (7), where θ is the angle between satellite location vector and the vector pointing towards perigee (See Figures 3.), $p = a(1\text{-}e^2)$, $n = \text{sqrt}(\mu/a^3)$, $r = p/(1+e\cos(\theta))$, and f_r, f_θ and f_z are the components of the perturbing force along the radius vector direction **r**, the transverse orbit direction and the direction of the normal to the orbit plane, respectively.

To avoid the singularity due to the poorly defined parameters, the Gauss equations can be rewritten in the terms of the equinoctial elements as in (8), where $b=a\cdot sqrt(1\text{-}P_1^2\text{-}P_1^2)$, $h=nab$, $p/r=1+P_1\sin(L)+P_2\cos(L)$, $L=\omega+\Omega+\theta$, and $K=\omega+\Omega+E$. (We and Roithmayr, C.M. 2001)



Figures 3. The spacecraft orbit and auxiliary circle.

Attitude Model

Dynamics

From equation (5) we get that the torque acting on the satellite body is equal to the derivative of the angular momentum of the spacecraft in the inertial coordinate system. Hence, in the rotating body coordinate system

$$\sum \mathbf{T} = \dot{\mathbf{h}}_I = \dot{\mathbf{h}} + \boldsymbol{\omega}\times\mathbf{h}\,.$$

If momentum exchange devices are used in the control, the angular momentum vector $\mathbf{h} = \mathbf{h}_B+\mathbf{h}_w$ where $\mathbf{h}_B$ is the angular momentum of satellite rigid body and $\mathbf{h}_w$ is the angular momentum of the momentum exchange devices. Hence, the time rate of angular velocity of the satellite body is like in equation (6).

$$\dot{\boldsymbol{\omega}} = \left(\mathbf{I}_s\right)^{-1}\cdot\left(-\dot{\mathbf{I}}_s\boldsymbol{\omega} + \sum_i \mathbf{T}_i - \dot{\mathbf{h}}_w - \boldsymbol{\omega}\times\left(\mathbf{I}_s\boldsymbol{\omega}\right) - \boldsymbol{\omega}\times\mathbf{h}_w\right) \quad (6)$$

Kinematics

The spacecraft attitude has been modeled as a quaternion representation $\mathbf{q}=(q_1, q_2, q_3, q_4)$. Hence, the equation (9), where ω_i is the satellite angular velocity about satellite body axis i, gives the derivative of quaternion vector.

$$\begin{aligned}
\dot{a} &= \frac{2a^2}{\sqrt{\mu p}}\left(f_r e\sin(\theta)+f_\theta\left(1+e\cos(\theta)\right)\right)\\
\dot{e} &= \sqrt{\frac{p}{\mu}}\left(f_r\sin(\theta)+f_\theta\left(\cos(\theta)+\frac{e+\cos(\theta)}{1+e\cos(\theta)}\right)\right)\\
\dot{i} &= \frac{rf_z\cos(\omega+\theta)}{\sqrt{\mu p}}\\
\dot{\Omega} &= \frac{rf_z\sin(\omega+\theta)}{\sqrt{\mu p}\sin(i)}\\
\dot{\omega} &= -\frac{f_z r\sin(\omega+\theta)\cos(i)}{\sqrt{\mu p}\sin(i)}\\
&\quad -\frac{1}{e}\sqrt{\frac{p}{\mu}}\left(f_r\cos(\theta)-f_\theta\left(1+\frac{r}{p}\right)\sin(\theta)\right)\\
\dot{M} &= n-\frac{2rf_r}{na^2}+\frac{1-e^2}{nae}\left(f_r\cos(\theta)-f_\theta\left(1+\frac{r}{p}\right)\sin(\theta)\right)
\end{aligned}\tag{7}$$

$$\begin{aligned}
\dot{a} &= \frac{2a^2}{h}\left(\left(P_2\sin(L)-P_1\cos(L)\right)f_r+\frac{p}{r}f_\theta\right)\\
\dot{P}_1 &= \frac{r}{h}\left(\frac{p}{r}\cos(L)f_r+\left(P_1+\left(1+\frac{p}{r}\right)\sin(L)\right)f_\theta\right.\\
&\quad\left.-P_2\left(Q_1\cos(L)-Q_2\sin(L)\right)f_z\right)\\
\dot{P}_2 &= \frac{r}{h}\left(\frac{p}{r}\sin(L)f_r+\left(P_2+\left(1+\frac{p}{r}\right)\cos(L)\right)f_\theta\right.\\
&\quad\left.+P_1\left(Q_1\cos(L)-Q_2\sin(L)\right)f_z\right)\\
\dot{Q}_1 &= \frac{r}{2h}\left(1+Q_1^2+Q_2^2\right)\sin(L)f_z\\
\dot{Q}_1 &= \frac{r}{2h}\left(1+Q_1^2+Q_2^2\right)\cos(L)f_z\\
\dot{l} &= n-\frac{r}{h}\left(\left(\frac{a}{a+b}\frac{p}{r}\left(P_1\sin(L)+P_2\cos(L)\right)+\frac{2b}{a}\right)f_r\right.\\
&\quad+\frac{a}{a+b}\left(1+\frac{p}{r}\right)\left(P_1\cos(L)-P_2\sin(L)\right)f_\theta\\
&\quad\left.+\left(Q_1\cos(L)-Q_2\sin(L)\right)f_z\right)
\end{aligned}\tag{8}$$

$$\frac{d}{dt}\begin{pmatrix}q_1\\q_2\\q_3\\q_4\end{pmatrix}=\frac{1}{2}\begin{pmatrix}0&\omega_3&-\omega_2&\omega_1\\-\omega_3&0&\omega_1&\omega_2\\\omega_2&-\omega_1&0&\omega_3\\-\omega_1&-\omega_2&-\omega_3&0\end{pmatrix}\begin{pmatrix}q_1\\q_2\\q_3\\q_4\end{pmatrix}\tag{9}$$

Actuators

The actuators are used to produce the control torques and forces for the satellite attitude control. The modeled actuators are:

- thruster,
- reaction-wheels, and
- magnetotorquer.

The thruster has been modeled as a thrust force vector $\mathbf{F}_t$ affecting the satellite in position $\mathbf{r}_t$. Hence, the torque about the center of the mass of the spacecraft is the cross product between the position and the force vectors (Equation (10)).

$$\mathbf{T}_t=\mathbf{r}_t\times\mathbf{F}_t\tag{10}$$

The idea of reaction wheels (or momentum exchange devices) is to transfer the angular momentum of the whole system between different parts of the spacecraft without changing its overall internal angular momentum. The achieved torque level is of the order of 0.05 – 2 Nm. (Sidi 1997)

The reaction-wheel is modeled as equation (11).

$$\begin{cases}\dot{\boldsymbol{\omega}}_w=\mathbf{f}\left(\mathbf{T}_{dem}\right)-\mathbf{f}_\mu\\ \mathbf{h}_w=I_w\boldsymbol{\omega}_w\end{cases}\tag{11}$$

In magnetotorquer, the control torque $\mathbf{T}_{mag}$ is generated by an interaction of the Earth's geomagnetic field $\mathbf{B}(t)$ with the magnetic dipole momentum $\mathbf{m}(t)$ (See equations (12) and (13)) where n_{coil} is the number of coil, $i_{coil}(t)$ is the magnetotorquer current, A_{coil} loop area, and $\hat{\mathbf{n}}$ is the unit normal vector to the plane of the loop.

$$\mathbf{T}_{mag}(t)=\mathbf{m}(t)\times\mathbf{B}(t)\tag{12}$$

$$\mathbf{m}(t)=n_{coil}i_{coil}(t)A_{coil}\cdot\hat{\mathbf{n}}\tag{13}$$

Sensors

In the simulation model the modeled sensors are:

- coarse Earth and Sun Sensor (CESS),
- star tracker,
- magnetometer,
- gyro, and
- GPS.

The CESS is modeled as a component that gives the direction of Sun and Earth in the body coordinate system.

The star tracker is modeled as a component that gives the satellite attitude contaminated with an uncertainty that depends on the satellite angular speed. Magnetometer is modeled as a component that gives the magnitude and direction of the Earth's magnetic field. WMM magnetic model is used in the simulator.

A gyroscope is modeled as an instrument that measures the angular speed of the spacecraft. The actual angular speed is contaminated with relatively small Gaussian random uncertainty. A GPS is modeled as an instrument that gives the satellite location in the inertial Earth centered coordinate system.

Faults

One of the main aims of the spacecraft simulation model is that it can be used in the FDIR-simulation. Hence, the faults and failures have to be taken into account already in the design and modeling phase. The faults can occur in any part of the model and any kind of faults are possible. Usually, the faults are either additive or parametric but also a total blackout of a component is possible.

Perhaps, the most prevalent fault is ice building on the surface of any optical instrument increasing the inaccuracy of this element.

Environmental Torques

The main sources of the environmental torques are represented in the Table 3.

Table 3. The main environmental torques (Wertz 1978).

Source	*Dependence*	*Dominant*
Aerodynamic	$e^{-\alpha r}$	below ~ 500 km
Magnetic	$1/r^3$	~ 500 - 35 000 km
Gravity Gradient	$1/r^3$	~ 500 - 35 000 km
Solar Radiation	Independent	Interplanetary space above synchronous altitude
Micrometeorites	independent	Normally negligible

The aerodynamic drag is one of the main environmental torques for the spacecraft in low orbit. The aerodynamic drag model has been explained extensively, for example, in the book (Wertz 1978).

The force $d\mathbf{f}_a$ on the surface elements dA is given by equation (14) where $\hat{\mathbf{N}}$ is a outward normal of the surface element dA, $\hat{\mathbf{V}}$ a unit vector of the translational velocity, ρ is the air density and C_D is the drag coefficient of the surface. In real terms, the drag coefficient C_D is a function of the surface structure and the local angle of attachment and its value is usually between 1 and 2. For all practical applications, the value C_D=2 can be used. (Wertz 1978)

$$d\mathbf{f}_{aero} = -\frac{1}{2} C_D \rho V_0^2 \left(\hat{\mathbf{N}} \cdot \hat{\mathbf{V}}\right) \hat{\mathbf{V}} dA \qquad (14)$$

In the simulation model, the satellite structure has been approximated by a collection of simple geometrical figures. Hence, the total aerodynamic torque is the sum over the torques acting on individual parts of the spacecraft.

$$\mathbf{T}_{aero} = \sum_i \mathbf{r}_i \times \mathbf{f}_i = \frac{1}{2} C_D \rho V_0^2 \sum_i A_i \left(\hat{\mathbf{N}}_i \cdot \hat{\mathbf{V}}\right) \hat{\mathbf{V}} \times \mathbf{r}_i$$

Any nonsymmetrical body in orbit is subject to a gravitational torque because of the variation in the Earth's gravitational force over the object. Usually in the literature, the gravity-gradient is only derived for the unrealistic spherical Earth model. Due to the non-sphericity and the non-homogenous mass distribution of the Earth the real gravitational field is granular.

For spherical Earth, the gravitational force $d\mathbf{f}_i$ acting on a s/c mass element dm_i located at a position $\mathbf{R}_i$ is

$$d\mathbf{f}_i = \frac{-\mu \mathbf{R}_i dm_i}{R_i^3}.$$

Hence, the torque about the satellite geometric center due to a force $d\mathbf{f}_i$, at position $\mathbf{r}_i$, is

$$d\mathbf{T}_i = \mathbf{r}_i \times d\mathbf{f}_i = \left(\boldsymbol{\rho} + \mathbf{r}'_i\right) \times d\mathbf{f}_i$$

where $\boldsymbol{\rho}$ is the vector from the geometric center to the cm and $\mathbf{r}_i$' from cm to the mass element dm_i. Assuming that the cm and the geometric center of the s/c lie in the same point the gravity-gradient is

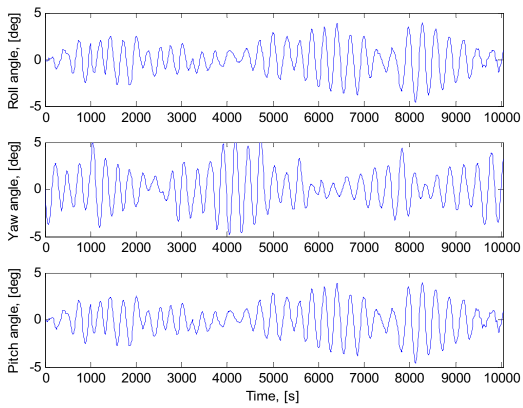
$$\mathbf{T}_{gg} = \frac{3\mu}{R_s^3} \left(\hat{\mathbf{R}}_s \times \mathbf{I}_s \hat{\mathbf{R}}_s\right)$$

where $\hat{\mathbf{R}}_s$ is a unit vector along $\mathbf{R}_s$ and $\mathbf{I}_s$ is the spacecraft inertial matrix. (Wertz 1978)

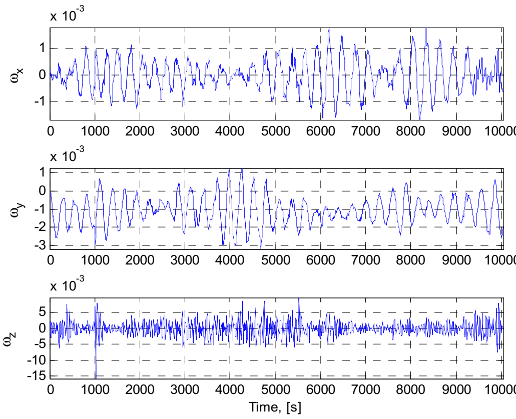
A SIMULATION CASE

In this section, some simulation results obtained by the above-described simulation model are presented. The simulation case is simple and fancied.

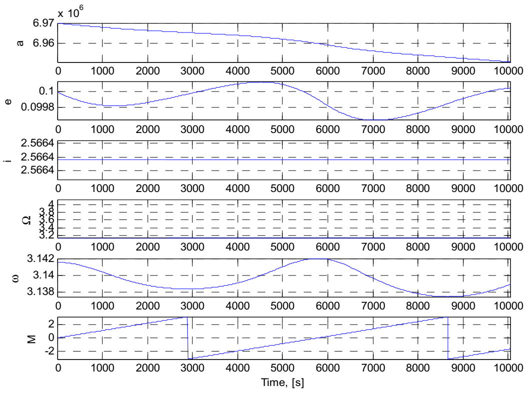
The orbit of the simulation case is circular with an altitude of 450 km and inclination 87°. The moments of inertia of the satellite are I_{xx}=36, I_{yy}=17, I_{zz}=26, and I_{xy}= I_{xy}= I_{xz} = I_{yz} =0 kgm^2. The aim is that the attitude control system shall ensure a three-axis stabilization of the satellite. The satellite attitude is measured by GPS sensor and only three reaction wheels are used in the control. The reaction wheels are mounted orthogonally such that the rotation axes are along **X**, **Y** and **Z**-axis of the satellite body. Three PID-controllers are used to calculate the control torques. The simulation results have been represented in Figures 4-8.



Figures 4. Attitude angles.



Figures 5. The satellite angular rates.

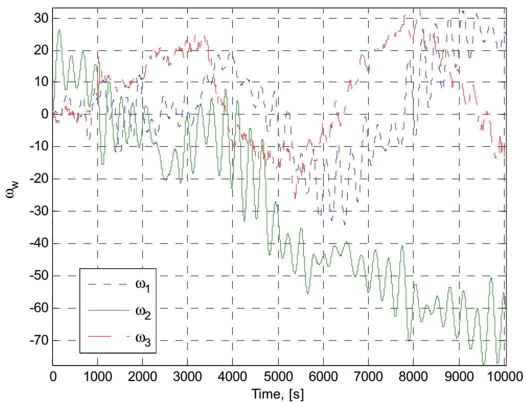


Figures 6. The classical orbital elements in simulation.

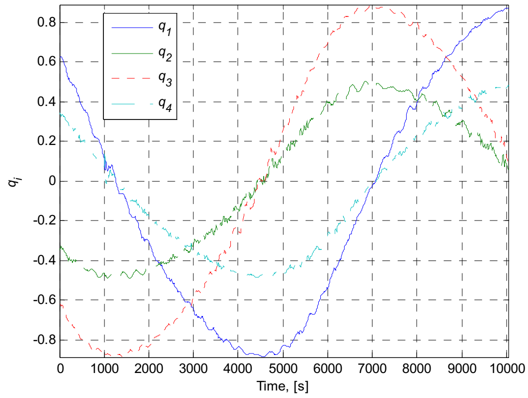
CONCLUSION

The satellite attitude and orbit simulation model with the most common actuators and sensors have been introduced in this paper. The simulation model can be utilized both in the control algorithm designs and in the development of FDIR methods.

The simulation models have been implemented in the MATLAB/SIMULINK environment.



Figures 7. The angular rates of reaction wheels.



Figures 8. The attitude quaternions.

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