DISTRIBUTED PARAMETER MODEL ORIENTED IDENTIFICATION

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ABSTRACT

By means of mathematically defined digital filters, the disturbance affecting the system response may be eliminated and the real response recovered. Thereby a multivariable exponential test signal is assumed, but any other type of input is also pertinent. This achievement involves both a functional and a stochastic component. The former consists in the proved property that a multivariable function given over a finite interval may be approximately expressed by a product of functions of only one variable. Further, each such factor is expandable in a finite sum of exponential terms. The latter component consists in the low probability of coincidence regarding the test signal exponent and any exponent of the disturbance (approximate) spectrum. The developed procedure enables to estimate the constant coefficients included in a distributed parameter model of the process.
Short observation time, test signal / noise small ration, the good accuracy of the model estimation and simple identification algorithm will be achieved. These features are due to the method peculiarity, to operate with any incipient segment of the response, and to the simple structure and good selectivity of the proposed digital filters. One may give up the test signal use, by having recoursed to exponential decomposition of the actual system input.

INTRODUCTION

Let us consider the dynamic process of a distributed parameter system, a process expressed by the partial differential eq.

$$\left(\sum_{k=0}^{N}\sum_{i,j=0}^{k} a_{ijk}\mathbf{D}_{ijk}\right)\varphi(t,x,y) = \left(\sum_{k=0}^{N'}\sum_{i,j=0}^{k} b_{ijk}\mathbf{D}_{ijk}\right)\psi(t,x,y) \quad (1)$$

Above $\mathbf{D}_{ijk}$ denotes the partial derivate operator

$$\mathbf{D}_{ijk} = \frac{\partial^k}{\partial t^i \partial x^j \partial y^{k-i-j}} \quad (2)$$

and a_{ijk}, b_{ijk} are some constant coefficients. This eq. relates the system response $\varphi(t,x,y)$ to the external action $\psi(t,x,y)$. The following considerations may be easily extended to the functions of arguments t,x,y,z.

We aim at producing an alternative to stochastic methods (Unbehauen 1990), (Chen and Wahlberg, Eds. 1997) in order to achieve a shorter observation time of the process.

The developed procedure is based on the exponential decomposition of a continuous function, providing an individual exponential spectrum (Cehan-Racovita 1999, 2002). The resulting sum is a generalization of the *Fourier*'s finite sum. In contrast to current methods, a higher efficiency of disturbance filtering is performed.

EXPONENTIAL DECOMPOSITION

The Approximate

Let $f(t)$ be a process and $f_n(t)$ an approximate of $f(t)$, over a finite interval [0,T], with the structure

$$f_m(t) = \sum_{i=1}^{m} A_i \exp \alpha_i t, \qquad A_i = const \quad (3)$$

where A_i and α_i are complex constants; exp(.) means $e^{(.)}$.

This approximate has to fulfill the conditions

$$f_m(kt_0) = f(kt_0), \qquad t_0 = const \quad (4)$$

for $k = \overline{0,k_M}$, $k_M t_0 \le T$. The approximate (3), (4) may be considered as an extension of a *Fourier*'s finite sum. The frequency spectrum of the last sum does not characterize $f(t)$. On the contrary, the complex spectrum $\{\alpha_i\}_{1\le i\le m}$ expresses the individual character of $f(t)$ over the given interval. Consequently this spectrum may separate components from a combined signal.

Spectrum Determination

We define the shifting operator $\mathbf{q}$ by its effect on any function $g(t)$, i.e.

$$\mathbf{q}g(t) = g(t+t_0), \quad (5)$$

$t_0 \ge 0$. One may prove that polynomials of $\mathbf{q}$ may be handled like algebraic polynomials. Note that

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$$(\mathbf{q}-q_i\mathbf{q}^0)\exp(\alpha_\mu t)=(q_\mu-q_i)\exp(\alpha_\mu t)\,, \quad (6)$$

where

$$q_i=\exp(\alpha_i t_0)\,. \quad (7)$$

Further observe that (6) vanishes if $q_\mu=q_i$.
Consequently, denoting

$$\rho(\xi)=\prod_{i=1}^{m}(\xi-q_i) \quad (8)$$

one may write $\rho(\mathbf{q})f(t)=0$, and further

$$\mathbf{q}^\lambda\rho(\mathbf{q})f(t)=0\,,\ \lambda=\overline{0,m-1}\,. \quad (9)$$

Adopting for $\rho(\xi)$ the alternate expression

$$\rho(\xi)=\sum_{i=0}^{m}\sigma_i\xi^{m-i}\,, \qquad (\sigma_0=1)$$

the eq.s (9) turn at $t=0$ into

$$\sum_{i=1}^{m}\sigma_i f(\theta_i)=0\,,\quad \theta_i=(m-i+\lambda)t_0 \quad (10)$$

$\lambda=\overline{0,m-1}$. Solving the system (10) one obtains all σ_i, $i=\overline{1,m}$, whereby the roots of the polynomial (10) are just the values (7). Note that the imaginary part of α_i is not a unique one. We have to take the smallest value of $|\alpha_i|$, in order to avoid oscillatory variation inside the incremental subintervals. One introduces the exponential spectrum

$$S(f(t))=\{q_1,q_2,...,q_m\};$$

its use avoiding nonunique values α_i. But the most advantageous is the parametric spectrum

$$S_p(f(t))=\{\sigma_1,\sigma_2,...,\sigma_m\}$$

which does not require the solving of any polynomial eq.

Amplitude Determination

We succeed in selectively eliminating any term of (3), except one chosen term. The procedure consists in applying to $f(t)$ the polynomial

$$\rho_\mu(\xi)=\rho(\xi)/(\xi-q_\mu) \quad (11)$$

as the operator $\rho_\mu(\mathbf{q})$. Indeed, due to (6) and (11) we have

$$\rho_\mu(\mathbf{q})f(t)=A_\mu\rho_\mu(q_\mu)\exp(\alpha_\mu t)\,,$$

This result was obtained observing that $\mathbf{q}^n$ applied to $\exp(\alpha_\mu t)$ yields just $q_\mu^n\exp(\alpha_\mu t)$. The coefficients of $\rho_\mu(\xi)$ may be derived from $\rho(\xi)$ using the *Horner*'s algorithm.

Disturbance Filtration by Means of Exponential Decompositon

A linear ordinary differential model with concentrated parameters is considered. The output will be

$$\varphi_d(t)=\varphi_f(t)+\varphi_h(t)+n(t)\,, \quad (12)$$

where $\varphi_d(t)$ is the disturbed output. The terms in the right side member are: φ_f = forced component (induced by the input $\psi(t)$, φ_h = homogeneous term (induced by the initial conditions), n = noise. The input is assumed to be a given exponential function of amplitude A_0. Considering the two last terms of (12) as a global disturbance

$$d(t)=\varphi_h(t)+n(t)\,,$$

we have to consider the eq.

$$\varphi_d(t)=\varphi_f(t)+d(t)\,.$$

No direct information concerning the components of $d(t)$ is available. Such information in a hidden form is included in $\varphi_d(t)$. If the test input is

$$\psi(t)=A_0\exp(\alpha_0 t) \quad (13a)$$

then $\varphi_f(t)$ will be an exponential function, too, subsequently.

$$\varphi_d(t)=A_0'\exp(\alpha_0 t)+d(t)\,. \quad (13b)$$

Denoting

$$q_0=\exp(\alpha_0 t)$$

we apply the operator $\mathbf{q}-q_0\mathbf{q}^0$ to $\varphi_d(t)$. As a result the first term in the right side member of (13b) is suppressed, i.e.

$$\hat{\varphi}_d(t)=(\mathbf{q}-q_0\mathbf{q}^\mathbf{0})d(t)\,, \quad (14)$$

where

$$\hat{\varphi}_d(t) = (\mathbf{q} - q_0\mathbf{q}^{\mathbf{0}})\varphi_d(t).$$

We infer that

$$S_p(\hat{\varphi}_d(t)) = S_p(d(t)),$$

as the operator present in (14) does not alterate the parametric spectrum of $d(t)$. Solving the system (10), the components σ_i of $S_p(d(t))$ will be determined. The essential information concerning $d(t)$ becomes available, thereby the polynomial $\rho(\xi)$ is established. With respect to (12), (13b) and (5) one derives

$$\rho(\mathbf{q})\varphi_d(t) = A_0'\rho(q_0)\exp(\alpha_0 t) \qquad (15)$$

for $t = kt_0$, $k = 0,1,\ldots$. The polynomial operator $\rho(\mathbf{q})$ mathematically defines a digital filter tuned on the spectrum of $d(t)$. The last result offers the value of A_0', morever the transfer function $Y(s)$ at $s = \alpha_0$ is equal to A_0'/A_0. Obiouvsly, the developed procedure performs correctly only if α_0 and any α_i, $i = \overline{1,m}$ are not identical.

Stochastic Behavior of the Exponential Decomposition

Numerical results produced by the decomposition show a marked spreading of exponents, if the decomposed $f(t)$ will be slightly modified. Consequently, the coincidence probability of α_0 and any α_i of $d(t)$ is very low. Therefore one has recoursed to a sequence of A_0' obtained for $t = t_0, 2t_0, \ldots$ in (15), or $m = m_1, m_2, \ldots$ in (3). The A_0' - values occurring with higher frequency should provide an accurate average value.

METHOD EXTENSION TO THE PARTIAL DERIVATIVE MODEL

Multivariable Exponential Decomposition

Let us consider a continuous function $f(t,x,y)$ and also the function

$$f_{mpq}(t,x,y) = [\sum_{i=1}^{m} A_i \exp(\alpha_i t_0)].g(x,y), \qquad (16)$$

where

$$g(x,y) = [\sum_{j=1}^{p} B_j \exp(\beta_j x)].[\sum_{k=1}^{q} C_k \exp(\gamma_k y)] \quad (17)$$

with $B_1 = C_1 = 1$. We require

$$f_{mpq}(P) = f(P), \qquad (18)$$

wherein $P = P_{k_1k_2k_3}$ denotes the point (k_1t_0, k_2x_0, k_3y_0) with k_1, k_2, k_3 some integers and t_0, x_0, y_0 the increments of the variables t, x, y. We shall prove that the approximate f_{mpq} within a subspace $[0,T]\times[0,X]\times[0,Y], T, X, Y > 0$ which should verify (18) really exists.

Similarly to (8) and (11) we consider the polynomials

$$\rho_j(\xi) = \prod_{i=1}^{m_j}(\xi - q_{ji}), \; \rho_{j\mu} = \rho_j(\xi)/(\xi - q_{j\mu}) \qquad (19a,b)$$

with $m_1 = m$, $m_2 = p$, $m_3 = q$ and

$$q_{ji} = \exp(\varepsilon_{ji}), \qquad (20)$$

where $\varepsilon_{1i} = \alpha_i t_0$, $\varepsilon_{2i} = \beta_i x_0$, $\varepsilon_{3i} = \gamma_i y_0$. Instead of (5) we define

$$\mathbf{q}_j f(t,x,y) = f(t+\Delta_1, x+\Delta_2, y+\Delta_3) \qquad (21)$$

wherein $\Delta_1 = t_0$, $\Delta_2 = \Delta_3 = 0$, if $j = 1$; $\Delta_2 = x_0$, $\Delta_1 = \Delta_3 = 0$, if $j = 2$; $\Delta_3 = y_0$, $\Delta_1 = \Delta_2 = 0$, if $j = 3$. Instead of (9) we have

$$\mathbf{q}_j^{\lambda_j}\rho_j(\mathbf{q}_j)f(t,x,y) = 0, \; \lambda_j = \overline{1,m_j}.$$

Further (19a) may be expressed as

$$\rho_j(\xi) = \sum_{i=0}^{m_j}\sigma_{ji}\xi^{m_j - i}, \qquad (22)$$

so that

$$\sum_{i=0}^{m_j}\sigma_{ji}q_j^{m_j - i + \lambda_j}f(t,x,y) = 0 \qquad (23)$$

$(\lambda_j = \overline{1,m_j}; j = 1,2,3)$ holds. Solving the three resulting algebraic systems, one gets three sequences of σ_{ji} forming together the parametric spectrum of $f(t,x,y)$:

$$S_p(f(t,x,y)) = \{\sigma_{j1}, \sigma_{j2}, \ldots, \sigma_{jm_j}\}_{j=1,2,3}.$$

By means of $\rho_{j\mu}(\mathbf{q}_j)$ we are filtering out all components of f_{npq}, except a selected one. Indeed,

$$\rho_{1\mu_1}(\mathbf{q}_1)\rho_{2\mu_2}(\mathbf{q}_2)\rho_{3\mu_3}(\mathbf{q}_3)f(t,x,y) = A_{\mu_1}\rho_{1\mu_1}(q_{1\mu_1})B_{\mu_2}\rho_{2\mu_2}(q_{2\mu_2})C_{\mu_3}\rho_{3\mu_3}(q_{3\mu_3}) \qquad (24)$$

By replacing above $\mu_2 = \mu_3 = 1$ and taking into account $B_1 = C_1 = 1$, one gets the expression for A_{μ_1}. Taking $\mu_1 = \mu_3 = 1$ and $\mu_1 = \mu_2 = 1$, we obtain the expressions for B_{μ_2} and C_{μ_3}, respectively. Thereby we proved that (for a given interval), a well determined $f_{npq}(t,x,y)$ in the form defined by (16), (17) may be found.

Multivariable Filtration

In order to accomplish this task, one goes through two filtrations, the first one eliminating the forced component $\varphi_f(x,y)$ from the disturbed system response, the second one eliminating the disturbance from the output. At the first stage, the parametric spectrum of the disturbance $d(t,x,y)$ is extracted. At the second stage, involving the just determined spectrum, the distrubance is filtered out and the amplitude of the forced component obtained.
In particular, we take

$$\psi(t,x,y) = A_0 \exp(\alpha_0 t + \beta_0 x + \gamma_0 y) \qquad (25)$$

and correspondingly

$$\varphi_d(t,x,y) = A_0' \exp(\alpha_0 t + \beta_0 x + \gamma_0 y) + d(t,x,y) . \qquad (26)$$

$d(t,x,y)$ includes the homogeneous component φ_h, due to the initial and boundary conditions and the noise $n(t,x,y)$. The disturbance $d(t,x,y)$ is approximated by f_{npq} given by (16), (17).

The filtration operator

$$\mathbf{Q} = \prod_{j=1}^{3} (\mathbf{q}_j - q_{j0}\mathbf{q}_j^0),$$

defining a digital filter, enables the elimination of $\varphi_f(t,x,y)$ (the first term in the right side member (26)) from the disturbed system response $\varphi_d(t,x,y)$. Denoting

$$\hat{\varphi}_d(t,x,y) = \mathbf{Q} d(t,x,y),$$

from (26) one gets

$$\hat{\varphi}_d(t,x,y) = \mathbf{Q}\varphi_d(t,x,y) . \qquad (27)$$

The operator $\mathbf{Q}$ is not altering the spectra $S(d(t,x,y))$ and $S_p(d(t,x,y))$. From the last eq. we infer that the spectrum of $\hat{\varphi}_d(t,x,y)$ is identical with that one of $d(t,x,y)$. In particular, the spectrum of $d(t,x,y)$ results from (23) by replacing $f(t,x,y)$ by $\hat{\varphi}_d(t,x,y)$, i.e.

$$\sum_{i=0}^{m_j} \sigma_{ji} \mathbf{q}_j^{m_j - i + \lambda_j} \hat{\varphi}_d(t,x,y) = 0 . \qquad (28)$$

By means of $\sigma_{ji}, j = 1,2,3$ and $i = \overline{1, m_j}$, $(\sigma_{j0} = 1)$, one builds up with respect to (22) the polynomials $\rho_{10}(\xi)$, $\rho_{20}(\xi)$ and $\rho_{30}(\xi)$. Now, by replacing $\mu_1 = \mu_2 = \mu_3 = 0$ and $A_{\mu_1} = A_0'$ in (24) one gets

$$A_0' = \rho_{10}(\mathbf{q}_1)\rho_{20}(\mathbf{q}_2)\rho_{30}(\mathbf{q}_3)\varphi_d(t,x,y) / \rho_{10}(q_{10})\rho_{20}(q_{20})\rho_{30}(q_{30}) exp(\alpha_0 t + \beta_0 x + \gamma_0 y) \qquad (29)$$

In particular, one may take the shifting origin $t = x = y = 0$. The numerator of (29) defines a digital filter tuned on the spectrum of the disturbance $d(t,x,y)$.
The operators $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$ are defined by (21) and the constants q_{10}, q_{20}, q_{30} are given by (20). Note that the numerator in (29) includes sampled data of the measured system output.

NUMERICAL RESULTS

Table I includes a sample of A_0' / A_0 - values obtained for $t = kt_0$, $k = 0,1,2,3,\ldots$.

Table I. Sequence of A_0' / A_0 - Values Obtained by Applying Equation (29)

k	0	1	2	3
A_0' / A_0	2.002	2.000	2.002	1.998
k	4	5	6	7
A_0' / A_0	2.416	2.003	1.197	2.001

The A_0' / A_0 ratio varies nearly 2.000, an estimate representing the accurate value at a given model and $A_0 = 1$. At $k = 4$ the computation yields a quite different estimate. This event reflects the exponent coincidence of $\psi(t,x,y)$ and $d(t,x,y)$ during the first filtration stage, i.e. (27). The ratio of the peak values of ψ and d was about 1:20. At other sequences a very great estimate may occur. Such events mark the exponent coincidence of ψ and d during the second filtration stage, i.e. (29).

POSSIBLE APPLICATIONS OF THE METHOD

The developed method may be applied to a significant variety of industrial equipment and technological processes. Let us mention chemical reactors (reactive vessels), heat exchangers, (induction) furnances. The method is also suitable for other technical components of industrial plants, involving (time depending) multivariable chemical, thermodynamical or electromagnetic phenomena. In particular, the control input casually may depend on the time only. E.g. the electric tension $u(t) = \varphi(t)$ applied to a furnance is producing a multidimensional electromagnetic field $\varphi(t, x, y, z)$. This one induces certain *Foucault* currents leading to a temperature distribution $\psi(t, x, y, z)$ within the melted metal. As a result, the global model is obtained by coupling both models of the two mentioned phenomena.

CONCLUSION

Some simple digital filters, mathematically defined, allow to suppress the disturbance in the system output. The last one consists of both homogeneous system response and the noise. No information on these components is previously given. The low probability of the coincidence between the exponent functionally characterizing the test signal and any exponent of the decomposed disturbance ensures the required accuracy. If, however, a coincidence is occurring, this event is easily detected, and any wrong partial estimate A_0' will be rejected. The short measuring and computing times avoid the drift (i.e. parameter and operation point changes) effects on the estimation. Consequently, real-time identification may be performed.

In order to estimate the coefficients a_{ijk}, b_{ijk} of the model (1), (2) a set of test signals of the form (25) is applied. Each given triplet $\alpha_0, \beta_0, \gamma_0$ generates an algebraic eq. of structure (1), where $\mathbf{D}_{ijk}$ are replaced by $\alpha_0^i \beta_0^j \gamma_0^k$, φ by A_0' and ψ by A_0. A_0 is known and A_0' will be estimated by means of (29). Finally an algebraic system is produced that yields the values of all coefficients a_{ijk}, b_{ijk}.

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