COLOR-TO-GRAYSCALE IMAGE TRANSFORMATION PRESERVING THE GRADIENT STRUCTURE

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ABSTRACT

A new algorithm for transforming a color image into a grayscale one is proposed. The algorithm accurately preserves the contrast range at the all image boundaries. Another application of the algorithm is visualization of vector fields.

INTRODUCTION

The need for proceeding from a more informative multi-component image to a less informative single-component one arises in a number of situations. First, for many high-performance technologies, the size of the data structures processed by the system is critical. The transition from a color to a monochrome image allows reducing the required computer memory size by a factor of 3. Second, some relevant image processing systems were initially designed to process only monochrome images. The conservative way of the evolution of such systems implies organizing an input filter that transforms the raw data to the form being standard for the system used. Third, effective expertise often requires visualizing a multi-component vector field, which is not so easy to achieve using conventional methods.

Obviously, there is no general solution of the problem formulated. The solution substantially depends on the specific information that must be preserved when reducing a multi-component image. For example, it is often important to preserve the edges between different objects, that is, to keep the objects distinguishable. We shall demonstrate below that the conventional method of image grayscaling does not preserve the object edges and shall propose an original algorithm that is free from this drawback.

PROBLEM STATEMENT

The problem of color-to-grayscale image conversion is often solved by color coordinates transformation. For example, it is possible to proceed to so-called «brightness»:

$$I(x_1, x_2) = \frac{\sum_{i=1}^{n} c_i(x_1, x_2)}{n}, \quad (1)$$

where $c_i(x_1, x_2)$ are the color components of the pixel with the coordinates (x_1, x_2) and n is the number of these components (three, in our case). However, such a monochrome image will fundamentally have small or even zero differences for some pairs of colors, which are quite dissimilar in the original color space (e.g. bright-green and ruby). The specific color combinations to form such an «indistinguishable pair» depend on the structure of the color-to-grayscale transformation chosen. For instance, a brightness-based monochrome image does not represent any variations of chromaticity, no matter how they are large.

Let us now formally define the problem of preserving all the contrast details of the color image being grayscaled. Consider an edge detection operator that yields maximally close results when processing the original color image and its grayscale “descendant”.

The natural edge detector for monochrome images is the gradient magnitude operator. Such a figure characterizes the rate of change of a scalar function along the direction of the steepest change. However, the classical gradient operator can be applied only to a scalar field, while a color image is a vector field.

The above drawback is eliminated in the vector analog of gradient proposed by Di Zenzo (Di Zenzo 1986). This is a quadratic form, Σ, defined on the image to characterize the rate of change of the vector function along the direction of the steepest change:

$$\Sigma_{jk}(x_1, x_2) = \sum_{i=1}^{n} \frac{\partial c_i(x_1, x_2)}{\partial x_j} \cdot \frac{\partial c_i(x_1, x_2)}{\partial x_k}, \quad (2)$$

where j and k may be either 1 or 2.

The directions of the maximum and the minimum variation of the considered function are the mutually orthogonal eigenvectors of Σ. Therefore, these two extremum variations can be calculated from the following expression

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$$\lambda_{\pm} = \frac{\Sigma_{11} + \Sigma_{22} \pm \sqrt{(\Sigma_{11} - \Sigma_{22})^2 + 4 \cdot \Sigma_{12}^2}}{2}, \quad (3)$$

where λ_+ / λ_- are the squared variation rates of the function along the direction of the maximum/minimum variation. The direction of the «color gradient» is defined by the following expression:

$$\varphi = \frac{1}{2} \cdot arctg\left(\frac{2 \cdot \Sigma_{12}}{\Sigma_{11} - \Sigma_{22}} \right). \quad (4)$$

Note that for a monochrome image ($n = 1$): $\lambda_+ = |\nabla c|^2$, $\lambda_- = 0$, and the «color gradient» coincides with the «classical» one.

Let us choose the signs of the components of the color gradient $\vec{g}(x, y)$ so that to match the vector of color variation along the gradient with the increase in brightness. In the case when $\partial \vec{c} / \partial \vec{g} \cdot (1,1,1) = 0$, we shall match $\vec{g}(x, y)$ with the change in the α-component of chromaticity (the unit vector $(1,-1,0)$). Should even that be impossible, the β-component (the unit vector $(-1,-1,2)$) can be used.

The general problem can now be reformulated as a variational one. Let us look for the function $f(x, y)$ that minimizes the functional

$$I = \iint_{\Omega} F(x, y, f, \partial f / \partial x, \partial f / \partial y) \cdot dx \cdot dy, \quad (5)$$

where Ω is the area occupied by the image, and F is a quality functional that characterizes the difference between $\nabla f(x, y)$ and the color gradient, $\vec{g}(x, y)$, calculated on the original image.

CHOICE OF THE QUALITY FUNCTIONAL

A simple quality functional meeting the above criteria is the quadratic functional

$$F_{sqr} = \left(\frac{\partial f(x, y)}{\partial x} - g_x(x, y) \right)^2 + \left(\frac{\partial f(x, y)}{\partial y} - g_y(x, y) \right)^2. \quad (6)$$

Using the necessary condition of extremum:

$$\frac{\partial}{\partial x} \frac{\partial F}{\partial (\partial f / \partial x)} + \frac{\partial}{\partial y} \frac{\partial F}{\partial (\partial f / \partial y)} = 0, \quad (7)$$

we obtain Poisson's equation:

$$\Delta f(x, y) = \partial g_x(x, y) / \partial x + \partial g_y(x, y) / \partial y. \quad (8)$$

This equation can be easily solved in a rectangular domain, for instance, by the Fourier transform method that requires $O(l^2 \log l)$ operations, where l is linear size of the image.

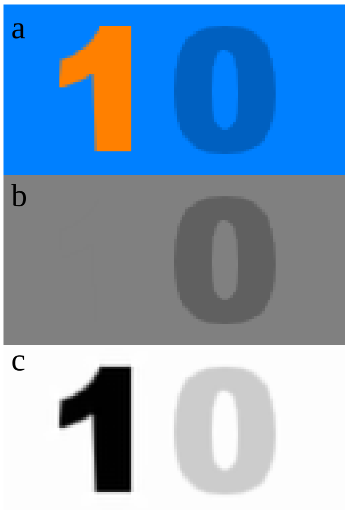

Figure 1: Color Reduction Methods Comparision

An example of color reduction by means of the proposed technique is shown in Fig. 1c. The original image (Fig. 1a) represents two digits: orange "1" (with RGB space coordinates (255, 128, 0)) and dark-blue "0" (0, 96, 192) located on the sky-blue background (0, 128, 255). Brightness of the original image is shown in Fig 1b. One can see that digit "1" disappeared on the brightness image in spite of the difference of color coordinates. On the contrary, proposed method retains boundaries contrast proportionally to the original color differences.

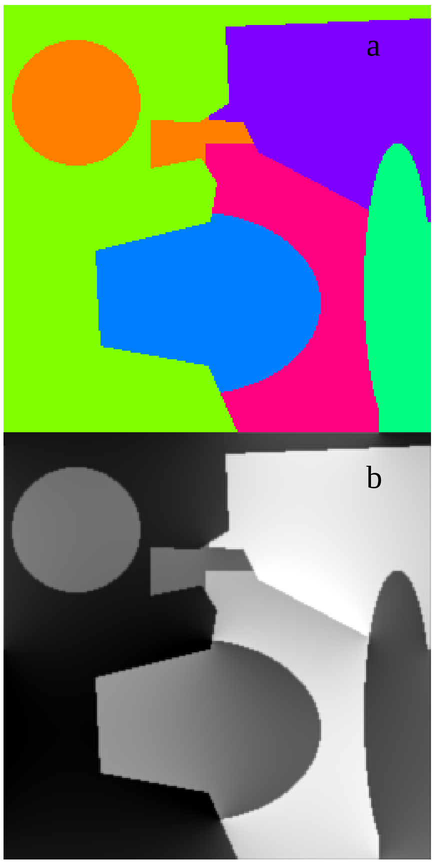

Figure 2: Artifacts of Color Reduction

However, some artifacts may arise at three-color junctions. Fig. 2b shows result of color reduction of a constant-brightness Mondrian-like image (Fig. 2a) by using proposed method. As can be seen, the brightness of resulting image is not a step function. Smooth variations of brightness appear because of impossibility to equalize the contrast along all the edges simultaneously.

To localize such artifacts, we suggest using a quality functional with a limited penalty. This can be, for example, the Gaussian functional

$$F_{gauss} = \exp\left(-F_{sqr} / 2 \cdot \sigma^2\right). \qquad (9)$$

Substituting (9) into (7), we obtain the following nonlinear equation:

$$\left(1-\frac{\alpha^2}{\sigma^2}\right)\frac{\partial\alpha}{\partial x}-\left(\frac{\partial\alpha}{\partial y}+\frac{\partial\beta}{\partial x}\right)\frac{\alpha\beta}{\sigma^2}+\left(1-\frac{\beta^2}{\sigma^2}\right)\frac{\partial\beta}{\partial y}=0, \qquad (10)$$

where $\alpha = \frac{\partial f(x,y)}{\partial x} - g_x(x,y)$, $\beta = \frac{\partial f(x,y)}{\partial y} - g_y(x,y)$.

It's worth noting that this equation is related to the equation governing the behavior of a non-viscous compressible fluid (Lavrentyev and Shabat 1958). For the equations of this class, a number of numerical methods have been developed. Some of these are iterative. In such a case, the solution of the corresponding linear equation (8) is used as the initial condition.

CONCLUSION

This paper describes the method for transforming a multi-component image into a single-component one. It allows to visualize boundary structure of the image. Comparative analysis for the case of RGB images indicates that proposed method preserves every boundary unlike the conventional method of image grayscaling. Our method consists in solving of the Poisson's equation with right-hand member calculated using Di Zenzo pseudo-gradient operator. This can be done, for example, using Fourier transform technique. Obviously, the proposed algorithm can be used to visualize the domain structure of all the functions of the class $F:\mathfrak{R}^2 \to \mathfrak{R}^n$, n being arbitrary (in the case of RGB color image $n=3$).

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